BC selected HW Problems Ch 2.6 Related Rates Day 3 Problems # 14, 20,24,28,29,31

14. Volume A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is (a) 30 centimeters and (b) 60 centimeters?

14. $V = \frac{4}{3}\pi r^3, \frac{dV}{dt} = 800$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\left(\frac{dV}{dt}\right) = \frac{1}{4\pi r^2}(800)$$

(a) When $r = 30$,

$$\frac{dr}{dt} = \frac{1}{4\pi(30)^2}(800) = \frac{2}{9\pi} \text{ cm/min.}$$

(b) When $r = 60$,

$$\frac{dr}{dt} = \frac{1}{4\pi(60)^2}(800) = \frac{1}{18\pi} \text{ cm/min.}$$

28. Sports For the baseball diamond in Exercise 27, suppose the player is running from first base to second base at a speed of 25 feet per second. Find the rate at which the distance from home plate is changing when the player is 20 feet from second base.

28. $s^2 = 90^2 + x^2$

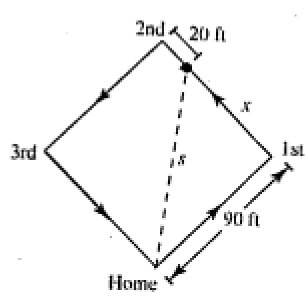
$$x = 90 - 20 = 70$$

$$\frac{dx}{dt} = 25$$

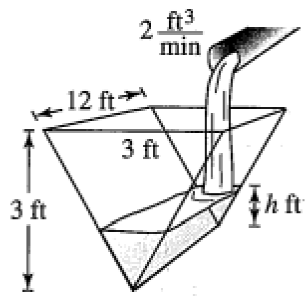
$$\frac{ds}{dt} = \frac{x}{s} \cdot \frac{dx}{dt}$$

When $x = 70$, $s = \sqrt{90^2 + 70^2} = 10\sqrt{130}$,

$$\frac{ds}{dt} = \frac{70}{10\sqrt{130}}(25) = \frac{175}{\sqrt{130}} \approx 15.35 \text{ ft/sec.}$$



20. Depth A trough is 12 feet long and 3 feet across the top (see figure). Its ends are isosceles triangles with altitudes of 3 feet.



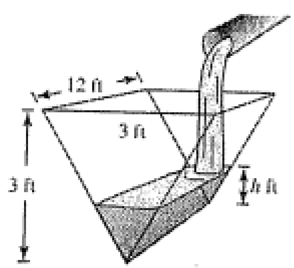
(a) Water is being pumped into the trough at 2 cubic feet per minute. How fast is the water level rising when the depth h is 1 foot?

(b) The water is rising at a rate of $\frac{3}{8}$ inch per minute when $h = 2$. Determine the rate at which water is being pumped into the trough.

20. $V = \frac{1}{2}bh(12) = 6bh = 6h^2 \quad (\text{since } b = h)$

(a) $\frac{dV}{dt} = 12h\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{12h}\frac{dV}{dt}$

When $h = 1$ and $\frac{dV}{dt} = 2$, $\frac{dh}{dt} = \frac{1}{12(1)}(2) = \frac{1}{6}$ ft/min.



(b) If $\frac{dh}{dt} = \frac{3}{8}$ in./min $= \frac{1}{32}$ ft/min and $h = 2$ ft, then $\frac{dV}{dt} = (12)(2)\left(\frac{1}{32}\right) = \frac{3}{4}$ ft³/min.

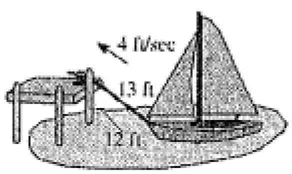
24. Boating A boat is pulled into a dock by means of a winch 12 feet above the deck of the boat (see figure).

(a) The winch pulls in rope at a rate of 4 feet per second. Determine the speed of the boat when there is 13 feet of rope out. What happens to the speed of the boat as it gets closer to the dock?

(b) Suppose the boat is moving at a constant rate of 4 feet per second. Determine the speed at which the winch pulls in rope when there is a total of 13 feet of rope out. What happens to the speed at which the winch pulls in rope as the boat gets closer to the dock?

24. Let L be the length of the rope.

(a)
$$L^2 = 144 + x^2$$
$$2L\frac{dL}{dt} = 2x\frac{dx}{dt}$$
$$\frac{dx}{dt} = \frac{L}{x}\cdot\frac{dL}{dt} = -\frac{4L}{x} \qquad \left(\text{since } \frac{dL}{dt} = -4 \text{ ft/sec}\right)$$



When $L = 13$:
$$x = \sqrt{L^2 - 144} = \sqrt{169 - 144} = 5$$
$$\frac{dx}{dt} = -\frac{4(13)}{5} = -\frac{52}{5} = -10.4 \text{ ft/sec}$$

Speed of the boat increases as it approaches the dock.

31. Machine Design The endpoints of a movable rod of length 1 meter have coordinates $(x, 0)$ and $(0, y)$ (see figure). The position of the end on the x-axis is

$$x(t) = \frac{1}{2}\sin\frac{\pi t}{6}$$

where t is the time in seconds.

(a) Find the time of one complete cycle of the rod.

(b) What is the lowest point reached by the end of the rod on the y-axis?

(c) Find the speed of the y-axis endpoint when the x-axis endpoint is $\left(\frac{1}{4}, 0\right)$.

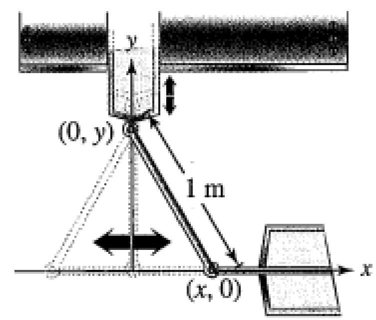
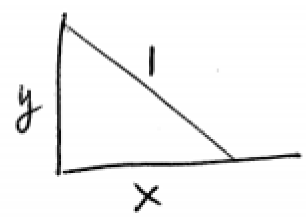


Figure for 31

31) $x(t) = \frac{1}{2}\sin\left(\frac{\pi t}{6}\right)$, $x^2 + y^2 = 1$



a) Period $= \frac{2\pi}{B} = \frac{2\pi}{\pi/6} = 12$ seconds

b) Greatest value for $x(t) = 1/2$, since largest value for $\sin\left(\frac{\pi t}{6}\right) = 1$, $x(t) = \frac{1}{2}(1) = \frac{1}{2}$

when $x = \frac{1}{2}$, $\left(\frac{1}{2}\right)^2 + y^2 = 1$ $\quad y^2 = 1 - \frac{1}{4}$, $y = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

Lowest point: $\left(0, \frac{\sqrt{3}}{2}\right)$ $\quad y = \frac{\sqrt{3}}{2}$, ~~$-\frac{\sqrt{3}}{2}$~~