Summer Packet

AP CALCULUS BC

Success in AP Calculus requires a solid and functional knowledge of all the Algebra, Precalculus, and Trigonometry topics and skills you have covered in previous classes. Due to the scope of the AP Calculus curriculum there is very little time to review/reteach previous skills; therefore as you enter AP Calculus these topics are treated as assumed knowledge since you have already taken the prerequisite classes.
This packet is a compilation of questions intended to refresh your Algebra/Precalculus knowledge so you start the year well prepared for the challenging AP Calculus curriculum

You are to complete ALL questions numbers which are a numerical **multiple of 5,** with all work shown on separate paper. Do not do work in this packet.
When doing these problems you must follow the following format:

1. All work must be done in **INK** (black or blue pen.)
2. Questions must be submitted in the order they appear in the packet.
3. You must show all work/reasoning, including any MC Questions.
4. You must leave at least one blank line between each question.
5. You may not do two columns of solutions on the same side of paper
6. Start the questions from each different section on a fresh sheet of paper.
7. You must not work with/copy from other students.

The solutions to these problems are due at the beginning of class on the second day of school, Tuesday August 8th.
The packet will be graded on accuracy as well as completeness, and will weighted as a test grade in the class. It is therefore vital that you complete all problems thoroughly and accurately with all work shown.
You will also have a test on the contents of this packet on Wednesday August 9th. All questions will be taken directly from this packet.

If you have any questions over the summer please email:

David Yang: yangd@fultonschools.org and/or Sim Jones: jonessg@fultonschools.org .

Or come in for help during pre-planning 8/1/16- 8/4/16

CHAPTER REVIEW

Things To Know

Formulas

Distance formula (p. 6) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint formula (p. 8) $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope (p. 61) $m = \frac{y_2 - y_1}{x_2 - x_1}$, if $x_1 \neq x_2$; undefined if $x_1 = x_2$

Parallel lines (p. 69) Equal slopes $(m_1 = m_2)$ and different y-intercepts

Perpendicular lines (p. 71) Product of slopes is -1 $(m_1 \cdot m_2 = -1)$

Equations of Lines and Circles

Vertical line (p. 65) $x = a$

Horizontal line (p. 67) $y = b$

Point–slope form of the equation of a line (p. 66) $y - y_1 = m(x - x_1)$; m is the slope of the line, (x_1, y_1) is a point on the line

Slope-intercept form of the equation of a line (p. 67) $y = mx + b$; m is the slope of the line, b is the y-intercept

General form of the equation of a line (p. 69) $Ax + By = C$, A, B not both 0

Standard form of the equation of a circle (p. 84) $(x - h)^2 + (y - k)^2 = r^2$; r is the radius of the circle, (h, k) is the center of the circle

Equation of the unit circle (p. 84) $x^2 + y^2 = 1$

General form of the equation of a circle (p. 86) $x^2 + y^2 + ax + by + c = 0$

Quadratic equation and quadratic formula (p. 30)

If $ax^2 + bx + c = 0$, $a \neq 0$, and if $b^2 - 4ac \geq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Discriminant (p. 30)

If $b^2 - 4ac > 0$, there are two distinct real solutions.

If $b^2 - 4ac = 0$, there is one repeated real solution.

If $b^2 - 4ac < 0$, there are no real solutions.

Properties of Inequalities

Addition property (p. 51)
If $a < b$, then $a + c < b + c$.
If $a > b$, then $a + c > b + c$.

Multiplication properties (p. 52)
(a) If $a < b$ and if $c > 0$, then $ac < bc$.
If $a < b$ and if $c < 0$, then $ac > bc$.
(b) If $a > b$ and if $c > 0$, then $ac > bc$.
If $a > b$ and if $c < 0$, then $ac < bc$.

Absolute Value Equations and Inequalities

If $|u| = a$, $a > 0$, then $u = -a$ or $u = a$. (p. 32)

If $|u| \leq a$, $a > 0$, then $-a \leq u \leq a$. (p. 56)

If $|u| \geq a$, $a > 0$, then $u \leq -a$ or $u \geq a$. (p. 57)

True/False Items

T F 1. The distance between two points is sometimes a negative number.
T F 2. The graph of the equation $y = x^4 + x^2 + 1$ is symmetric with respect to the y-axis.
T F 3. Vertical lines have undefined slope.
T F 4. The slope of the line $2y = 3x + 5$ is 3.
T F 5. Perpendicular lines have slopes that are reciprocals of one another.
T F 6. The radius of the circle $x^2 + y^2 = 9$ is 3.
T F 7. Equations can have no solutions, one solution, or more than one solution.
T F 8. Quadratic equations always have two real solutions.
T F 9. If the discriminant of a quadratic equation is positive, then the equation has two real solutions that are unequal.
T F 10. The square of any real number is always nonnegative.
T F 11. The expression $x^2 + x + 1$ is positive for any real number x.
12. If $a < b$ and $c < 0$, which of the following statements are true?
T F (a) $a \pm c < b \pm c$
T F (b) $a \cdot c < b \cdot c$
T F (c) $a/c > b/c$

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–28, find all real solutions, if any, of each equation (a) graphically and (b) algebraically.

1. $2 - \dfrac{x}{3} = 6$
2. $\dfrac{x}{4} - 2 = 6$
3. $-2(5 - 3x) + 8 = 4 + 5x$
4. $(6 - 3x) - 2(1 + x) = 6x$
5. $\dfrac{3x}{4} - \dfrac{x}{3} = \dfrac{1}{12}$
6. $\dfrac{4 - 2x}{3} + \dfrac{1}{6} = 2x$
7. $\dfrac{x}{x - 1} = \dfrac{5}{6}, \quad x \neq 1$
8. $\dfrac{4x - 5}{3 - 7x} = 4, \quad x \neq \frac{3}{7}$
9. $x(1 - x) = -6$
10. $x(1 + x) = 6$
11. $\dfrac{1}{2}\left(x - \dfrac{1}{3}\right) = \dfrac{3}{4} - \dfrac{x}{6}$
12. $\dfrac{1 - 3x}{4} = \dfrac{x + 6}{3} + \dfrac{1}{2}$
13. $(x - 1)(2x + 3) = 3$
14. $x(2 - x) = 3(x - 4)$
15. $4x + 3 = 4x^2$
16. $5x = 4x^2 + 1$
17. $\sqrt[3]{x - 1} = 2$
18. $\sqrt[4]{1 + x} = 3$
19. $x(x + 1) - 2 = 0$
20. $2x^2 - 3x + 1 = 0$
21. $\sqrt{2x - 3} + x = 3$
22. $\sqrt{2x - 1} = x - 2$
23. $\sqrt{x + 1} + \sqrt{x - 1} = \sqrt{2x + 1}$
24. $\sqrt{2x - 1} - \sqrt{x - 5} = 3$
25. $|2x + 3| = 7$
26. $|3x - 1| = 5$
27. $|2 - 3x| = 7$
28. $|1 - 2x| = 3$

In Problems 29–38, solve each inequality (a) graphically and (b) algebraically. Graph the solution set.

