Answers and Answer Explanations

Using the table below, score your test. Determine how many questions you answered correctly and how many you answered incorrectly. Additional information about scoring is at the end of the Practice Test.

1. E	2. B	3. C	4. D	5. C
6. E	7. E	8. C	9. B	10. C
11. A	12. A	13. A	14. A	15. A
16. D	17. C	18. D	19. A	20. C
21. A	22. C	23. B	24. D	25. D
26. B	27. C	28. E	29. B	30. D
31. D	32. C	33. C	34. E	35. B
36. A	37. C	38. B	39. D	40. E
41. D	42. E	43. A	44. B	45. D

MULTIPLE-CHOICE QUESTIONS

***Note*: Asterisks (*) indicate BC questions and solutions.**

1. ANSWER: (E) $f'(x) = 2x\,(-\sin x) + \cos x\,(2) = -2x\sin x + 2\cos x$
(*Calculus* 8th ed. pages 119–125 / 9th ed. pages 119–125)

2. ANSWER: (B) $f'(x)$ increasing $\Rightarrow f''(x) > 0 \Rightarrow f(x)$ is concave up.
$f'(x)$ decreasing $\Rightarrow f''(x) < 0 \Rightarrow f(x)$ is concave down.
(B) is concave up for $x < 2$ and concave down for $x > 2$.
(*Calculus* 8th ed. pages 209–214 / 9th ed. pages 209–214)

3. ANSWER: (C) If the interval from 0 to 1 is partitioned into n subintervals, then each one has width $\Delta x = \frac{1}{n}$ and their x-coordinates are $\frac{0}{n}, \frac{1}{n}, \frac{2}{n}, \cdots, \frac{k}{n}, \cdots, \frac{n}{n}$. Thus $x_k = \frac{k}{n}$. Recall that a definite integral is defined as the limit of a Riemann sum, $\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x$. In this problem, $f(x) = \sqrt{x}$. Therefore
$$\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}\cdot\frac{1}{n} = \lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{x_k}\,\Delta x = \int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}.$$
(*Calculus* 8th ed. pages 271–278 / 9th ed. pages 271–278)

*4. **ANSWER: (D)** Using Euler's method, $y_{n+1} \approx y_n + \left.\dfrac{dy}{dx}\right|_{(x_n, y_n)} \cdot \Delta x$

$x_0 = 2$ $\quad y_0 = 5$ $\quad \left.\dfrac{dy}{dx}\right|_{(2,\,5)} = 5 - 4 + 3 = 4$

$x_1 = 2.5$ $\quad y_1 = 5 + 4(0.5) = 7$ $\quad \left.\dfrac{dy}{dx}\right|_{(2.5,\,7)} = 7 - 5 + 3 = 5$

$x_2 = 3$ $\quad y_2 = 7 + 5(0.5) = 9.5$ $\quad$ Therefore $f(3) \approx 9.5$.

(*Calculus* 8th ed. pages 404–408 / 9th ed. pages 406–410)

5. **ANSWER: (C)** $y(5) = 8\sqrt{3 \cdot 5 + 1} = 32$

$y' = \dfrac{8 \cdot 3}{2\sqrt{3x+1}} = \dfrac{12}{\sqrt{3x+1}} \Rightarrow y'(5) = \dfrac{12}{\sqrt{3 \cdot 5 + 1}} = 3$

The equation of the tangent line is $y - 32 = 3(x - 5)$, or $y = 3x + 17$.

(*Calculus* 8th ed. pages 107–118 / 9th ed. pages 107–118)

*6. **ANSWER: (E)** $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{\dfrac{16}{2\sqrt{t}}}{\dfrac{10}{1+t}} = \dfrac{16(1+t)}{20\sqrt{t}}$. Therefore

$\left.\dfrac{dy}{dx}\right|_{t=4} = \dfrac{80}{40} = 2.$

(*Calculus* 8th ed. pages 719–724 / 9th ed. pages 721–726)

7. **ANSWER: (E)** $\int_0^1 \dfrac{3}{x}\,dx = \lim\limits_{b \to 0} \int_b^1 \dfrac{3}{x}\,dx = \lim\limits_{b \to 0} 3\ln|x|\Big|_b^1 =$

$3\ln|1| - \lim\limits_{b \to 0} 3\ln|b| = 0 - (-\infty) = \infty$

(*Calculus* 8th ed. pages 578–584 / 9th ed. pages 580–586)

8. **ANSWER: (C)** Since $f(x)$ is strictly increasing, left end points produce inscribed rectangles and an underapproximation. Right end points produce circumscribed rectangles and an overapproximation. Since $f(x)$ is concave down, a trapezoidal approximation consists of line segments which are below $f(x)$, producing an underapproximation. So I and II are false and III is true, making (C) correct.

(*Calculus* 8th ed. pages 309–313 / 9th ed. pages 311–315)

9. **ANSWER: (B)** Separating variables,

$\int \dfrac{dy}{y} = \int \left(1 + \dfrac{1}{x^2}\right) dx \Rightarrow \ln|y| = x - \dfrac{1}{x} + C_1 \Rightarrow y = e^{x - \frac{1}{x} + C_1} \Rightarrow y = Ce^{x - \frac{1}{x}}$

(*Calculus* 8th ed. pages 421–428 / 9th ed. pages 423–430)

10. ANSWER: (C) Differentiating both sides implicitly,
$$2x \cdot 2y\frac{dy}{dx} + 2y^2 = 6x - 3y^2\frac{dy}{dx}.$$

At point (1, 1), this equation is $4\frac{dy}{dx} + 2 = 6 - 3\frac{dy}{dx}$. Therefore
$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{6-2}{4+3} = \frac{4}{7}.$$
(*Calculus* 8th ed. pages 141–145 / 9th ed. pages 141–145)

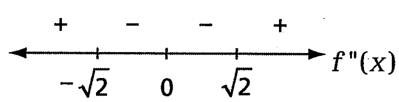
11. ANSWER: (A) $f(x) = \int 12x^2 \sin\left(2x^3 - 16\right)dx$. Let
$u = 2x^3 - 16 \Rightarrow du = 6x^2dx$.
$f(x) = \int 2\sin u\, du = -2\cos u + C = -2\cos\left(2x^3 - 16\right) + C.$
$5 = -2\cos(2 \cdot 2^3 - 16) + C \Rightarrow 5 = -2\cos(0) + C = -2 + C \Rightarrow C = 7.$
$f(x) = -2\cos\left(2x^3 - 16\right) + 7.$
(*Calculus* 8th ed. pages 248–255 / 9th ed. pages 248–255)

