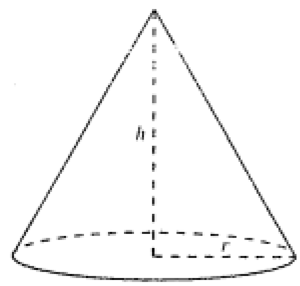
**17. Volume** At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high? (*Hint:* The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{9}{4}h^2\right)h \qquad [\text{because } 2r = 3h]$$

$$= \frac{3\pi}{4}h^3$$




$$\frac{dV}{dt} = 10$$

$$\frac{dV}{dt} = \frac{9\pi}{4}h^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4(dV/dt)}{9\pi h^2}$$

When $h = 15$,

$$\frac{dh}{dt} = \frac{4(10)}{9\pi(15)^2} = \frac{8}{405\pi} \text{ ft/min.}$$

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**22. Construction** A construction worker pulls a five-meter plank up the side of a building under construction by means of a rope tied to one end of the plank (see figure). Assume the opposite end of the plank follows a path perpendicular to the wall of the building and the worker pulls the rope at a rate of 0.15 meter per second. How fast is the end of the plank sliding along the ground when it is 2.5 meters from the wall of the building?

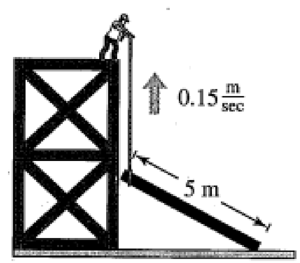


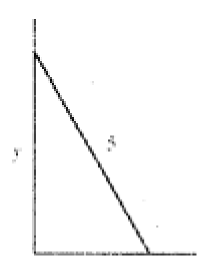

Figure for 22

**22.** $x^2 + y^2 = 25$

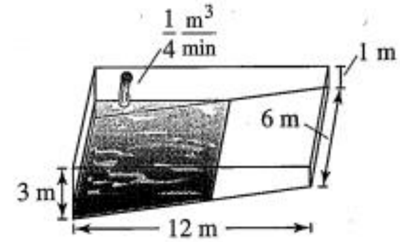
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y}{x}\cdot\frac{dy}{dt} = -\frac{0.15y}{x} \qquad \left(\text{because } \frac{dy}{dt} = 0.15\right)$$

When $x = 2.5$, $y = \sqrt{18.75}$, $\frac{dx}{dt} = -\frac{\sqrt{18.75}}{2.5}0.15 \approx -0.26$ m/sec.



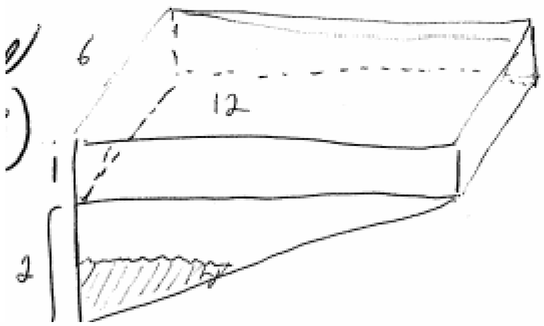
**19. Depth** A swimming pool is 12 meters long, 6 meters wide, 1 meter deep at the shallow end, and 3 meters deep at the deep end (see figure). Water is being pumped into the pool at $\frac{1}{4}$ cubic meter per minute, and there is 1 meter of water at the deep end.




(a) What percent of the pool is filled?

(b) At what rate is the water level rising?

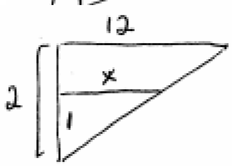
a)




Rectangular prism $1 \cdot 6 \cdot 12 = 72$

Triangular prism $= \frac{1}{2} \cdot 2 \cdot 12 \cdot 6 = 72$

Total $= 144 \, m^3$




$\frac{1}{2} = \frac{x}{12}$

$x = 6$

Volume of water $= \frac{1}{2} \cdot 1 \cdot 6 \cdot 6 = 18 \, m^3$

% Filled $= \frac{18}{144} = \frac{1}{8}$ or $12.5\%$

b) $\frac{dV}{dt} = \frac{1}{4} \, m^3/min$

$h = 1$

$V = \frac{1}{2} h \cdot l \cdot w$

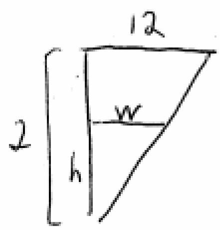
$V = \frac{1}{2} \cdot h \cdot 6 \cdot w$

$V = 3hw$

$V = 3h(6h)$

$V = 18h^2$

$\frac{dV}{dt} = 36h\left(\frac{dh}{dt}\right)$




$\frac{w}{12} = \frac{h}{2}$

$2w = 12h$

$w = 6h$

$\frac{1}{4} = 36(1)\left(\frac{dh}{dt}\right)$

$\frac{1}{36} \cdot \frac{1}{4} = \frac{dh}{dt}$

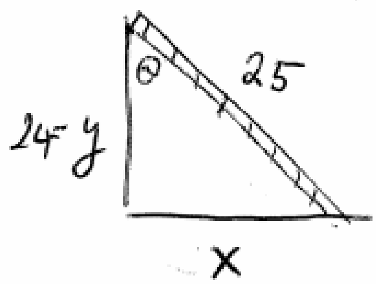
$\boxed{\frac{dh}{dt} = \frac{1}{144} \, m/min}$

**21. Moving Ladder** A ladder 25 feet long is leaning against the wall of a house (see figure). The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

(c) Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall.

21 c) Find rate of change of angle b/t wall and ladder

$$\frac{d\theta}{dt} = \_\_\_\_$$




$x = 7$

$y = 24$

$\frac{dx}{dt} = 2$

$\frac{dy}{dt} = -7/12$

$$\tan\theta = \frac{x}{y}$$

$$\frac{\sec^2\theta}{\sec^2\theta}\left(\frac{d\theta}{dt}\right) = \frac{\left(\frac{dx}{dt}\right)y - x\left(\frac{dy}{dt}\right)}{y^2} \cdot \frac{1}{\sec^2\theta}$$

$$\frac{d\theta}{dt} = \frac{(\cos\theta)^2\left[\frac{dx}{dt}y - x\left(\frac{dy}{dt}\right)\right]}{y^2} \qquad \cos\theta = \frac{24}{25}$$

$$\frac{d\theta}{dt} = \frac{\left(\frac{24}{25}\right)^2\left[2(24) - 7\left(-7/12\right)\right]}{24^2} = \boxed{\frac{1}{12}\ \text{rad/sec}}$$