29. $\dfrac{2x - 3}{5} + 2 \leq \dfrac{x}{2}$
30. $\dfrac{5 - x}{3} \leq 6x - 4$
31. $-9 \leq \dfrac{2x + 3}{-4} \leq 7$
32. $-4 < \dfrac{2x - 2}{3} < 6$
33. $6 > \dfrac{3 - 3x}{12} > 2$
34. $6 > \dfrac{5 - 3x}{2} \geq -3$
35. $|3x + 4| < \frac{1}{2}$
36. $|1 - 2x| < \frac{1}{3}$
37. $|2x - 5| \geq 9$
38. $|3x + 1| \geq 10$

In Problems 39–48, find an equation of the line having the given characteristics. Express your answer using either the general form or the slope–intercept form of the equation of a line, whichever you prefer.

39. Slope $= -2$; containing the point $(3, -1)$
40. Slope $= 0$; containing the point $(-5, 4)$
41. Slope undefined; containing the point $(-3, 4)$
42. x-Intercept $= 2$; containing the point $(4, -5)$
43. y-Intercept $= -2$; containing the point $(5, -3)$
44. Containing the points $(3, -4)$ and $(2, 1)$
45. Parallel to the line $2x - 3y = -4$; containing the point $(-5, 3)$
46. Parallel to the line $x + y = 2$; containing the point $(1, -3)$

47. Perpendicular to the line $x + y = 2$; containing the point $(4, -3)$
48. Perpendicular to the line $3x - y = -4$; containing the point $(-2, 4)$

In Problems 49–54, graph each line by hand, labeling any intercepts.

49. $4x - 5y = -20$
50. $3x + 4y = 12$
51. $\frac{1}{2}x - \frac{1}{3}y = -\frac{1}{6}$
52. $-\frac{3}{4}x + \frac{1}{2}y = 0$
53. $\sqrt{2}x + \sqrt{3}y = \sqrt{6}$
54. $\frac{x}{3} + \frac{y}{4} = 1$

In Problems 55–62, list the x- and y-intercepts and test for symmetry.

55. $2x = 3y^2$
56. $y = 5x$
57. $4x^2 + y^2 = 1$
58. $x^2 - 9y^2 = 9$
59. $y = x^4 + 2x^2 + 1$
60. $y = x^3 - x$
61. $x^2 + x + y^2 + 2y = 0$
62. $x^2 + 4x + y^2 - 2y = 0$

In Problems 63–66, find the center and radius of each circle. Graph each circle by hand.

63. $x^2 + y^2 - 2x + 4y - 4 = 0$
64. $x^2 + y^2 + 4x - 4y - 1 = 0$
65. $3x^2 + 3y^2 - 6x + 12y = 0$
66. $2x^2 + 2y^2 - 4x = 0$

67. Find the slope of the line containing the points $(7, 4)$ and $(-3, 2)$. What is the distance between these points? What is their midpoint?
68. Find the slope of the line containing the points $(2, 5)$ and $(6, -3)$. What is the distance between these points? What is their midpoint?
69. Show that the points $A = (3, 4)$, $B = (1, 1)$, and $C = (-2, 3)$ are the vertices of an isosceles triangle.
70. Show that the points $A = (-2, 0)$, $B = (-4, 4)$, and $C = (8, 5)$ are the vertices of a right triangle in two ways:
    (a) By using the converse of the Pythagorean Theorem
    (b) By using the slopes of the lines joining the vertices
71. Show that the points $A = (2, 5)$, $B = (6, 1)$, and $C = (8, -1)$ lie on a line by using slopes.
72. Show that the points $A = (1, 5)$, $B = (2, 4)$, and $C = (-3, 5)$ lie on a circle with center $(-1, 2)$. What is the radius of this circle?
73. The endpoints of the diameter of a circle are $(-3, 2)$ and $(5, -6)$. Find the center and radius of the circle. Write the general equation of this circle.
74. Find two numbers y such that the distance from $(-3, 2)$ to $(5, y)$ is 10.

For Problems 75–78, (a) use a graphing utility to draw a scatter diagram, (b) use a graphing utility to find the line of best fit to the data, and (c) interpret the slope.

75.

x	3	4	5	6	7	8	9
y	3	5	6	8	10	11	13

76.

x	10	12	13	15	16	18	20
y	34	27	26	23	20	18	17

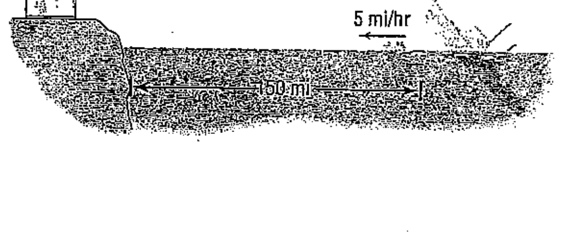
77.

x	100	110	125	130	140	145	150	160	170	175
y	300	340	365	380	400	410	425	430	450	460

78.

x	200	220	230	235	245	250	265	275	280	300
y	1000	990	975	960	955	940	935	920	910	895

79. **Lightning and Thunder** A flash of lightning is seen and the resulting thunderclap is heard 3 seconds later. If the speed of sound averages 1100 feet per second, how far away is the storm?
80. **Physics: Intensity of Light** The intensity I (in candlepower) of a certain light source obeys the equation $I = 900/x^2$, where x is the distance (in meters) from the light. Over what range of distances can an object be placed from this light source so that the range of intensity of light is from 1600 to 3600 candlepower, inclusive?
81. **Extent of Search and Rescue** A search plane has a cruising speed of 250 miles per hour and carries enough fuel for at most 5 hours of flying. If there is a wind that averages 30 miles per hour and the direction of search is with the wind one way and against it the other, how far can the search plane travel?
82. **Extent of Search and Rescue** If the search plane described in Problem 81 is able to add a supplementary fuel tank that allows for an additional 2 hours of flying, how much farther can the plane extend its search?
83. **Rescue at Sea** A life raft, set adrift from a sinking ship 150 miles offshore, travels directly toward a Coast Guard station at the rate of 5 miles per hour. At the time that the raft is set adrift, a rescue helicopter is dispatched from the Coast Guard station. If the helicopter's average speed is 90 miles per hour, how long will it take the helicopter to reach the life raft?



84. Physics: Uniform Motion Two bees leave two locations 150 meters apart and fly, without stopping, back and forth between these two locations at average speeds of 3 meters per second and 5 meters per second, respectively. How long is it until the bees meet for the first time? How long is it until they meet for the second time?

85. Working Together to Get a Job Done Clarissa and Shawna, working together, can paint the exterior of a house in 6 days. Clarissa by herself can complete this job in 5 days less than Shawna. How long will it take Clarissa to complete the job by herself?

86. Emptying a Tank Two pumps of different sizes, working together, can empty a fuel tank in 5 hours. The larger pump can empty this tank in 4 hours less than the smaller one. If the larger one is out of order, how long will it take the smaller one to do the job alone?

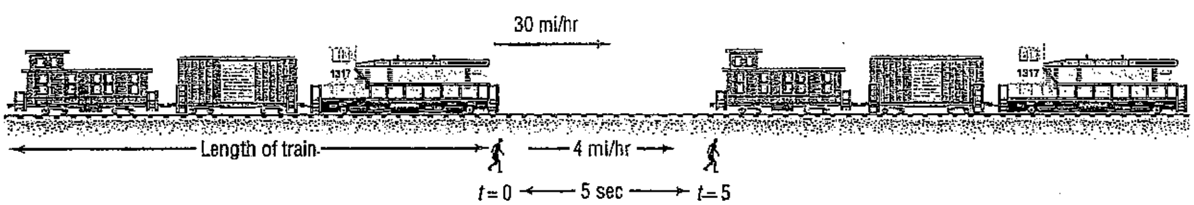
87. Chemistry: Mixing Acids For a certain experiment, a student requires 100 cubic centimeters of a solution that is 8% HCl. The storeroom has only solutions that are 15% HCl and 5% HCl. How many cubic centimeters of each available solution should be mixed to get 100 cubic centimeters of 8% HCl?