*12. ANSWER: (A) The Taylor expansion for a function about $x = a$ is defined as $\sum_{k=0}^{\infty}\frac{f^{(k)}(a)(x-a)^k}{k!}$. Therefore,
$$\frac{f''(3)(x-3)^2}{2!} = -\frac{7(x-3)^2}{3}.\ \text{Solving, } f''(3) = -\frac{7}{3}\cdot 2! = -\frac{14}{3}.$$

Equivalently, differentiating the given polynomial twice and substituting $x = 3$ produces $f''(3) = -14/3$.
(*Calculus* 8th ed. pages 648–655 / 9th ed. pages 650–657)

13. ANSWER: (A) $f'(x) = 6x^5 - 20x^3 \Rightarrow f''(x) = 30x^4 - 60x^2 = 30x^2\left(x^2 - 2\right)$

$30x^2\left(x^2 - 2\right) = 0 \Rightarrow x = 0,\ \pm\sqrt{2}$



The sign of $f''(x)$ changes at $x = \pm\sqrt{2}$ only, so these are the locations of the inflection points.
(*Calculus* 8th ed. pages 190–194 / 9th ed. pages 190–194)

*14. ANSWER: (A) $\frac{\cos(0) - e^0}{\ln(1+0)} = \frac{1-1}{0} = \frac{0}{0}$. This is a quotient indeterminate form, so L'Hôpital's rule applies.
$$\lim_{x\to 0}\frac{\cos x - e^x}{\ln(1+x)} = \lim_{x\to 0}\frac{-\sin x - e^x}{\frac{1}{1+x}} = \frac{-0-1}{\frac{1}{1+0}} = -1$$
(*Calculus* 8th ed. pages 567–573 / 9th ed. pages 569–575)

*15. **ANSWER: (A)** I: The series is geometric with $r = 3/4$, so it converges.

II: $\lim\limits_{k\to\infty}\dfrac{k^2}{(2k+1)^2} = \dfrac{1}{4} \neq 0$, so the series diverges by the nth-Term Test.

III: $|\sec k| \geq 1$, so $\dfrac{|\sec k|}{k} \geq \dfrac{1}{k}$. Since $\sum\limits_{k=1}^{\infty}\dfrac{1}{k}$ is a divergent p-series

(harmonic series, $p = 1$), the series $\sum\limits_{k=1}^{\infty}\dfrac{|\sec k|}{k}$ diverges by the

Direct Comparison Test. Therefore (A) is correct.
(*Calculus* 8th ed. pages 606–611, 624–627 / 9th ed. pages 608–613, 626–629)

16. **ANSWER: (D)** Separating variables,

$$\int\frac{dy}{y-2} = \int k\,dt \Rightarrow \ln|y-2| = kt + C_1 \Rightarrow y = e^{kt+C_1} + 2 \Rightarrow y = Ce^{kt} + 2.$$

(*Calculus* 8th ed. pages 421–428 / 9th ed. pages 423–430)

17. **ANSWER: (C)** By the Second Fundamental Theorem, the domain is the largest continuous interval of $f(t)$ containing the lower limit of the integral. Since the upper limit is a function of x, solve the inequality $-1 < 2x - 1 < 5$. The solution is $0 < x < 3$.
(*Calculus* 8th ed. pages 282–290 / 9th ed. pages 282–290)

18. **ANSWER: (D)** $F'(x) = f(2x-1)\cdot 2 \Rightarrow F'(2) = 2f(2\cdot 2 - 1) = 2f(3) = 2\cdot 2 = 4$
(*Calculus* 8th ed. pages 282–290 / 8th ed. pages 282–290)

19. **ANSWER: (A)** $f'(x) = \dfrac{3}{\sqrt{1-x^2}} \Rightarrow f'(0) = \dfrac{3}{1} = 3 \Rightarrow -\dfrac{1}{f'(0)} = -\dfrac{1}{3}$
(*Calculus* 8th ed. pages 371–379 / 9th ed. pages 373–381)

*20. **ANSWER: (C)** $\dfrac{dx}{dt} = \sec^2 t$ and $\dfrac{dy}{dt} = \dfrac{1}{2}e^{\frac{1}{2}t}$

$$\text{Length} = \int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1\sqrt{\sec^4 t + \frac{1}{4}e^t}\,dt$$

(*Calculus* 8th ed. pages 719–724 / 9th ed. pages 721–726)

*21. **ANSWER: (A)** The Maclaurin series for e^x is $\sum\limits_{k=0}^{\infty}\dfrac{x^k}{k!}$. So the series for

e^{x^2} is $\sum\limits_{k=0}^{\infty}\dfrac{\left(x^2\right)^k}{k!} = \sum\limits_{k=0}^{\infty}\dfrac{x^{2k}}{k!}$.

$$\text{Area} = \int_0^1 e^{x^2}dx = \int_0^1\left(\sum_{k=0}^{\infty}\frac{x^{2k}}{k!}\right)dx \approx \int_0^1\left(\frac{x^0}{0!} + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!}\right)dx$$

$$= \left.\frac{x^1}{1\cdot 0!} + \frac{x^3}{3\cdot 1!} + \frac{x^5}{5\cdot 2!} + \frac{x^7}{7\cdot 3!}\right|_0^1 = 1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42}$$

(*Calculus* 8th ed. pages 676–684 / 9th ed. pages 678–686)

22. **ANSWER: (C)** The slopes are positive in quadrants I and II and negative in quadrants III and IV. This indicates no change in sign

on opposite sides of the y-axis, thus x has an even power. There is a change in sign on opposite sides of the x-axis, thus y has an odd power. Therefore (C) is correct.
(*Calculus* 8th ed. pages 404–408 / 9th ed. pages 406–410)

*23. **ANSWER: (B)** The answer can be determined in two ways:
1. Find the series for $f(x) = x \sin x$ and differentiate. 2. Differentiate $f(x) = x \sin x$ and find its series.

$$1:\quad f(x) = x\sin x \approx x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right)$$

$$= x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!}.\quad f'(x) \approx 2x - \frac{4x^3}{3!} + \frac{6x^5}{5!} - \frac{8x^7}{7!}$$

$$2:\quad f'(x) = x\cos x + \sin x \approx x\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right) + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$= x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

$$= 2x - \frac{4x^3}{3!} + \frac{6x^5}{5!} - \frac{8x^7}{7!}$$

(*Calculus* 8th ed. pages 676–687 / 9th ed. pages 678–689)

