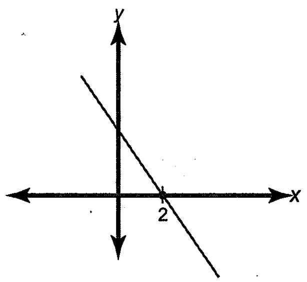
BC PRACTICE TEST 1
Section I, Part A: Multiple-Choice Questions
Time: 55 minutes
Number of Questions: 28

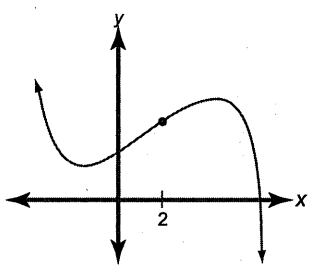
A calculator may not be used on this part of the examination.

1. If $f(x) = 2x\cos x$, then $f'(x) =$
   (A) $-2\sin x$
   (B) $2x\sin x + 2\cos x$
   (C) $2x\sin x - 2\cos x$
   (D) $-2x\sin x$
   (E) $-2x\sin x + 2\cos x$

2. If $f(x)$ is a function such that $f'(x)$ is increasing for $x < 2$ and $f'(x)$ is decreasing for $x > 2$, then which of the following could be the graph of $f(x)$?
   (A)



   (B)



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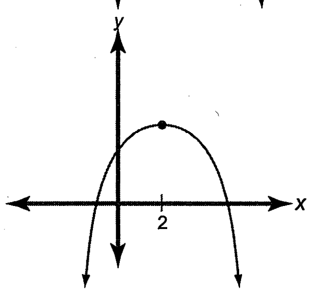

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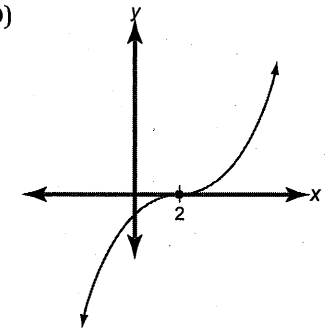
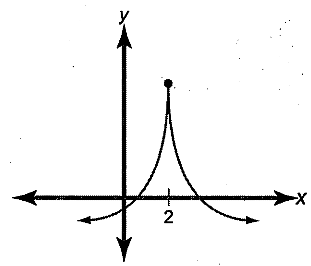


   (E)



3. Find the limit $\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}\cdot\frac{1}{n} =$.
   (A) 0
   (B) $\frac{1}{2}$
   (C) $\frac{2}{3}$
   (D) 1
   (E) ∞

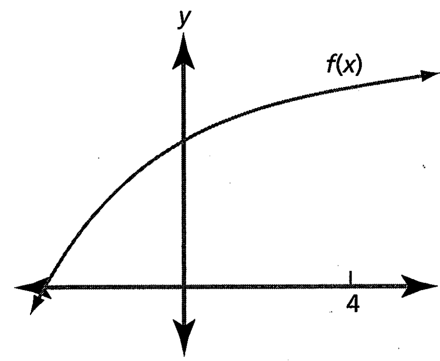
4. Consider the differential equation $\frac{dy}{dx} = y - 2x + 3$, where $y = f(x)$ is the solution to the equation and $f(2) = 5$. Using Euler's method starting at $x_0 = 2$ with step size $\Delta x = 0.5$, what is the approximation for $f(3)$?
   (A) 7
   (B) 8.5
   (C) 9
   (D) 9.5
   (E) 11

5. The equation of the tangent line to the function $y = 8\sqrt{3x+1}$ at $x = 5$ is
(A) $y = 3x + 27$
(B) $y = x + 27$
(C) $y = 3x + 17$
(D) $y = 6x + 12$
(E) $y = 6x + 2$

6. The position vector for a particle moving in the xy-plane for $t \geq 0$ is $\left(10\ln(1+t),\ 16\sqrt{t}\right)$. The slope of the tangent line to the path of the particle at $t = 4$ is
(A) $\frac{16}{5\ln 5}$
(B) $\frac{1}{2}$
(C) $2\sqrt{5}$
(D) $\frac{8}{5}$
(E) 2

7. Evaluate $\int_0^1 \frac{3}{x}\,dx$.
(A) 0
(B) 1
(C) $3e$
(D) e^3
(E) ∞

8. The graph of $f(x)$ is pictured below. Which of the following statements about $\int_0^4 f(x)\,dx$ are true?



I. A left endpoint approximation is greater than $\int_0^4 f(x)\,dx$.
II. A right endpoint approximation is less than $\int_0^4 f(x)\,dx$.
III. A trapezoidal approximation is less than $\int_0^4 f(x)\,dx$.