88. Chemistry: Salt Solutions How much water must be evaporated from 32 ounces of a 4% salt solution to make a 6% salt solution?

89. Business: Theater Attendance The manager of the Coral Theater wants to know whether the majority of its patrons is adults or children. During a week in July, 5200 tickets were sold and the receipts totaled $20,335. The adult admission is $4.75, and the children's admission is $2.50. How many adult patrons were there?

90. Business: Blending Coffee A coffee manufacturer wants to market a new blend of coffee that will cost $6 per pound by mixing two coffees that sell for $4.50 and $8 per pound, respectively. What amounts of each coffee should be blended to obtain the desired mixture?
[**Hint:** Assume that the total weight of the desired blend is 100 pounds.]

91. Physics: Uniform Motion Refer to the figure below. A man is walking at an average speed of 4 miles per hour alongside a railroad track. A freight train, going in the same direction at an average speed of 30 miles per hour, requires 5 seconds to pass the man. How long is the freight train? Give your answer in feet.



92. One formula stating the relationship between the length l and width w of a rectangle of "pleasing proportion" is $l^2 = w(l + w)$. How should a 4 foot by 8 foot sheet of plasterboard be cut so that the result is a rectangle of "pleasing proportion" with a width of 4 feet?

93. Business: Determining the Cost of a Charter A group of 20 senior citizens can charter a bus for a one-day excursion trip for $15 per person. The charter company agrees to reduce the price of each ticket by 10¢ for each additional passenger in excess of 20 who goes on the trip, up to a maximum of 44 passengers (the capacity of the bus). If the final bill from the charter company was $482.40, how many seniors went on the trip, and how much did each pay?

94. Time Required for Copying A new copying machine can do a certain job in 1 hour less than an older copier. Together they can do this job in 72 minutes. How long would it take the older copier by itself to do the job?

95. Relating Algebra and Calculus Scores The following data represent scores in an algebra achievement test and calculus achievement test for the same student. Treat the algebra test score as the independent variable and the calculus test score as the dependent variable.

Algebra Score	Calculus Score
17	73
21	66
11	64
16	61
15	70
11	71
24	90
27	68
19	84
8	52

(a) Use a graphing utility to draw a scatter diagram.
(b) Use a graphing utility to find the line of best fit to the data.

True/False Items

T F 1. Every relation is a function.
T F 2. Vertical lines intersect the graph of a function in no more than one point.
T F 3. The y-intercept of the graph of the function $y = f(x)$, whose domain is all real numbers, is $f(0)$.
T F 4. A function f is decreasing on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.
T F 5. Even functions have graphs that are symmetric with respect to the origin.
T F 6. The graph of $y = f(-x)$ is the reflection about the y-axis of the graph of $y = f(x)$.
T F 7. $f(g(x)) = f(x) \cdot g(x)$.
T F 8. The domain of the composite function $(f \circ g)(x)$ is the same as that of $g(x)$.

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

1. Given that f is a linear function, $f(4) = -5$ and $f(0) = 3$, write the equation that defines f.
2. Given that g is a linear function with slope $= -4$ and $g(-2) = 2$, write the equation that defines g.
3. A function f is defined by

$$f(x) = \frac{Ax + 5}{6x - 2}$$

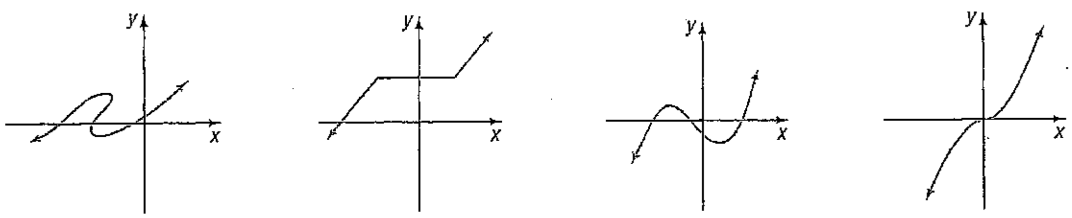
If $f(1) = 4$, find A.

4. A function g is defined by

$$g(x) = \frac{A}{x} + \frac{8}{x^2}$$

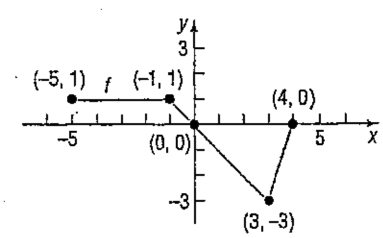
If $g(-1) = 0$, find A.

5. Tell which of the following graphs are graphs of functions.



(a) (b) (c) (d)

6. Use the graph of the function f shown to find:
   (a) The domain and range of f
   (b) $f(-1)$
   (c) The intercepts of f
   (d) The intervals on which f is increasing, decreasing, or constant
   (e) Whether the function is even, odd, or neither



In Problems 7–12, find the following for each function:

(a) $f(-x)$ *(b)* $-f(x)$ *(c)* $f(x + 2)$ *(d)* $f(x - 2)$ *(e)* $f(2x)$

7. $f(x) = \dfrac{3x}{x^2 - 4}$
8. $f(x) = \dfrac{x^2}{x + 2}$
9. $f(x) = \sqrt{x^2 - 4}$
10. $f(x) = |x^2 - 4|$
11. $f(x) = \dfrac{x^2 - 4}{x^2}$
12. $f(x) = \dfrac{x^3}{x^2 - 4}$

In Problems 13–20, find the domain of each function.

13. $f(x) = \dfrac{x}{x^2 - 9}$
14. $f(x) = \dfrac{3x^2}{x - 2}$
15. $f(x) = \sqrt{2 - x}$
16. $f(x) = \sqrt{x + 2}$
17. $h(x) = \dfrac{\sqrt{x}}{|x|}$
18. $g(x) = \dfrac{|x|}{x}$
19. $f(x) = \dfrac{x}{x^2 + 2x - 3}$
20. $F(x) = \dfrac{1}{x^2 - 3x - 4}$

In Problems 21–24:

(a) Find the domain of each function.
(b) Locate any intercepts.
(c) Graph each function by hand.
(d) Based on the graph, find the range.
(e) Verify your results using a graphing utility.

21. $f(x) = \begin{cases} 3x & -2 < x \le 1 \\ x + 1 & x > 1 \end{cases}$

22. $f(x) = \begin{cases} x - 1 & -3 < x < 0 \\ 3x - 1 & x \ge 0 \end{cases}$

23. $f(x) = \begin{cases} x & -4 \le x < 0 \\ 1 & x = 0 \\ 3x & x > 0 \end{cases}$

24. $f(x) = \begin{cases} x^2 & -2 \le x \le 2 \\ 2x - 1 & x > 2 \end{cases}$

In Problems 25–28, find the average rate of change from 2 to x for each function f. Be sure to simplify.