*24. **ANSWER: (D)** $r = \dfrac{1}{\cos\left(\frac{1}{2}\theta\right)} = \sec\left(\frac{1}{2}\theta\right)$

$$\text{Polar area} = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sec^2 \frac{1}{2}\theta\, d\theta$$

$$= 2\cdot\frac{1}{2}\tan\frac{1}{2}\theta\Big|_0^{\frac{\pi}{2}}$$

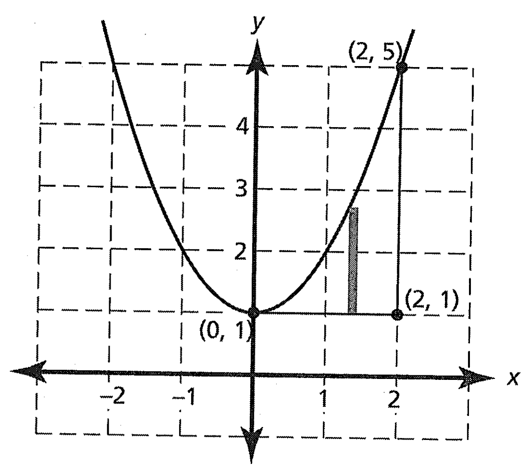
$$= \tan\frac{\pi}{4} - \tan 0$$

$$= 1 - 0 = 1$$

(*Calculus* 8th ed. pages 739–744 / 9th ed. pages 741–746)

25. **ANSWER: (D)** By the Washer Method,
Volume $= \pi\int_0^2\left[\left(1 + x^2\right)^2 - 1^2\right]dx$.

(*Calculus* 8th ed. pages 456–462 / 9th ed. pages 458–464)



*26. **ANSWER: (B)** $\int \frac{2x-3}{x^2+9x+18}\,dx = \int \frac{2x-3}{(x+6)(x+3)}\,dx$. Integrate by partial fractions. $\frac{2x-3}{(x+6)(x+3)} = \frac{A}{x+6} + \frac{B}{x+3} \Rightarrow 2x-3 = A(x+3)+B(x+6)$

Let $x=-6$: $2(-6)-3 = A(-6+3) \Rightarrow -15 = 3A \Rightarrow A = 5$

Let $x=-3$: $2(-3)-3 = B(-3+6) \Rightarrow -9 = 3B \Rightarrow B = -3$

Therefore,

$$\int \frac{2x-3}{(x+6)(x+3)}\,dx = \int\left(\frac{5}{x+6} - \frac{3}{x+3}\right)dx$$
$$= 5\ln|x+6| - 3\ln|x+3| + C$$
$$= \ln\left|\frac{(x+6)^5}{(x+3)^3}\right| + C.$$

(*Calculus* 8th ed. pages 552–558 / 9th ed. pages 554–560)

*27. **ANSWER: (C)** To get the velocity vector, integrate the coordinates of the acceleration vector.

$\int -\pi \sin \pi t\,dt = \cos \pi t + C_1$. $\cos(\pi \cdot 0) + C_1 = 1 \Rightarrow 1 + C_1 = 1 \Rightarrow C_1 = 0$.

$\int (2t+1)\,dt = t^2 + t + C_2$. $0^2 + 0 + C_2 = 0 \Rightarrow C_2 = 0$. The velocity vector is $(\cos \pi t,\ t^2+t)$. Therefore the speed of the particle when $t = 2$ is $\sqrt{\cos^2(\pi \cdot 2) + (2^2+2)^2} = \sqrt{1^2+6^2} = \sqrt{37}$.

(*Calculus* 8th ed. pages 719–724 / 9th ed. pages 721–726)

*28. **ANSWER: (E)** This is a variation on a *p*-series, $\sum_{k=1}^{\infty} \frac{1}{k^p}$, so the Limit Comparison Test should be used. If $\lim_{k\to\infty} \frac{u_k}{v_k}$ is finite and positive, then the original series and the comparison series will both converge or both diverge. A *p*-series converges if $p > 1$. Compare to $\sum_{k=1}^{\infty} \frac{1}{k^{2a-5}}$, because the difference in degree (denominator minus numerator) of the original series is $2a-3-2 = 2a-5$.

$$\lim_{k\to\infty} \frac{\frac{k^2}{k^{2a-3}+4}}{\frac{1}{k^{2a-5}}} = \lim_{k\to\infty} \frac{k^2 \cdot k^{2a-5}}{k^{2a-3}+4} = \lim_{k\to\infty} \frac{k^{2a-3}}{k^{2a-3}+4} = 1,$$ which is finite and positive. $\sum_{k=1}^{\infty} \frac{1}{k^{2a-5}}$ converges if $2a-5 > 1 \Rightarrow 2a > 6 \Rightarrow a > 3$.

(*Calculus* 8th ed. pages 617–630 / 9th ed. pages 619–632)

29. **ANSWER: (B)** Using the Trapezoid Rule,

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}(y_0+2y_1+2y_2+\cdots+2y_{n-1}+y_n)$$
$$=\frac{10-4}{2\cdot 3}(24+2\cdot 37+2\cdot 47+58)=1(250)=250$$

(*Calculus* 8th ed. pages 309–313 / 9th ed. pages 311–315)

30. **ANSWER: (D)** $\lim\limits_{x\to -2} f(x)=+\infty$, which is nonexistent; the graph is closed at $x = 1$, thus $f(1)$ exists; $f(x)$ is not continuous at $x = 1$, and therefore cannot be differentiable at $x = 1$; $\lim\limits_{x\to\infty} f(x)=0$, indicated by the horizontal asymptote $y = 0$. Thus (A), (B), (C), and (E) are all false. $\lim\limits_{x\to 1^+} f(x)$ is a finite value, even though it is not the same value as $f(1)$. Therefore (D) is the true statement.
(*Calculus* 8th ed. pages 70–78 / 9th ed. pages 70–78)