(A) None are true
(B) I and II only
(C) III only
(D) I and III only
(E) I, II, and III

9. The general solution to the differential equation $\frac{dy}{dx} = y\left(1 + \frac{1}{x^2}\right)$ is $y =$
(A) $Ce^{\tan^{-1} x}$
(B) $Ce^{x - \frac{1}{x}}$
(C) $e^{x + \frac{1}{x}} + C$
(D) $\sqrt{2x - \frac{2}{x} + C}$
(E) $e^{x - \frac{1}{x}}$

10. What is the slope of the curve $2xy^2 = 3x^2 - y^3$ at the point (1, 1)?
(A) -3
(B) $\frac{1}{7}$
(C) $\frac{4}{7}$
(D) $\frac{6}{7}$
(E) $\frac{6}{5}$

11. If $f'(x) = 12x^2 \sin(2x^3 - 16)$ and $f(2) = 5$, then $f(x) =$
(A) $-2\cos(2x^3 - 16) + 7$
(B) $-4x^3 \cos(2x^3 - 16) + 5$
(C) $2\cos(2x^3 - 16) + 3$
(D) $-2\cos(2x^3 - 16) + 5$
(E) $24x\cos(2x^3 - 16) + 5$

12. The first four terms of the Taylor expansion for $f(x)$ about $x = 3$ are $5 - \frac{x-3}{4} - \frac{7(x-3)^2}{3} + \frac{9(x-3)^3}{2}$. What is the value of $f''(3)$?
(A) $-\frac{14}{3}$
(B) $-\frac{7}{3}$
(C) $-\frac{7}{6}$
(D) $-\frac{1}{2}$
(E) $-\frac{1}{4}$

13. The graph of $f(x) = x^6 - 5x^4$ has inflection points at $x =$
(A) $-\sqrt{2}$ and $\sqrt{2}$ only.
(B) 0 and $\sqrt{2}$ only.
(C) 0 and $\sqrt{\frac{10}{3}}$ only.
(D) $-\sqrt{\frac{10}{3}}$, 0, and $\sqrt{\frac{10}{3}}$.
(E) $-\sqrt{2}$, 0, and $\sqrt{2}$.

14. $\lim_{x \to 0} \frac{\cos x - e^x}{\ln(1+x)} =$
(A) -1
(B) 0
(C) 1
(D) e
(E) ∞

15. Which of the following series converge?

I. $\sum_{k=0}^{\infty} \frac{3^{k+1}}{4^k}$

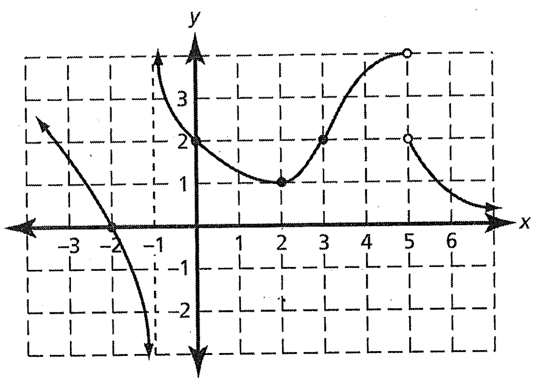
II. $\sum_{k=0}^{\infty} (-1)^k \frac{k^2}{(2k+1)^2}$

III. $\sum_{k=1}^{\infty} \frac{|\sec k|}{k}$

(A) I only
(B) I and II
(C) I and III
(D) II and III
(E) I, II, and III

16. If $\frac{dy}{dt} = k(y-2)$, then $y =$
(A) Ce^{t-2}
(B) $e^{kt} + C$
(C) $\frac{k}{2}(t-2)^2 + C$
(D) $Ce^{kt} + 2$
(E) $\ln|kt + C| + 2$

Questions 17 and 18 refer to the following information:



Let $F(x) = \int_0^{2x-1} f(t)\,dt$, where $f(t)$ is pictured above.