25. $f(x) = 2 - 5x$ **26.** $f(x) = 2x^2 + 7$ **27.** $f(x) = 3x - 4x^2$ **28.** $f(x) = x^2 - 3x + 2$

In Problems 29–36, determine (algebraically) whether the given function is even, odd, or neither.

29. $f(x) = x^3 - 4x$ **30.** $g(x) = \dfrac{4 + x^2}{1 + x^4}$ **31.** $h(x) = \dfrac{1}{x^4} + \dfrac{1}{x^2} + 1$ **32.** $F(x) = \sqrt{1 - x^3}$

33. $G(x) = 1 - x + x^3$ **34.** $H(x) = 1 + x + x^2$ **35.** $f(x) = \dfrac{x}{1 + x^2}$ **36.** $g(x) = \dfrac{1 + x^2}{x^3}$

In Problems 37–48, graph each function using the techniques of shifting, compressing or stretching, and reflections. Identify any intercepts on the graph. State the domain and, based on the graph, find the range.

37. $F(x) = |x| - 4$ **38.** $f(x) = |x| + 4$ **39.** $g(x) = -2|x|$ **40.** $g(x) = \frac{1}{2}|x|$

41. $h(x) = \sqrt{x - 1}$ **42.** $h(x) = \sqrt{x} - 1$ **43.** $f(x) = \sqrt{1 - x}$ **44.** $f(x) = -\sqrt{x + 3}$

45. $h(x) = (x - 1)^2 + 2$ **46.** $h(x) = (x + 2)^2 - 3$ **47.** $g(x) = 3(x - 1)^3 + 1$ **48.** $g(x) = -2(x + 2)^3 - 8$

In Problems 49–52, use a graphing utility to graph each function over the indicated interval. Approximate any local maxima and local minima. Determine where the function is increasing and where it is decreasing.

49. $f(x) = 2x^3 - 5x + 1 \quad (-3, 3)$

50. $f(x) = -x^3 + 3x - 5 \quad (-3, 3)$

51. $f(x) = 2x^4 - 5x^3 + 2x + 1 \quad (-2, 3)$

52. $f(x) = -x^4 + 3x^3 - 4x + 3 \quad (-2, 3)$

In Problems 53–58, for the given functions f and g find:

(a) $(f \circ g)(2)$ *(b)* $(g \circ f)(-2)$ *(c)* $(f \circ f)(4)$ *(d)* $(g \circ g)(-1)$

Verify your results using a graphing utility.

53. $f(x) = 3x - 5; \quad g(x) = 1 - 2x^2$

54. $f(x) = 4 - x; \quad g(x) = 1 + x^2$

55. $f(x) = \sqrt{x + 2}; \quad g(x) = 2x^2 + 1$

56. $f(x) = 1 - 3x^2; \quad g(x) = \sqrt{4 - x}$

57. $f(x) = \dfrac{1}{x^2 + 4}; \quad g(x) = 3x - 2$

58. $f(x) = \dfrac{2}{1 + 2x^2}; \quad g(x) = 3x$

In Problems 59–64, find $f \circ g$, $g \circ f$, $f \circ f$, *and* $g \circ g$ *for each pair of functions. State the domain of each.*

59. $f(x) = 2 - x; \quad g(x) = 3x + 1$

60. $f(x) = 2x - 1; \quad g(x) = 2x + 1$

61. $f(x) = 3x^2 + x + 1; \quad g(x) = |3x|$

62. $f(x) = \sqrt{3x}; \quad g(x) = 1 + x + x^2$

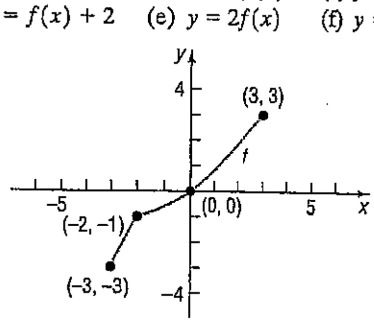
63. $f(x) = \dfrac{x + 1}{x - 1}; \quad g(x) = \dfrac{1}{x}$

64. $f(x) = \sqrt{x - 3}; \quad g(x) = \dfrac{3}{x}$

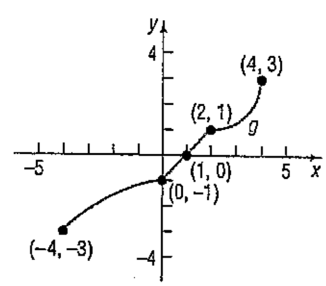
65. For the following graph of the function f draw the graph of:

(a) $y = f(-x)$ (b) $y = -f(x)$ (c) $y = f(x + 2)$

(d) $y = f(x) + 2$ (e) $y = 2f(x)$ (f) $y = f(3x)$



66. Repeat Problem 65 for the following graph of the function g.



Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–10, (a) graph each quadratic function by hand by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any; (b) verify your results using a graphing utility.

1. $f(x) = \frac{1}{4}x^2 - 16$ **2.** $f(x) = -\frac{1}{2}x^2 - 2$ **3.** $f(x) = -4x^2 + 4x$

4. $f(x) = 9x^2 - 6x + 3$ **5.** $f(x) = \frac{9}{2}x^2 + 3x + 1$ **6.** $f(x) = -x^2 + x + \frac{1}{2}$

7. $f(x) = 3x^2 - 4x - 1$ **8.** $f(x) = -2x^2 - x + 4$ **9.** $f(x) = x^2 - 4x + 6$

10. $f(x) = x^2 + 2x - 3$

In Problems 11–16, graph each function using transformations (shifting, compressing, stretching, and reflection). Show all the stages. Verify your results using a graphing utility.

11. $f(x) = (x + 2)^3$ **12.** $f(x) = -x^3 + 3$ **13.** $f(x) = -(x - 1)^4$

14. $f(x) = (x - 1)^4 - 2$ **15.** $f(x) = (x - 1)^4 + 2$ **16.** $f(x) = (1 - x)^3$

In Problems 17–22, determine whether the given quadratic function has a maximum value or a minimum value, and then find the value.

17. $f(x) = 3x^2 - 6x + 4$ **18.** $f(x) = 2x^2 + 8x + 5$ **19.** $f(x) = -x^2 + 8x - 4$

20. $f(x) = -x^2 - 10x - 3$ **21.** $f(x) = -3x^2 + 12x + 4$ **22.** $f(x) = -2x^2 + 4$

In Problems 23–30, for each polynomial function f:

(a) Using a graphing utility, graph f.
(b) Find the x- and y-intercepts.
(c) Determine whether each x-intercept is of odd or even multiplicity.
(d) Find the power function that the graph of f resembles for large values of $|x|$.
(e) Determine the number of turning points on the graph of f.
(f) Determine the local maxima and local minima, if any exist, rounded to two decimal places.

23. $f(x) = x(x + 2)(x + 4)$ **24.** $f(x) = x(x - 2)(x - 4)$ **25.** $f(x) = (x - 2)^2(x + 4)$

26. $f(x) = (x - 2)(x + 4)^2$ **27.** $f(x) = x^3 - 4x^2$ **28.** $f(x) = x^3 + 4x$

29. $f(x) = (x - 1)^2(x + 3)(x + 1)$ **30.** $f(x) = (x - 4)(x + 2)^2(x - 2)$

In Problems 31–42, discuss each rational function following the seven steps on page 220.