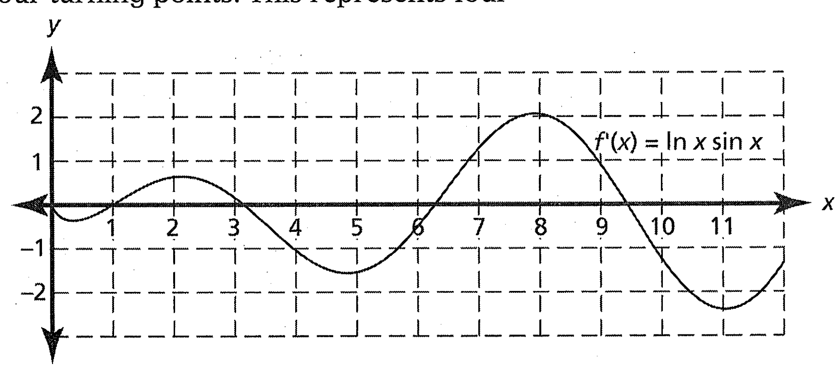
31. **ANSWER: (D)** $f(3)=0+\int_{-3}^{3} f'(x)\,dx$ and represents the accumulated area under the curve from $x = -3$ to $x = 3$. The net signed areas of the triangles are $\frac{1}{2}\cdot 3\cdot 2+\frac{1}{2}\cdot 2\cdot 2-\frac{1}{2}.1\cdot 1=3+2-\frac{1}{2}=4.5$.
(*Calculus* 8th ed. pages 282–290 / 9th ed. pages 282–290)

*32. **ANSWER: (C)** $dy/dx=\dfrac{dy/dt}{dx/dt}\Rightarrow dx/dt=\dfrac{dy/dt}{dy/dx}$. For the given function, $\dfrac{dy}{dx}=x\cos x+\sin x\Rightarrow \left.\dfrac{dy}{dx}\right|_{x=3}=3\cos 3+\sin 3$. Therefore,

$$\left.\frac{dx}{dt}\right|_{x=3}=\frac{-2}{3\cos 3+\sin 3}\approx 0.707.$$

(*Calculus* 8th ed. pages 719–724 / 9th ed. pages 721–726)

33. **ANSWER: (C)** $1.6\le x\le 11.6$. In looking at the graph of $f'(x)$ on the given window, there are four turning points. This represents four points at which $f''(x)$ (or the slope of $f'(x)$) is equal to 0 and changes sign from either positive to negative or negative to positive. So these represent four changes in concavity, hence four inflection points.
(*Calculus* 8th ed. pages 190–194 / 9th ed. pages 190–194)



34. **ANSWER: (E)** The average value of a function in an interval is the value of the definite integral divided by its length, that is, $f_{ave}=\dfrac{1}{b-a}\int_a^b f(x)\,dx$. A look at the graphs of $y = \cos x$, $y = \cos 2x$, and $y = \sin x$ reveals that the areas under the curves can be easily

compared without computation. (A) and (C) have positive areas, (B) and (D) are zero, and (E) is negative. Therefore (E) is the only choice to have a negative average value, so it is the smallest. Alternatively, calculate the five average values on the calculator and see that (E) is the smallest.
(*Calculus* 8th ed. pages 119–129 / 9th ed. pages 282–296)

35. **ANSWER: (B)** Use the graphing calculator to graph $v(t)$. Use the derivative feature to graph $v'(t)$. Then $v'(t) = a(t) = 0$ at $t = 2.35619$, $v(2.35619) = -0.670$.
(*Calculus* 8th ed. pages 119–129 / 9th ed. pages 282–296)

*36. **ANSWER: (A)** Series I and II are essentially p-series, so the convergence of the series of absolute values can be obtained by using the Limit Comparison Test. Recall that $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p > 1$ and diverges if $0 < p \leq 1$. Compare I to $\sum_{k=1}^{\infty} \frac{1}{k}$, which is divergent. $\lim_{k\to\infty} \left| \frac{(-1)^k \frac{k^2}{k^3+1}}{\frac{1}{k}} \right| = \lim_{k\to\infty} \left| \frac{k^3}{k^3+1} \right| = 1$, which is finite and positive, thus I does not converge absolutely. But the sequence of positive terms decreases to a limit of zero, so as an alternating series, I converges by the Alternating Series Test. Therefore, I is conditionally convergent. Compare II to $\sum_{k=1}^{\infty} \frac{1}{k^2}$. Using the same limit procedure, II converges absolutely. The sequence in III has limit 1, so it is divergent. In summary, I is the only series that converges conditionally.
(*Calculus* 8th ed. pages 631–636 / 9th ed. pages 633–638)

37. **ANSWER: (C)** $\tan\theta = \frac{h}{8} \Rightarrow h = 8\tan\theta \Rightarrow \frac{dh}{dt} = 8\sec^2\theta \frac{d\theta}{dt}$. When $h = 13$, $8^2 + 13^2 = z^2 \Rightarrow z = \sqrt{64+169} = \sqrt{233}$.
$\left.\frac{dh}{dt}\right|_{h=13} = 8 \cdot \frac{233}{64}(0.03) = 0.874$ cm/sec.
(*Calculus* 8th ed. pages 149–153 / 9th ed. pages 149–153)

38. **ANSWER: (B)** $f''(x) = e^x + 1$, which is positive everywhere. Therefore $f(x)$ is concave up everywhere, so any tangent line to $f(x)$ will be below the curve except at the point of tangency. Thus $H(a + 0.1) < f(a + 0.1)$ for all values of a. [Note: If, for example, $f''(x) = e^x - 1$, there would be a sign change in $f''(x)$, hence an inflection point on the graph of $f(x)$. In that case (C) would be the correct answer.]
(*Calculus* 8th ed. pages 235–239 / 9th ed. pages 235–239)

*39. **ANSWER: (D)** Using the Ratio Test for Absolute Convergence,

$$\lim_{k\to\infty}\left|\frac{\frac{(2x)^{k+1}}{k+2}}{\frac{(2x)^k}{k+1}}\right| = \lim_{k\to\infty}\left|\frac{(2x)^{k+1}\cdot(k+1)}{(k+2)\cdot(2x)^k}\right| < 1$$

$$\Rightarrow \lim_{k\to\infty}|2x| < 1 \Rightarrow |2x| < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$$

End points must be checked separately.

$x = -\frac{1}{2}$: $\sum_{k=0}^{\infty}\frac{(-1)^k}{k+1}$ converges by the Alternating Series Test, since the series of positive terms is decreasing and $\lim_{k\to\infty}\frac{1}{k+1} = 0$.

$x = \frac{1}{2}$: $\sum_{k=0}^{\infty}\frac{1}{k+1}$ diverges, since it is a p-series (harmonic series, $p = 1$). Therefore, the interval of convergence is $-\frac{1}{2} \le x < \frac{1}{2}$.