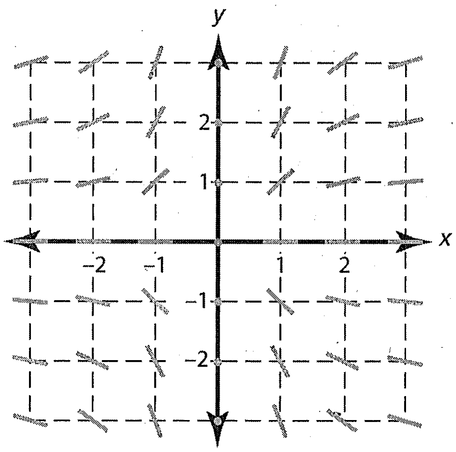
17. What is the domain of $F(x)$?
    (A) All real numbers except -1 and 5
    (B) $-1 < x < 5$
    (C) $0 < x < 3$
    (D) $x < -1$
    (E) $x > 5$

18. What is the value of $F'(2)$?
    (A) 0
    (B) 1
    (C) 2
    (D) 4
    (E) undefined

19. The slope of the normal line to $f(x) = 3\sin^{-1} x$ at $x = 0$ is
    (A) $-\frac{1}{3}$
    (B) 0
    (C) $\frac{1}{3}$
    (D) 3
    (E) undefined

20. A particle moves in the xy-plane according to the parametric equations $x = \tan t$ and $y = e^{\frac{1}{2}t}$. An expression for the length of the path of the particle from $t = 0$ to $t = 1$ is
    (A) $\int_0^1 \left[\sec^2 t + \frac{1}{2}e^{\frac{1}{2}t}\right] dt$
    (B) $\int_0^1 \sqrt{\tan^2 t + e^t}\, dt$
    (C) $\int_0^1 \sqrt{\sec^4 t + \frac{1}{4}e^t}\, dt$
    (D) $\int_0^1 \left[\sec^4 t + \frac{1}{4}e^t\right] dt$
    (E) $\int_0^1 \sqrt{\sec^2 t + \frac{1}{2}e^{\frac{1}{2}t}}\, dt$

21. Which expression below represents the first four terms of the Maclaurin approximation to the area under the curve $f(x) = e^{x^2}$ from $x = 0$ to $x = 1$?
    (A) $1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42}$
    (B) $1 + 1 + \frac{1}{4} + \frac{1}{36}$
    (C) $1 + 1 + \frac{1}{2} + \frac{1}{6}$
    (D) $1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$
    (E) $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10}$



22. The slope field above represents an approximation to the general solution to which differential equation?

(A) $\frac{dy}{dx}=\frac{y}{x}$

(B) $\frac{dy}{dx}=\frac{x}{y^2}$

(C) $\frac{dy}{dx}=\frac{y}{x^2}$

(D) $\frac{dy}{dx}=\frac{y^3}{x}$

(E) $\frac{dy}{dx}=\frac{y^2}{x^2}$

23. Let $f(x)=x\sin(x)$. The first four nonzero terms of the Taylor approximation for $f'(x)$ about $x=0$ are

(A) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}$

(B) $2x-\frac{4x^3}{3!}+\frac{6x^5}{5!}-\frac{8x^7}{7!}$

(C) $x^2+\frac{x^4}{3!}+\frac{x^6}{5!}+\frac{x^8}{7!}$

(D) $1+2x+\frac{3x^2}{2!}+\frac{4x^3}{3!}$

(E) $x^2-\frac{x^4}{3!}+\frac{x^6}{5!}-\frac{x^8}{7!}$

24. The area enclosed by the polar curve $r\cos\frac{1}{2}\theta=1$ in the interval $0\le\theta\le\frac{\pi}{2}$ is

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{\pi}{4}$

(D) 1

(E) 2

25. The volume of the solid generated by revolving the region enclosed between the graph of $y=1+x^2$ and the lines $y=1$ and $x=2$ about the x-axis is given by which integral expression?