31. $R(x) = \frac{2x - 6}{x}$ **32.** $R(x) = \frac{4 - x}{x}$ **33.** $H(x) = \frac{x + 2}{x(x - 2)}$

34. $H(x) = \frac{x}{x^2 - 1}$ **35.** $R(x) = \frac{x^2 + x - 6}{x^2 - x - 6}$ **36.** $R(x) = \frac{x^2 - 6x + 9}{x^2}$

37. $F(x) = \frac{x^3}{x^2 - 4}$ **38.** $F(x) = \frac{3x^3}{(x - 1)^2}$ **39.** $R(x) = \frac{2x^4}{(x - 1)^2}$

40. $R(x) = \frac{x^4}{x^2 - 9}$ **41.** $G(x) = \frac{x^2 - 4}{x^2 - x - 2}$ **42.** $F(x) = \frac{(x - 1)^2}{x^2 - 1}$

43. Landscaping A landscape engineer has 200 feet of border to enclose a rectangular pond. What dimensions will result in the largest pond?

44. Geometry Find the length and width of a rectangle whose perimeter is 20 feet and whose area is 16 square feet.

45. A rectangle has one vertex on the line $y = 10 - x$, $x > 0$, another at the origin, one on the positive x-axis, and one on the positive y-axis. Find the largest area A that can be enclosed by the rectangle.

46. Minimizing Cost Callaway Golf Company has determined that the daily per unit cost C of manufacturing x additional Big Bertha-type golf clubs may be expressed by the quadratic function

$$C(x) = 5x^2 - 620x + 20{,}000$$

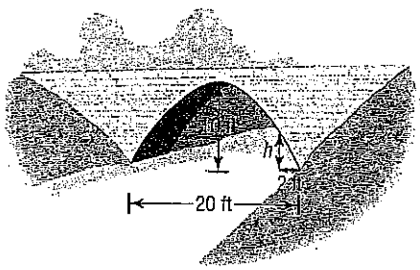
(a) How many clubs should be manufactured to minimize the additional cost per club?
(b) At this level of production, what is the additional cost per club?

47. **Minimizing Cost** Scott–Jones Publishing Company has found that the per unit cost C for paper, printing, and binding of x additional textbooks is given by the quadratic function

$$C(x) = 0.003x^2 - 30x + 111{,}800$$

(a) How many books should be manufactured for the additional cost per text to be a minimum?
(b) At this level of production, what is the additional cost per text?

48. **Parabolic Arch Bridge** A horizontal bridge is in the shape of a parabolic arch. Given the information shown in the figure, what is the height h of the arch 2 feet from shore?



49. **Life Cycle Hypothesis** An individual's income varies with his or her age. The following table shows the median income I of individuals of different age groups within the United States for 1997. For each age group, the midpoint represents the independent variable, x. For the age group "65 years and older," we assume that the midpoint is 69.5.

Age	Midpoint, x	Median Income, I
15–24 years	19.5	\$22,583
25–34 years	29.5	\$38,174
35–44 years	39.5	\$46,359
45–54 years	49.5	\$51,875
55–64 years	59.5	\$41,356
65 years and older	69.5	\$20,761

urce: U.S. Census Bureau

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) Find the quadratic function of best fit.
(c) Graph the quadratic function of best fit on the scatter diagram.
(d) Using the function found in part (b), determine the age at which an individual can expect to earn the most income.
(e) Predict the peak income earned.
(f) Compare your results in parts (d) and (e) to the data and comment.
(g) Compare the results obtained in 1997 to those of 1995 and 1996 found in Problems 73 and 74 in Section 3.1.

50. **Relating the Length and Period of a Pendulum** Tom constructs simple pendulums with different lengths, l, and uses a light probe to record the corresponding period, T. The following data are collected:

Length l (in feet)	Period T (in seconds)
0.5	0.79
1.2	1.20
1.8	1.51
2.3	1.65
3.7	2.14
4.9	2.43

(a) Using a graphing utility, draw a scatter diagram of the data with length as the independent variable and period as the dependent variable.
(b) Find the power function of best fit.
(c) Graph this power function on the scatter diagram.
(d) Use the function $T(l)$ to predict the period of a pendulum whose length is 4 feet.

51. **AIDS Cases in the United States** The following data represent the cumulative number of reported AIDS cases in the United States for 1990–1997.

Year, t	Number of AIDS Cases, A
1990, 1	193,878
1991, 2	251,638
1992, 3	326,648
1993, 4	399,613
1994, 5	457,280
1995, 6	528,215
1996, 7	594,760
1997, 8	653,253

Source: U.S. Center for Disease Control and Prevention

(a) Using a graphing utility, draw a scatter diagram of the data, treating year as the independent variable.
(b) Find the cubic function of best fit.
(c) Graph the cubic function of best fit on the scatter diagram given in part (b).
(d) Use the function $A(t)$ to predict the cumulative number of AIDS cases reported in the United States in 2000.
(e) Use the function $A(t)$ given in part (b) to predict the year in which the cumulative number of AIDS cases reported in the United States reaches 850,000.
(f) Do you think that the function given in part (b) will be useful in predicting the number of AIDS cases in 2010?

68. Business The monthly revenue achieved by selling x boxes of candy is figured to be $x(5 - 0.05x)$ dollars. The wholesale cost of each box of candy is \$1.50.
(a) How many boxes must be sold each month to achieve a profit of at least \$60?
(b) Using a graphing utility, graph the revenue function.
(c) What is the maximum revenue that this firm could earn?
(d) How many boxes of candy should the firm sell to maximize revenue?
(e) Using a graphing utility, graph the profit function.
(f) What is the maximum profit that this firm can earn?
(g) How many boxes of candy should the firm sell to maximize profit?
(h) Provide a reasonable explanation as to why the answers found in parts (d) and (g) differ. Is the shape of the revenue function reasonable in your opinion? Why?

69. Cost of Manufacturing In Problem 67 of Section 3.3, a cubic function of best fit relating the cost C of manufacturing x Chevy Cavaliers in a day was found. Budget constraints will not allow Chevy to spend more than \$97,000 per day. Determine the number of Cavaliers that could be produced in a day.

70. Cost of Printing In Problem 64 of Section 3.3, a cubic function of best fit relating the cost C of printing x textbooks in a week was found. Budget constraints will not allow the printer to spend more than \$170,000 per week. Determine the number of textbooks that could be printed in a week.

71. Minimum Sales Requirements Marissa is thinking of leaving her \$1000 a week job and buying a computer resale shop. According to the financial records of the firm, the profits (in dollars) of the company for different amounts of computers sold and the corresponding profits are as follows:

Number of Computers Sold, x	Profit, p
0	−1500
4	−522
7	54
12	775
18	1184
23	1132
29	653

(a) Using a graphing utility, draw a scatter diagram of the data with the number of computers sold as the independent variable.
(b) Find the quadratic function of best fit using a graphing utility.
(c) Using the function found in part (b), determine the number of computers that Marissa must sell in order for the profits to exceed \$1000 per week and therefore to make it worthwhile for her to quit her job.
(d) Using the function found in part (b), determine the number of computers that Marissa should sell in order to maximize profits.
(e) Using the function found in part (b), determine the maximum profit that Marissa can expect to earn.