(*Calculus* 8th ed. pages 659–665 / 9th ed. pages 661–667)

40. **ANSWER: (E)** The functions intersect at $x = -6$, 0, and 4 and enclose two regions.

Area = $\int_{-6}^{0}[f(x)-g(x)]\,dx + \int_{0}^{4}[g(x)-f(x)]\,dx = 31.5 + 10.667 = 42.167$. Alternatively, Area = $\int_{-6}^{4}|f(x)-g(x)|\,dx = 42.167$.

(*Calculus* 8th ed. pages 446–451 / 9th ed. pages 448–453)

41. **ANSWER: (D)** The total sales figure is represented by $\int_{0}^{30}(0.32x^2 - 0.01x^3)\,dx = 855$.

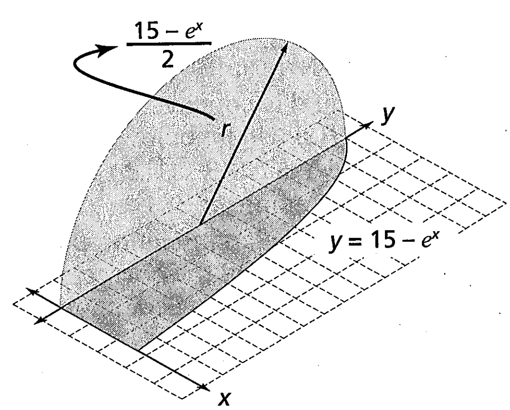
(*Calculus* 8th ed. pages 282–290 / 9th ed. pages 282–290)

42. **ANSWER: (E)** By the product and chain rules, $g'(x) = x^2\cdot 3f'(3x) + 2x\cdot f(3x)$. Therefore, $g'(-1) = (-1)^2\cdot 3f'(-3) - 2f(-3) = 3\cdot 7 - 2(-2) = 25$.

(*Calculus* 8th ed. pages 119–125, 130–136 / 9th ed. pages 119–125, 130–136)

43. **ANSWER: (A)** $15 - e^x = 0 \Rightarrow x = 2.70805$. The radius of each cross section is $\frac{15-e^x}{2}$, so the area of each cross section is $\frac{1}{2}\pi\left(\frac{15-e^x}{2}\right)^2 = \frac{\pi}{8}\left(15-e^x\right)^2$. Therefore, $V = \int_{0}^{2.70805}\frac{\pi}{8}\left(15-e^x\right)^2\,dx = 118.325$.

(*Calculus* 8th ed. pages 456–462 / 9th ed. pages 458–464)



*44. **ANSWER: (B)** This is the indeterminate form ∞^0. Let $y = [f(x)]^{\frac{1}{x}}$.

Then $\ln y = \frac{1}{x}\ln[f(x)] = \frac{\ln[f(x)]}{x}$.

Thus $\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln[f(x)]}{x}$. This is the indeterminate form $\frac{\infty}{\infty}$, so use L'Hôpital's rule. $\operatorname{Lim}_{x\to\infty} \frac{\ln[f(x)]}{x} = \lim_{x\to\infty} \frac{\frac{f'(x)}{f(x)}}{1} = \frac{\frac{3}{\infty}}{1} = 0$. Therefore, $\lim_{x\to\infty} \ln y = 0 \Rightarrow \lim_{x\to\infty} y = e^0 = 1$.

(*Calculus* 8th ed. pages 567–573 / 9th ed. pages 569–575)

45. **(D)** The Mean Value Theorem guarantees at least one value c in the interval $0.5 < x < 3.5$ such that $f'(c) = \frac{f(3.5) - f(0.5)}{3.5 - 0.5}$. For the given function, $f'(x) = \frac{1}{x} - 1$. Therefore $f'(c) = \frac{1}{c} - 1 = \frac{0.75276 - 1.80685}{3.5 - 0.5} = -\frac{1.05409}{3} = -0.35136$. So $\frac{1}{c} - 1 = -0.35136 \Rightarrow c = 1.542$.

(*Calculus* 8th ed. pages 172–175 / 9th ed. pages 172–175)

FREE-RESPONSE QUESTIONS

*1. A particle moves in the xy-plane with position vector $(x(t), y(t))$ such that $x(t) = t^3 - 6t^2 + 9t + 1$ and $y(t) = -t^2 + 6t + 2$ in the time interval $0 \le t \le 5$.

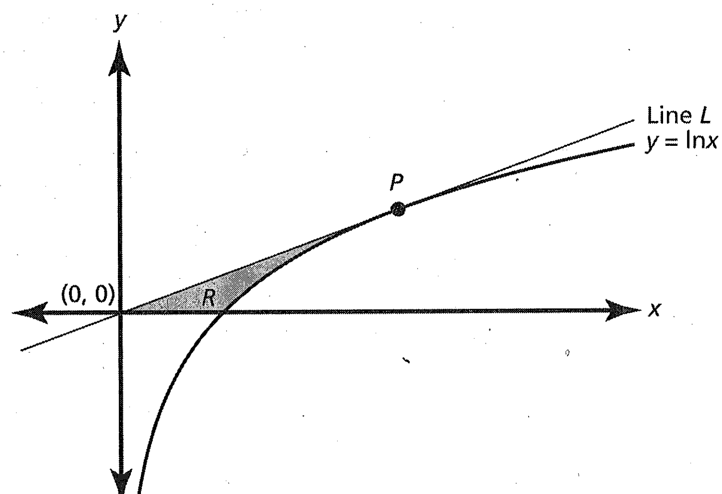
a. At what time t is the particle at rest? Justify your answer.
b. Give the velocity vector at $t = 5$.
c. How fast is the particle moving when $t = 5$?
d. Is the speed of the particle increasing or decreasing when $t = 5$? Justify your answer.
e. What is the average speed of the particle for the time interval $0 \le t \le 5$?