(A) $\pi\int_0^2 x^4\,dx$

(B) $\pi\int_0^2 \left(1+x^2\right)^2 dx$

(C) $\pi\int_1^5 \left(1-\sqrt{y-1}\right)^2 dy$

(D) $\pi\int_0^2 \left[\left(1+x^2\right)^2-1^2\right] dx$

(E) $2\pi\int_0^2 x^3\,dx$

26. $\int\frac{2x-3}{x^2+9x+18}\,dx=$

(A) $\ln\left|(x+9)^3(x+2)\right|+C$

(B) $\ln\left|\frac{(x+6)^5}{(x+3)^3}\right|+C$

(C) $3\ln|x+9|-\ln|x+2|+C$

(D) $\ln\left|x^2+9x+18\right|+C$

(E) $5\ln|x+6|+3\ln|x+3|+C$

27. The acceleration vector of a particle moving in the xy-plane is $(-\pi \sin \pi t,\ 2t+1)$, for $t \geq 0$. If the velocity vector at $t = 0$ is $(1, 0)$, then how fast is the particle moving when $t = 2$?
(A) 5
(B) 6
(C) $\sqrt{37}$
(D) $\sqrt{40}$
(E) $\sqrt{\pi^4 + 4}$

28. What are all the values of a for which the series $\sum_{k=1}^{\infty} \frac{k^2}{k^{2a-3}+4}$ converges?
(A) $a > 2$
(B) $a \geq 3$
(C) $a < 3$
(D) $a > 1$
(E) $a > 3$

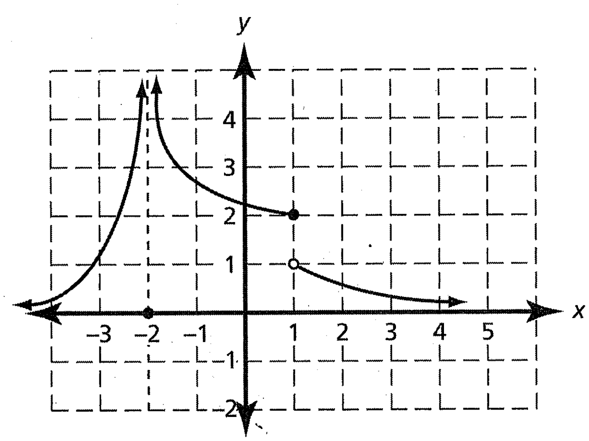
Section I, Part B: Multiple-Choice Questions
Time: 50 minutes
Number of Questions: 17

A calculator may be used on this part of the examination.

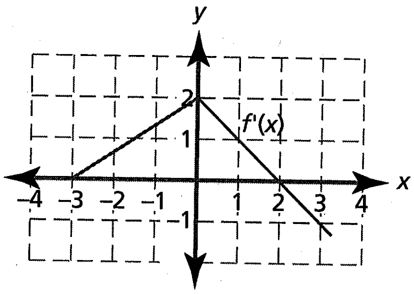
29. Let $f(x)$ be a continuous function defined on the interval $4 \leq x \leq 10$. A table of selected values of $f(x)$ is given below.

x	$f(x)$
4	24
6	37
8	47
10	58

What is the estimate of $\int_4^{10} f(x)\,dx$ produced by a trapezoidal approximation with $n = 3$?
(A) 216
(B) 250
(C) 262
(D) 270
(E) 284



30. The graph of a function $f(x)$ is shown above. Which of the following statements is true?
(A) $\lim_{x \to -2} f(x)$ exists.
(B) $f(1)$ does not exist.
(C) $f'(1)$ exists.
(D) $\lim_{x \to 1^+} f(x)$ exists.
(E) $\lim_{x \to \infty} f(x)$ does not exist.