72. Minimum Sales Requirements Barry is considering the purchase of a gas station. According to the financial records of the gas station, its monthly sales (in thousands of gallons of gasoline) and the corresponding profits are as follows:

Thousands of Gallons of Gasoline, x	Profit, p
50	3947
54	4214
74	4942
92	4838
82	5003
75	4965
100	4521
88	4933
63	4665

(a) Using a graphing utility, draw a scatter diagram of the data with the number of gallons of gasoline sold as the independent variable.
(b) Find the quadratic function of best fit using a graphing utility.
(c) Using the function found in part (b), determine the number of gallons of gasoline that Barry must sell in order for the profits to exceed \$4000 a month and therefore to make it worthwhile for him to quit his job.
(d) Using the function found in part (b), determine the number of gallons of gasoline that Barry should sell in order to maximize profits.
(e) Using the function found in part (b), determine the maximum profit Barry can expect to earn.

73. Prove that if a, b are real numbers and $a \geq 0, b \geq 0$, then

$$a \leq b \quad \text{is equivalent to} \quad \sqrt{a} \leq \sqrt{b}$$

[**Hint:** $b - a = (\sqrt{b} - \sqrt{a})(\sqrt{b} + \sqrt{a})$.]

74. Make up an inequality that has no real solution. Make up one that has exactly one real solution.

75. The inequality $x^2 + 1 < -5$ has no real solution. Explain why.

CHAPTER REVIEW

Things To Know

Zeros of a polynomial f (p. 238) — Numbers for which $f(x) = 0$; these are the x-intercepts of the graph of f.

Remainder Theorem (p. 240) — If a polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

Factor Theorem (p. 240) — $x - c$ is a factor of a polynomial $f(x)$ if and only if $f(c) = 0$.

Descartes' Rule of Signs (p. 242) — Let f denote a polynomial function. The number of positive zeros of f either equals the number of variations in sign of the nonzero coefficients of $f(x)$ or else equals that number less some even integer. The number of negative zeros of f either equals the number of variations in sign of the nonzero coefficients of $f(-x)$ or else equals that number less some even integer.

Rational Zeros Theorem (p. 243) — Let f be a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0, \quad a_n \neq 0, a_0 \neq 0$$

where each coefficient is an integer. If p/q, in lowest terms, is a rational zero of f, then p must be a factor of a_0 and q must be a factor of a_n.

Quadratic equation and quadratic formula (p. 259) — If $ax^2 + bx + c = 0$, $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Discriminant (p. 260) — If $b^2 - 4ac > 0$, there are two distinct real solutions.
If $b^2 - 4ac = 0$, there is one repeated real solution.
If $b^2 - 4ac < 0$, there are two distinct complex solutions that are not real; the solutions are conjugates of each other.

Fundamental Theorem of Algebra (p. 262) — Every complex polynomial function $f(x)$ of degree $n \geq 1$ has at least one complex zero.

Conjugate Pairs Theorem (p. 263) — Let $f(x)$ be a polynomial whose coefficients are real numbers. If $r = a + bi$ is a zero of f, then its complex conjugate $\bar{r} = a - bi$ is also a zero of f.

How To

Use the Remainder and Factor Theorems (p. 239)

Use Descartes' Rule of Signs (p. 241)

Use the Rational Zeros Theorem (p. 243)

Find the real zeros of a polynomial function (p. 244)

Solve polynomial equations (p. 245)

Use the Theorem for Bounds on Zeros (p. 246)

Use the Intermediate Value Theorem (p. 249)

Add, subtract, multiply and divide complex numbers (p. 253)

Solve quadratic equations with a negative discriminant (p. 258)

Utilize the conjugate pairs theorem (p. 263)

Find a polynomial function with specified zeros (p. 264)

Find the complex zeros of a polynomial (p. 265)

Solve polynomial inequalities graphically and algebraically (p. 267)

Solve rational inequalities graphically and algebraically (p. 270)

Fill-in-the-Blank Items

1. In the process of polynomial division, (Divisor)(Quotient) + ________ = ________.
2. When a polynomial function f is divided by $x - c$, the remainder is ________.
3. A polynomial function f has the factor $x - c$ if and only if ________.
4. The polynomial function $f(x) = x^5 - 2x^3 + x^2 - x + 1$ has at most ________ real zeros.
5. The possible rational zeros of $f(x) = 2x^5 - x^3 + x^2 - x + 1$ are ________.
6. In the complex number $5 + 2i$, the number 5 is called the ________ part; the number 2 is called the ________ part; the number i is called the ________.
7. If $3 + 4i$ is a zero of a polynomial of degree 5 with real coefficients, then so is ________.
8. The equation $|x^2| = 4$ has four solutions: ________, ________, ________, and ________.
9. If a function f whose domain is all real numbers is even and 4 is a zero of f, then ________ is also a zero.

True/False Items

T F 1. Every polynomial of degree 3 with real coefficients has exactly three real zeros.

T F 2. If $2 - 3i$ is a zero of a polynomial with real coefficients, then so is $-2 + 3i$.

T F 3. If f is a polynomial function of degree 4 and if $f(2) = 5$, then

$$\frac{f(x)}{x-2} = p(x) + \frac{5}{x-2}$$

where $p(x)$ is a polynomial of degree 3.

T F 4. The conjugate of $2 + 5i$ is $-2 - 5i$.

T F 5. A polynomial of degree n with real coefficients has exactly n complex zeros. At most n of them are real numbers.

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1 and 2, use Descartes' Rule of Signs to determine how many positive and negative zeros each polynomial function may have. Do not attempt to find the zeros.

1. $f(x) = 12x^8 - x^7 + 8x^4 - 2x^3 + x + 3$

2. $f(x) = -6x^5 + x^4 + 5x^3 + x + 1$

3. List all the potential rational zeros at $f(x) = 12x^8 - x^7 + 6x^4 - x^3 + x - 3$.

4. List all the potential rational zeros of $f(x) = -6x^5 + x^4 + 2x^3 - x + 1$.

In Problems 5–10, follow the steps on page 248 to find all the real zeros of each polynomial function.

5. $f(x) = x^3 - 3x^2 - 6x + 8$

6. $f(x) = x^3 - x^2 - 10x - 8$

7. $f(x) = 4x^3 + 4x^2 - 7x + 2$

8. $f(x) = 4x^3 - 4x^2 - 7x - 2$

9. $f(x) = x^4 - 4x^3 + 9x^2 - 20x + 20$

10. $f(x) = x^4 + 6x^3 + 11x^2 + 12x + 18$

In Problems 11–16, determine the real zeros of the polynomial function. Approximate all irrational zeros rounded to two decimal places.

11. $f(x) = 2x^3 - 11.84x^2 - 9.116x + 82.46$

12. $f(x) = 12x^3 + 39.8x^2 - 4.4x - 3.4$

13. $g(x) = 15x^4 - 21.5x^3 - 1718.3x^2 + 5308x + 3796.8$

14. $g(x) = 3x^4 + 67.93x^3 + 486.265x^2 + 1121.32x + 412.195$

15. $f(x) = 3x^3 + 18.02x^2 + 11.0467x - 53.8756$

16. $f(x) = x^3 - 3.16x^2 - 39.4611x + 151.638$

In Problems 17–20, find the real solutions of each equation.

17. $2x^4 + 2x^3 - 11x^2 + x - 6 = 0$

18. $3x^4 + 3x^3 - 17x^2 + x - 6 = 0$

19. $2x^4 + 7x^3 + x^2 - 7x - 3 = 0$

20. $2x^4 + 7x^3 - 5x^2 - 28x - 12 = 0$

In Problems 21–24, find bounds to the zeros of each polynomial function. Obtain a complete graph of f.