	Solution	Possible points
a.	$x'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3) = 0$ $t = 1$ or 3 $y'(t) = -2t + 6 = -2(t-3) = 0$ $t = 3$ The particle is at rest at $t = 3$ because both $x'(3) = 0$ and $y'(3) = 0$.	3: $\begin{cases} \text{1: } x'(t) \text{ and } y'(t) \\ \text{1: zeros of } x'(t) \text{ and } y'(t) \\ \text{1: answer with reason} \end{cases}$
b.	$x'(5) = 3\cdot 25 - 12\cdot 5 + 9 = 24$ $y'(5) = -10 + 6 = -4$ The velocity vector at $t = 5$ is $(24, -4)$.	1: answer

	Solution	Possible points
c.	The speed of the particle is $\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$ $=\sqrt{(3t^2-12t+9)^2+(-2t+6)^2}$. At $t=5$, the speed is $\sqrt{(24)^2+(-4)^2}=\sqrt{592}=24.331$.	2: $\begin{cases}\text{1: expression for speed}\\ \text{1: answer}\end{cases}$
d.	Using the calculator, $\frac{d}{dt}\left(\sqrt{(3t^2-12t+9)^2+(-2t+6)^2}\right)\Big\vert_{t=5}=18.084>0$. Since the derivative of the speed function is positive when $t=5$, the speed is increasing at $t=5$.	2: $\begin{cases}\text{1: use of derivative of speed}\\ \text{1: answer with reason}\end{cases}$
e.	$\frac{1}{5-0}\int_0^5\sqrt{(3t^2-12t+9)^2+(-2t+6)^2}\,dt=6.609$	1: answer

1. a, b, c (*Calculus* 8th ed. pages 719–724 / 9th ed. pages 721–726)
1. d (*Calculus* 8th ed. pages 179–185 / 9th ed. pages 179–185)
1. e (Calculus 8th ed. pages 282–290 / 9th ed. pages 282–290)

2. Shown at the right are the graphs of $y=\ln x$ and line L. Line L is tangent to $y=\ln x$ at point P and passes through the point (0, 0). Region R is bounded by the graphs of $y=\ln x$, line L, and the x-axis.
   a. Find the equation of line L.
   b. Find the area of region R.
   c. Find the volume of the solid generated by revolving region R about the line $y=-1$.



	Solution	Possible points
a.	Label the point of tangency P $(a, \ln a)$. $y'(x)=\frac{1}{x}\Rightarrow y'(a)=\frac{1}{a}$ $m_{\tan}=\frac{\ln a-0}{a-0}=\frac{1}{a}\Rightarrow \ln a=1\Rightarrow a=e$ $y(e)=1$ and $y'(e)=\frac{1}{e}$. Since L contains (0, 0), the equation of L is $y=\frac{1}{e}x\Rightarrow y=\frac{x}{e}$. Therefore the equation of L is $y=\frac{1}{e}x\Rightarrow y=\frac{x}{e}$.	3: $\begin{cases}\text{1: slope of } L\\ \text{1: point of tangency}\\ \text{1: equation of } L\end{cases}$
	Solution	**Possible points**

b.	Area $= \int_0^1 \frac{x}{e}\,dx + \int_1^e \left(\frac{x}{e} - \ln x\right)\,dx = 0.359$	3: $\begin{cases} 2\text{: integrands} \\ \quad <-1> \text{ each error} \\ 1\text{: answer} \end{cases}$
c.	Volume $= \pi \int_0^1 \left[\left(\frac{x}{e}+1\right)^2 - 1^2\right] dx$ $+\pi \int_1^e \left[\left(\frac{x}{e}+1\right)^2 - (\ln x + 1)^2\right] dx$ $= 2.847.$	3: $\begin{cases} 2\text{: integrands} \\ \quad <-1> \text{ each error} \\ 1\text{: answer} \end{cases}$

2. a (*Calculus* 8th ed. pages 107–118 / 9th ed. pages 107–118)
2. b (*Calculus* 8th ed. pages 446–451 / 9th ed. pages 448–453)
2. c (*Calculus* 8th ed. pages 456–462 / 9th ed. pages 458–464)

*3. The Taylor expansion for a function $f(x)$ about $x = 4$ is given by

$$f(x) = 1 + \frac{1}{2}(x-4) + \frac{1}{4}(x-4)^2 + \frac{1}{8}(x-4)^3 + \cdots = \sum_{k=0}^{\infty} \frac{(x-4)^k}{2^k}.$$

a. What are all values of x for which $f(x)$ converges?

b. Find the first three nonzero terms and the general term of $f'(x)$. Use the first three terms to estimate the value of $f'(3.9)$.

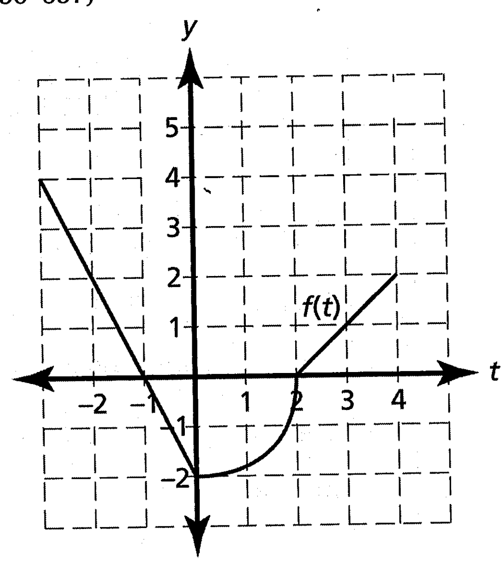
c. Let $g(x)$ be the second degree Taylor polynomial for $f(x)$, and let $h(x)$ be the function such that $h'(x) = g(x)$. If $h(5) = 0$, find $h(x)$.

	Solution	**Possible points**
a.	Using the Ratio Test for Absolute Convergence, $\lim_{k\to\infty}\left\lvert\frac{a_{k+1}}{a_k}\right\rvert < 1 \Rightarrow \lim_{k\to\infty}\left\lvert\frac{\frac{(x-4)^{k+1}}{2^{k+1}}}{\frac{(x-4)^k}{2^k}}\right\rvert < 1 \Rightarrow$ $\lim_{k\to\infty}\left\lvert\frac{(x-4)^{k+1}}{2^{k+1}} \cdot \frac{2^k}{(x-4)^k}\right\rvert < 1 \Rightarrow \lim_{k\to\infty}\left\lvert\frac{x-4}{2}\right\rvert < 1.$ Therefore $\lvert x-4\rvert < 2 \Rightarrow 2 < x < 6$. End points must be tested separately: Let $x = 2$: $\sum_{k=0}^{\infty}\frac{(-2)^k}{2^k} = \sum_{k=0}^{\infty}(-1)^k$ which diverges by the nth-Term Test. Let $x = 6$: $\sum_{k=0}^{\infty}\frac{(2)^k}{2^k} = \sum_{k=0}^{\infty}(1)^k$ which diverges by the nth-Term Test. The interval of convergence is $2 < x < 6$.	4: $\begin{cases} 1\text{: use of RTAC} \\ 1\text{: open interval} \\ 1\text{: test for } x = 2 \\ 1\text{: test for } x = 6 \end{cases}$