31. The graph of $f'(x)$, consisting of a pair of line segments, is pictured above. If $f(-3)=0$, then $f(3)=$
(A) -1
(B) 3
(C) 4
(D) 4.5
(E) 5.5

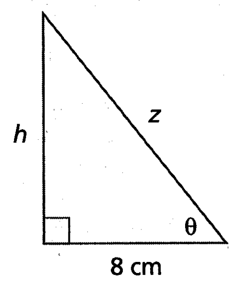
32. A particle moves in the xy-plane along the path of the curve $y = x\sin x$ for time $t \ge 0$. When the particle is at the point $(3,\ 3\sin 3)$, $\frac{dy}{dt} = -2$. What is the value of $\frac{dx}{dt}$ at the same point?
(A) -2.829
(B) 0.423
(C) 0.707
(D) 2.020
(E) 5.658

33. Let $f(x)$ be a function defined for $1.6 \le x \le 11.6$ such that $f'(x) = \ln x \sin x$. How many inflection points does the graph of $f(x)$ have on this interval?
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

34. Which of the following functions has the smallest average value on the given interval?
(A) $f(x) = \cos x$ on $0 \le x \le \frac{3\pi}{4}$
(B) $f(x) = \cos 2x$ on $0 \le x \le \pi$
(C) $f(x) = \cos x$ on $0 \le x \le \frac{\pi}{2}$
(D) $f(x) = \sin x$ on $0 \le x \le 2\pi$
(E) $f(x) = \cos 2x$ on $0 \le x \le \frac{3\pi}{4}$

35. A particle moves along a line for time $t \ge 0$ such that its velocity is $v(t) = 10e^{-t}\cos t$. What is the velocity of the particle when its acceleration is zero for the first time?
(A) -2.709
(B) -0.670
(C) 2.356
(D) 3.185
(E) 10.000

36. Which of the following series are conditionally convergent?
I. $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k^3+1}$
II. $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{k^4+1}$
III. $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^3}{k^3+1}$
(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III



37. The base of the right triangle pictured above is 8 centimeters and the angle θ is increasing at the constant rate of 0.03 radians per second. How fast, in centimeters per second, is the altitude h of the triangle increasing when $h = 13$?
(A) 0.458 cm/sec
(B) 0.744 cm/sec
(C) 0.874 cm/sec
(D) 12.626 cm/sec
(E) 29.125 cm/sec

38. Let $f'(x) = e^x + x$ and let $H(x)$ be the equation of the tangent line to $f(x)$ at $x = a$. If $H(x)$ is used to produce an estimate for $f(a + 0.1)$, then which of the following statements is true?
(A) $H(a + 0.1) > f(a + 0.1)$ for all values of a.
(B) $H(a + 0.1) < f(a + 0.1)$ for all values of a.
(C) $H(a + 0.1) > f(a + 0.1)$ for some values of a and $H(a + 0.1) < f(a + 0.1)$ for other values of a.
(D) $H(a + 0.1) = f(a + 0.1)$ for at least one value of a.
(E) No conclusion can be drawn about the relative values of $H(a + 0.1)$ and $f(a + 0.1)$.

39. What are all values of x for which the series $\sum_{k=0}^{\infty} \frac{(2x)^k}{k+1}$ converges?
(A) $x = 0$
(B) $-\frac{1}{2} \le x \le \frac{1}{2}$
(C) $-2 < x < 2$
(D) $-\frac{1}{2} \le x < \frac{1}{2}$
(E) x can be any real number.

40. What is the total area enclosed between the graphs of the functions $f(x) = \frac{1}{8}x^3 + \frac{1}{4}x^2 - \frac{5}{2}x + 1$ and $g(x) = \frac{1}{2}x + 1$?
(A) 10.667
(B) 20.833
(C) 31.500
(D) 35.333
(E) 42.167

41. A large auto dealer is running a special sales promotion. They expect to sell cars at the rate of $0.32x^2 - 0.01x^3$ cars per day for the first x days of the sale. According to the model, about how many cars will the dealer sell in the first 30 days of the sale?
(A) 18
(B) 29
(C) 722
(D) 855
(E) 863

x	$f(x)$	$f'(x)$
-1	4	3
-3	-2	7

42. The table above contains values of $f(x)$ and $f'(x)$ for certain values of x. If $g(x) = x^2 f(3x)$, then $g'(-1) =$
(A) -14
(B) 11
(C) 17
(D) 21
(E) 25