21. $f(x) = x^3 - x^2 - 4x + 2$

22. $f(x) = x^3 + x^2 - 10x - 5$

23. $f(x) = 2x^3 - 7x^2 - 10x + 35$

24. $f(x) = 3x^3 - 7x^2 - 6x + 14$

In Problems 25–28, use the Intermediate Value Theorem to show that each polynomial has a zero in the given interval. Approximate the zero rounded to two decimal places.

25. $f(x) = 3x^3 - x - 1$; $[0, 1]$

26. $f(x) = 2x^3 - x^2 - 3$; $[1, 2]$

27. $f(x) = 8x^4 - 4x^3 - 2x - 1$; $[0, 1]$

28. $f(x) = 3x^4 + 4x^3 - 8x - 2$; $[1, 2]$

In Problems 29–38, write each expression in the standard form $a + bi$. Verify your results using a graphing utility.

29. $(6 + 3i) - (2 - 4i)$

30. $(8 - 3i) + (-6 + 2i)$

31. $4(3 - i) + 3(-5 + 2i)$

32. $2(1 + i) - 3(2 - 3i)$

33. $\dfrac{3}{3 + i}$

34. $\dfrac{4}{2 - i}$

35. i^{50}

36. i^{29}

37. $(2 + 3i)^3$

38. $(3 - 2i)^3$

In Problems 39–42, information is given about a complex polynomial $f(x)$ whose coefficients are real numbers. Find the remaining zeros of f.

39. Degree 3; zeros: $4 + i, 6$

40. Degree 3; zeros: $3 + 4i, 5$

41. Degree 4; zeros: $i, 1 + i$

42. Degree 4; zeros: $1, 2, 1 + i$

In Problems 43–56, solve each equation in the complex number system.

43. $x^2 + x + 1 = 0$
44. $x^2 - x + 1 = 0$
45. $2x^2 + x - 2 = 0$
46. $3x^2 - 2x - 1 = 0$
47. $x^2 + 3 = x$
48. $2x^2 + 1 = 2x$
49. $x(1 - x) = 6$
50. $x(1 + x) = 2$
51. $x^4 + 2x^2 - 8 = 0$
52. $x^4 + 8x^2 - 9 = 0$
53. $x^3 - x^2 - 8x + 12 = 0$
54. $x^3 - 3x^2 - 4x + 12 = 0$
55. $3x^4 - 4x^3 + 4x^2 - 4x + 1 = 0$
56. $x^4 + 4x^3 + 2x^2 - 8x - 8 = 0$

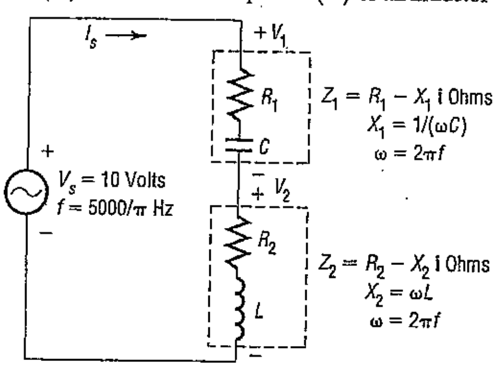
In Problems 57–66, solve each inequality (a) graphically and (b) algebraically.

57. $2x^2 + 5x - 12 < 0$
58. $3x^2 - 2x - 1 \geq 0$
59. $\dfrac{6}{x + 3} \geq 1$
60. $\dfrac{-2}{1 - 3x} < 1$
61. $\dfrac{2x - 6}{1 - x} < 2$
62. $\dfrac{3 - 2x}{2x + 5} \geq 2$
63. $\dfrac{(x - 2)(x - 1)}{x - 3} > 0$
64. $\dfrac{x + 1}{x(x - 5)} \leq 0$
65. $\dfrac{x^2 - 8x + 12}{x^2 - 16} > 0$
66. $\dfrac{x(x^2 + x - 2)}{x^2 + 9x + 20} \leq 0$

PROJECT AT MOTOROLA

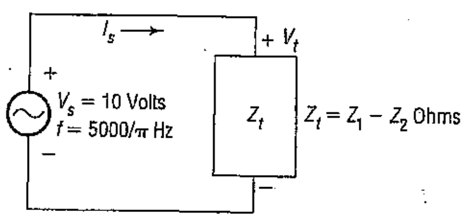
Alternating Current (AC) Circuit Analysis

AC circuit analysis techniques are used on electrical circuits that are energized with sinusoidal sources. A common example of an AC source is your electric power at home. Its effective amplitude is 120 volts and its frequency (f) is 60 cycles/second (60 Hz). The figure below is an electric circuit with sinusoidal voltage source V_s with amplitude 10 volts and frequency $5000/\pi$ Hz. It is connected to two complex load "impedances" $Z_1 = R_1 - X_1 i$ and $Z_2 = R_2 + X_2 i$ that are connected in "series" with each other. Each load contains a resistor (R) connected to a capacitor (C) or an inductor (L).



Resistors are electronic components designed to resist or limit electric current flow. The greater the resistance the more effectively it limits current. Electric power is lost or dissipated as heat in resistors. Capacitors and inductors limit current also and their ability to do this is measured as "reactance". Electric power is not lost but is stored in ideal capacitors and inductors. The capacitive and inductive reactance values X_1 and X_2 are given by $X_1 = 1/(\omega C)$ and $X_2 = \omega L$, where $\omega = 2\pi f$, C is capacitance in Farads (F), and L is inductance in Henrys (H). Reactance is unique to resistance in that it is dependent upon the frequency of operation.

1. Given resistors R_1 and R_2 are each 5 Ohms, capacitor C is F, and inductor L is 0.0015 H calculate X_1 and X_2 for the given frequency of operation. Write out Z_1 and Z_2 in standard complex form. Impedance, Z_t, is formed by adding Z_1 and Z_2 as shown in the figure below. What is Z_t? What two circuit elements (resistors, capacitors, inductors) do you think Z_t contains?



2. The current flow (I_s) is sinusoidal and its frequency is the same as the voltage source. Calculate I_s using the formula $I_s = V_s/Z_t$ (Amps).
3. As I_s flows through Z_1 a voltage drop V_1 is created as given by $V_1 = I_s Z_1$. Calculate the voltage across Z_1.
4. Find the voltage drop V_2 across $Z_2 (V_2 = I_s Z_2)$.
5. Does $V_s = V_1 + V_2$? Do you think this is reasonable?
6. Find the voltage drop V_t across $Z_t (V_t = I_s Z_t)$. Does $V_t = V_1 + V_2$? Is this a reasonable result?
7. The power dissipated in load Z_1 in Watts is given by $P_1 = (1/2)R_e\{V_1 I_{s^*}\}$ where I_{s^*} is the conjugate of I_s and $R_e\{V_1 I_{s^*}\}$ is the real part of $V_1 I_{s^*}$. Calculate P_1.
8. The power dissipated in load Z_2 in Watts is given by $P_2 = (1/2)R_e\{V_2 I_{s^*}\}$. Calculate P_2.
9. Calculate P_t the power dissipated in load Z_t.
10. Does $P_t = P_1 + P_2$? Do you think this is reasonable?
11. What do you think will happen to Z_t, I_s, and P_t as frequency goes to zero? To infinity?