	Solution	Possible points
b.	$f'(x)=\frac{1}{2}+\frac{2}{4}(x-4)+\frac{3}{8}(x-4)^2+\cdots=\sum_{k=0}^{\infty}\frac{k(x-4)^{k-1}}{2^k}$ $f'(3.9)\approx\frac{1}{2}+\frac{2}{4}(3.9-4)+\frac{3}{8}(3.9-4)^2=$ $\frac{1}{2}+\frac{1}{2}(-0.1)+\frac{3}{8}(-0.1)^2=0.454.$	3: { 1: first 3 terms <−1> first error 1: general term 1: $f'(3.9)$
c.	$g(x)=1+\frac{1}{2}(x-4)+\frac{1}{4}(x-4)^2$ Therefore, $h(x)=\int g(x)\,dx\Rightarrow$ $h(x)=(x-4)+\frac{1}{4}(x-4)^2+\frac{1}{12}(x-4)^3+C$ $h(5)=(5-4)+\frac{1}{4}(5-4)^2+\frac{1}{12}(5-4)^3+C=0\Rightarrow$ $1+\frac{1}{4}+\frac{1}{12}+C=0\Rightarrow\frac{4}{3}+C=0\Rightarrow C=-\frac{4}{3}.$ $h(x)=(x-4)+\frac{1}{4}(x-4)^2+\frac{1}{12}(x-4)^3-\frac{4}{3}$	2: { 1: antiderivative with constant 1: solves for constant

3. a (*Calculus* 8th ed. pages 659–665 / 9th ed. pages 661–667)
3. b (*Calculus* 8th ed. pages 648–655 / 9th ed. pages 650–657)
3. c (*Calculus* 8th ed. pages 248–255 / 9th ed. pages 248–255)

4. The graph of $f(t)$, a continuous function defined in the interval $-3\le t\le 4$, consists of two line segments and a quarter circle, as shown in the figure to the right. Let $g(x)=\int_{-3}^{x} f(t)\,dt$.

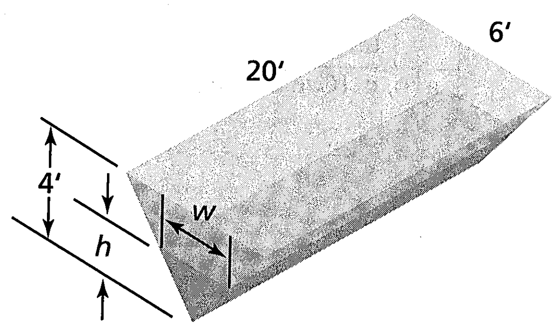


   a. Evaluate $g(0)$ and $g(4)$.
   b. Find the x-coordinate of the absolute maximum and absolute minimum of $g(x)$. Justify your answer.
   c. Does $\lim_{x\to 2} g''(x)$ exist? Give a reason for your answer.
   d. Find the x-coordinates of all inflection points of $g(x)$. Justify your answer.

	Solution	Possible points
a.	$g(x)$ represents the net signed area of the region between the graph of f and the horizontal axis. Therefore $g(0)=\frac{1}{2}\cdot 2\cdot 4-\frac{1}{2}\cdot 1\cdot 2=4-1=3$ and $g(4)=\frac{1}{2}\cdot 2\cdot 4-\frac{1}{2}\cdot 1\cdot 2-\frac{1}{4}\pi(2)^2+\frac{1}{2}\cdot 2\cdot 2=5-\pi$.	$2: \begin{cases} 1: g(0) \\ 1: g(4) \end{cases}$
b.	$g'(x)=f(x)=0$ at $x=-1,\ 2$. The candidates for absolute extremes are $x=-3,\ -1,\ 2,\ 4$. $g(-3)=0,\ g(-1)=4,\ g(2)=3-\pi,\ g(4)=5-\pi$ Therefore the absolute maximum of $g(x)$ occurs at $x=-1$ and the absolute minimum of $g(x)$ occurs at $x=2$.	$2: \begin{cases} \text{1: finds critical values} \\ \text{1: considers critical values and end points} \\ \text{1: answers} \end{cases}$
c.	$g''(x)=f'(x)$ which represents the slopes of the pictured graph. $\lim\limits_{x\to 2^-} f'(x)=+\infty$ and $\lim\limits_{x\to 2^+} f'(x)=1$. Since $+\infty\neq 1$, the limit does not exist.	$2: \begin{cases} \text{1: answer} \\ \text{1: reason} \end{cases}$
d.	$g''(x)=f'(x)$ which represents the slope of the pictured graph. $f'(x)<0$ when $x<0$ and $f'(x)>0$ when $x>0$. Therefore, the graph of $g(x)$ has an inflection point at $x=0$.	$2: \begin{cases} \text{1: answer} \\ \text{1: reason} \end{cases}$

4. a (*Calculus* 8th ed. pages 282–290 / 9th ed. pages 282–290)
4. b (*Calculus* 8th ed. pages 164–168 / 9th ed. pages 164–168)
4. c (*Calculus* 8th ed. pages 70–82, 96–106 / 9th ed. pages 70–82, 96–106)
4. d (*Calculus* 8th ed. pages 190–194 / 9th ed. pages 190–194)

5. A concrete reservoir in the shape of a triangular prism is being filled with water at the constant rate of 2 cubic feet per minute. The reservoir is 4 feet deep, measures 6 feet across the top, and is 20 feet long, as shown in the figure to the right. For any time $t\geq 0$, let h represent the depth of the water in the reservoir, and let w represent the width of the rectangular region of water at the top.



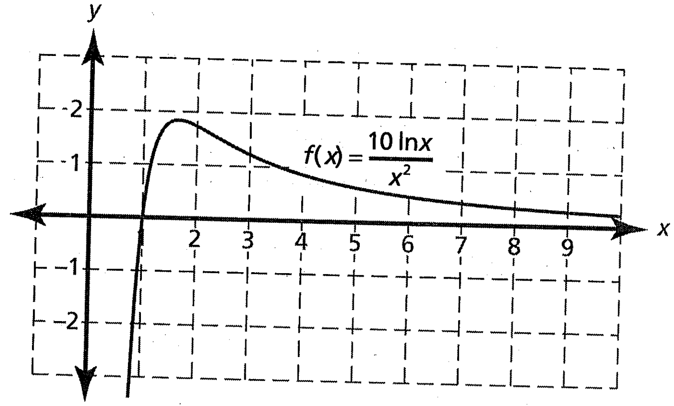
   a. If the reservoir is initially empty, how long will it take to fill completely?
   b. How fast is the depth of the water in the reservoir changing when the reservoir is half full? Indicate units of measure.
   c. How fast is the rectangular area of the surface of the water changing when the reservoir is half full? Indicate units of measure.