43. The base of a certain solid is the region in the first quadrant bounded by the x- and y-axes and the curve $y = 15 - e^x$. If each plane cross section of the solid perpendicular to the x-axis is a semicircle with diameter across the base, then the volume of the solid is
(A) 118.325
(B) 155.287
(C) 236.649
(D) 371.728
(E) 473.299

44. Let $f(x)$ be a continuous function with the properties that $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} f'(x) = 3$. What is the value of $\lim_{x\to\infty} [f(x)]^{\frac{1}{x}}$?
(A) 0
(B) 1
(C) 3
(D) e^3
(E) ∞

45. Consider the differentiable function $f(x) = \ln x - x + 3$ on the closed interval $0.5 \le x \le 3.5$. What is the value of c in the interval $0.5 < x < 3.5$ that satisfies the Mean Value Theorem?
(A) 1
(B) 1.484
(C) 1.507
(D) 1.542
(E) 2

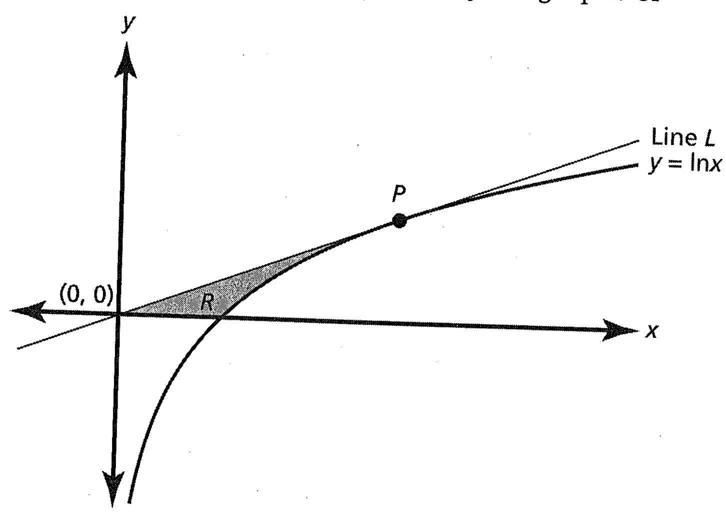
Section II
Free-Response Questions
Time: 1 hour and 30 minutes
Number of Problems: 6

Part A
Time: 45 minutes
Number of Problems: 3

You may use a calculator for any problem in this section.

1. A particle moves in the xy-plane with position vector $\langle x(t), y(t)\rangle$ such that $x(t) = t^3 - 6t^2 + 9t + 1$ and $y(t) = -t^2 + 6t + 2$ in the time interval $0 \le t \le 5$.
   a. At what time t is the particle at rest? Justify your answer.
   b. Give the velocity vector at $t = 5$.
   c. How fast is the particle moving when $t = 5$?
   d. Is the speed of the particle increasing or decreasing when $t = 5$? Justify your answer.
   e. What is the average speed of the particle for the time interval $0 \le t \le 5$?

2. Shown at the right are the graphs of $y = \ln x$ and line L. Line L is tangent to $y = \ln x$ at point P and passes through the point (0, 0). Region R is bounded by the graphs of $y = \ln x$, line L, and the x-axis.
   a. Find the equation of line L.
   b. Find the area of region R.
   c. Find the volume of the solid generated by revolving region R about the line $y = -1$.



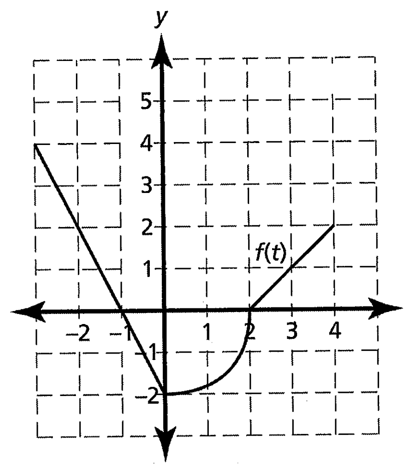
3. The Taylor expansion for a function $f(x)$ about $x = 4$ is given by
$$f(x) = 1 + \frac{1}{2}(x-4) + \frac{1}{4}(x-4)^2 + \frac{1}{8}(x-4)^3 + \cdots = \sum_{k=0}^{\infty} \frac{(x-4)^k}{2^k}.$$
   a. What are all values of x for which $f(x)$ converges?
   b. Find the first three nonzero terms and the general term of $f'(x)$. Use the first three terms to estimate the value of $f'(3.9)$.
   c. Let $g(x)$ be the second degree Taylor polynomial for $f(x)$, and let $h(x)$ be the function such that $h'(x) = g(x)$. If $h(5) = 0$, find $h(x)$.