	Solution	Possible points
a.	$V=\frac{1}{2}\cdot 6\cdot 4\cdot 20=240\ ft^3$ $\frac{240\ ft^3}{2\ ft^3/\text{min}}=120$ min. The reservoir will be full in 120 minutes.	2: $\begin{cases}1\text{: volume}\\1\text{: answer}\end{cases}$
b.	Using similar triangles, $\frac{h}{w}=\frac{4}{6}\Rightarrow w=\frac{3}{2}h.$ Therefore $V=\frac{1}{2}\cdot\frac{3}{2}h\cdot h\cdot 20=15h^2.$ $15h^2=120\Rightarrow h^2=8\Rightarrow h=2\sqrt{2}$ $\frac{dV}{dt}=30h\frac{dh}{dt}\Rightarrow\left.\frac{dh}{dt}\right\|_{\substack{V=120\\h=2\sqrt{2}}}=\frac{2}{30\cdot 2\sqrt{2}}=\frac{1}{30\sqrt{2}}$ The depth of water in the reservoir is increasing at $\frac{1}{30\sqrt{2}}$ ft/min when the reservoir is half full.	4: $\begin{cases}1\text{: } V \text{ vs. } h \text{ relationship}\\1\text{: depth when half full}\\1\text{: } \frac{dV}{dt}\\1\text{: answer}\end{cases}$
c.	$A=w\cdot 20=\frac{3}{2}h\cdot 20=30h$ Therefore $\frac{dA}{dt}=30\frac{dh}{dt}.$ $\left.\frac{dA}{dt}\right\|_{\substack{V=120\\h=2\sqrt{2}}}=30\cdot\frac{1}{30\sqrt{2}}=\frac{1}{\sqrt{2}}$ The area of the rectangular region of water is increasing at $\frac{1}{\sqrt{2}}$ ft^2/min when the reservoir is half full.	2: $\begin{cases}1\text{: } \frac{dA}{dt}\\1\text{: answer}\end{cases}$
	units	1: correct units in both b and c

5. a, b, c (*Calculus* 8th ed. pages 149–153 / 9th ed. pages 149–153)

*6. Consider the function $f(x)=\dfrac{10\ln x}{x^2}$, for $x\geq 1$. The graph of $f(x)$ is pictured below along with a table of values of $f(x)$.

x	$f(x)$
1	0
2	1.733
3	1.221
4	0.866
5	0.644
6	0.498
7	0.397



a. Evaluate $\lim\limits_{x\to\infty} f(x)$.

b. Find the x-coordinate of the relative maximum of $f(x)$. Justify your answer.

c. Use a midpoint Riemann sum with $n = 3$ to estimate the value of $\int_1^7 f(x)\,dx$.

d. Evaluate $\int_1^\infty f(x)\,dx$.

	Solution	Possible points
a.	Indeterminate form $\dfrac{\infty}{\infty}$ $\lim\limits_{x\to\infty}\dfrac{10\ln x}{x^2}=\lim\limits_{x\to\infty}\dfrac{10/x}{2x}=\lim\limits_{x\to\infty}\dfrac{5}{x^2}=0$	1: answer
b.	$f'(x)=\dfrac{x^2(10/x)-10\ln x(2x)}{x^4}=\dfrac{10x-20x\ln x}{x^4}$ $=\dfrac{10(1-2\ln x)}{x^3}=0$ $1-2\ln x=0\Rightarrow \ln x=\dfrac{1}{2}\Rightarrow x=e^{\frac{1}{2}}=\sqrt{e}$ Using values less than and greater than $\sqrt{e}$ to test signs, $f'(1)=\dfrac{10(1-0)}{1^3}=10$ and $f'(e)=\dfrac{10(1-2)}{e^3}=-\dfrac{10}{e^3}$. $\begin{array}{ccc} & + & - \\ 1 & \sqrt{e} & f'(x)\end{array}$ There is a relative maximum at $x=\sqrt{e}$ because $f'(x)$ changes from positive to negative at that point.	3: $\begin{cases}\text{1: derivative}\\ \text{1: solution}\\ \text{1: justification}\end{cases}$

	Solution	Possible points
c.	$\int_1^7 \frac{10\ln x}{x^2} \approx \frac{7-1}{3}(1.733+0.866+0.498)=6.194$	2: $\begin{cases}\text{1: sum of correct midpoints}\\ \text{1: answer including constant}\end{cases}$
d.	Integration by parts $u=\ln x \quad v=-\frac{10}{x}$ $du=\frac{dx}{x} \quad dv=\frac{10}{x^2}dx$ $\int\frac{10\ln x}{x^2}dx=-\frac{10\ln x}{x}-\int-\frac{10}{x^2}dx$ Since this is an improper integral, $\int_1^\infty\frac{10\ln x}{x^2}=\lim_{b\to\infty}\left(-\frac{10\ln x}{x}-\frac{10}{x}\right)\Bigg\vert_1^b$ $=\lim_{b\to\infty}\left[\left(-\frac{10\ln b}{b}-\frac{10}{b}\right)-\left(-\frac{10\ln 1}{1}-\frac{10}{1}\right)\right]$ $=\lim_{b\to\infty}\left(-\frac{10\ln b}{b}-0+0+10\right)$ $=\lim_{b\to\infty}-\frac{10\ln b}{b}+10.$ The limit is indeterminate of the form $\frac{\infty}{\infty}$. $\lim_{b\to\infty}-\frac{10\ln b}{b}+10=\lim_{b\to\infty}-\frac{10/b}{1}+10=\frac{0}{1}+10=10$	3: $\begin{cases}\text{1: integration by parts setup}\\ \text{1: antiderivative}\\ \text{1: answer}\end{cases}$

6. a (*Calculus* 8th ed. pages 567–573 / 9th ed. pages 569–575)
6. b (*Calculus* 8th ed. pages 164–168 / 9th ed. pages 164–168)
6. c (*Calculus* 8th ed. pages 271–278 / 9th ed. pages 271–278)
6. d (*Calculus* 8th ed. pages 525–530, 567–573, 578–584 / 9th ed. pages 527–532, 569–575, 580–586)