Part B
Time: 45 minutes
Number of Problems: 3

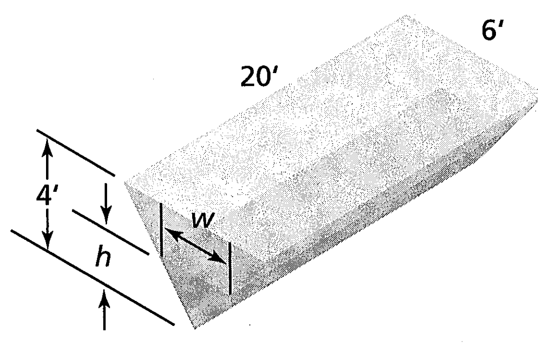
You may not use a calculator for any problem in this section.

During the timed portion for Section II, Part B, you may continue to work on the problems in Part A without the use of a calculator.

4. The graph of $f(t)$, a continuous function defined on the interval $-3 \le t \le 4$, consists of two line segments and a quarter circle, as shown in the figure to the right. Let $g(x) = \int_{-3}^{x} f(t)\,dt$
   a. Evaluate $g(0)$ and $g(4)$.
   b. Find the x-coordinate of the absolute maximum and absolute minimum of $g(x)$. Justify your answer.
   c. Does $\lim_{x\to 2} g''(x)$ exist? Give a reason for your answer.
   d. Find the x-coordinates of all inflection points of $g(x)$. Justify your answer.

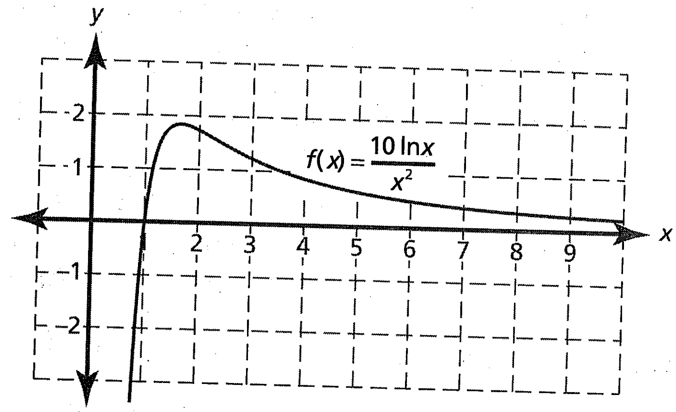


5. A concrete reservoir in the shape of a triangular prism is being filled with water at the constant rate of 2 cubic feet per minute. The reservoir is 4 feet deep, measures 6 feet across the top, and is 20 feet long, as shown in the figure below. For any time $t \ge 0$, let h represent the depth of the water in the reservoir, and let w represent the width of the rectangular region of water at the top.
   a. If the reservoir is initially empty, how long will it take to fill completely?
   b. How fast is the depth of the water in the reservoir changing when the reservoir is half full? Indicate units of measure.
   c. How fast is the rectangular area of the surface of the water changing when the reservoir is half full? Indicate units of measure.



6. Consider the function $f(x) = \frac{10 \ln x}{x^2}$, for $x \geq 1$. The graph of $f(x)$ is pictured below along with a table of values of $f(x)$.

x	$f(x)$
1	0
2	1.733
3	1.221
4	0.866
5	0.644
6	0.498
7	0.397



a. Evaluate $\lim_{x \to \infty} f(x)$.

b. Find the x-coordinate of the relative maximum of $f(x)$. Justify your answer.

c. Use a midpoint Riemann sum with $n = 3$ to estimate the value of $\int_1^7 f(x)\,dx$.

d. Evaluate $\int_1^{\infty} f(x)\,dx$.