CHAPTER REVIEW

Things To Know

One-to-one function f (p. 282) — If $f(x_1) = f(x_2)$, then $x_1 = x_2$ for any choice of x_1 and x_2 in the domain.

Horizontal-line test (p. 282) — If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

Inverse function f^{-1} **of** f (p. 283) — Domain of f = Range of f^{-1}; Range of f = Domain of f^{-1}.
$f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.
Graphs of f and f^{-1} are symmetric with respect to the line $y = x$.

Properties of the exponential function (p. 294–295 and 296)

$f(x) = a^x, \quad a > 1$ — Domain: $(-\infty, \infty)$; Range: $(0, \infty)$; x-intercepts: none; y-intercept: 1; horizontal asymptote: x-axis as $x \to -\infty$; increasing; one-to-one; smooth; continuous
See Figure 13 for a typical graph.

$f(x) = a^x, \quad 0 < a < 1$ — Domain: $(-\infty, \infty)$; Range: $(0, \infty)$; x-intercepts: none; y-intercept: 1; horizontal asymptote: x-axis as $x \to \infty$; decreasing; one-to-one; smooth; continuous
See Figure 17 for a typical graph.

Number e (p. 297) — Value approached by the expression $\left(1 + \frac{1}{n}\right)^n$ as $n \to \infty$;
that is, $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e$

Properties of the logarithmic function (p. 308)

$f(x) = \log_a x, \quad a > 1$ $(y = \log_a x$ means $x = a^y)$ — Domain: $(0, \infty)$; Range: $(-\infty, \infty)$; x-intercept: 1; y-intercept: none; vertical asymptote: $x = 0$ (y-axis); increasing; one-to-one; smooth; continuous
See Figure 26(b) for a typical graph.

$f(x) = \log_a x, 0 < a < 1$ $(y = \log_a x$ means $x = a^y)$ — Domain: $(0, \infty)$; Range: $(-\infty, \infty)$; x-intercept: 1; y-intercept: none; vertical asymptote: $x = 0$ (y-axis); decreasing; one-to-one; smooth; continuous
See Figure 26(a) for a typical graph.

Natural logarithm (p. 308) — $y = \ln x$ means $x = e^y$.

Properties of logarithms (p. 316–317)

$\log_a 1 = 0 \qquad \log_a a = 1 \qquad a^{\log_a M} = M \qquad \log_a a^r = r$

$\log_a(MN) = \log_a M + \log_a N \qquad \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

$\log_a\left(\frac{1}{N}\right) = -\log_a N \qquad \log_a M^r = r\log_a M$

Formulas

Change-of-Base Formula (p. 320)	$\log_a M = \dfrac{\log_b M}{\log_b a}$
Compound interest (p. 330)	$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$
Continuous compounding (p. 332)	$A = Pe^{rt}$
Present value (p. 333)	$P = A\left(1 + \dfrac{r}{n}\right)^{-nt}$ or $P = Ae^{-rt}$
Growth and decay (p. 339)	$A(t) = A_0 e^{kt}$
Logistic growth (p. 344)	$P(t) = \dfrac{c}{1 + ae^{-bt}}$

T F 5. The present value of \$1000 to be received after 2 years at 10% per annum compounded continuously is approximately \$1205.

T F 6. If $y = \log_a x$, then $y = a^x$.

T F 7. The graph of every logarithmic function $f(x) = \log_a x$, $a > 0$, $a \neq 1$, will contain the points $(1, 0)$ and $(a, 1)$.

T F 8. $a^{\log_M a} = M$, where $a > 0$, $a \neq 1$, $M > 0$.

T F 9. $\log_a(M + N) = \log_a M + \log_a N$, where $a > 0$, $a \neq 1$, $M > 0$, $N > 0$.

T F 10. $\log_a M - \log_a N = \log_a(M/N)$, where $a > 0$, $a \neq 1$, $M > 0$, $N > 0$.

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–6, the function f is one-to-one. Find the inverse of each function and check your answer. Find the domain and range of f and f^{-1}. Use a graphing utility to simultaneously graph f, f^{-1}, and $y = x$ on the same square screen.

1. $f(x) = \dfrac{2x + 3}{5x - 2}$
2. $f(x) = \dfrac{2 - x}{3 + x}$
3. $f(x) = \dfrac{1}{x - 1}$
4. $f(x) = \sqrt{x - 2}$
5. $f(x) = \dfrac{3}{x^{1/3}}$
6. $f(x) = x^{1/3} + 1$

In Problems 7–12, evaluate each expression. Do not use a graphing utility.

7. $\log_2\left(\frac{1}{8}\right)$
8. $\log_3 81$
9. $\ln e^{\sqrt{2}}$
10. $e^{\ln 0.1}$
11. $2^{\log_2 0.4}$
12. $\log_2 2^{\sqrt{3}}$

In Problems 13–18, write each expression as the sum and/or difference of logarithms. Express powers as factors.

13. $\log_3\left(\dfrac{uv^2}{w}\right)$
14. $\log_2(a^2\sqrt{b})^4$
15. $\log(x^2\sqrt{x^3 + 1})$
16. $\log_5\left(\dfrac{x^2 + 2x + 1}{x^2}\right)$
17. $\ln\left(\dfrac{x\sqrt[3]{x^2 + 1}}{x - 3}\right)$
18. $\ln\left(\dfrac{2x + 3}{x^2 - 3x + 2}\right)^2$

In Problems 19–24, write each expression as a single logarithm.

19. $3\log_4 x^2 + \dfrac{1}{2}\log_4 \sqrt{x}$
20. $-2\log_3\left(\dfrac{1}{x}\right) + \dfrac{1}{3}\log_3 \sqrt{x}$
21. $\ln\left(\dfrac{x - 1}{x}\right) + \ln\left(\dfrac{x}{x + 1}\right) - \ln(x^2 - 1)$
22. $\log(x^2 - 9) - \log(x^2 + 7x + 12)$
23. $2\log 2 + 3\log x - \dfrac{1}{2}[\log(x + 3) + \log(x - 2)]$
24. $\dfrac{1}{2}\ln(x^2 + 1) - 4\ln\dfrac{1}{2} - \dfrac{1}{2}[\ln(x - 4) + \ln x]$

In Problems 25 and 26, use the Change-of-Base Formula and a calculator to evaluate each logarithm. Round your answer to three decimal places.

25. $\log_4 19$
26. $\log_2 21$

In Problems 27–32, find y as a function of x. The constant C is a positive number.

27. $\ln y = 2x^2 + \ln C$
28. $\ln(y - 3) = \ln 2x^2 + \ln C$
29. $\ln(y - 3) + \ln(y + 3) = x + C$
30. $\ln(y - 1) + \ln(y + 1) = -x + C$
31. $e^{y + C} = x^2 + 4$
32. $e^{3y - C} = (x + 4)^2$

In Problems 33–42, use transformations to graph each function. Determine the domain, range, and any asymptotes. Verify your results using a graphing utility.

33. $f(x) = 2^{x-3}$
34. $f(x) = -2^x + 3$
35. $f(x) = \dfrac{1}{2}(3^{-x})$
36. $f(x) = 1 + 3^{2x}$

37. $f(x) = 1 - e^x$

38. $f(x) = 3 + \ln x$

39. $f(x) = 3e^x$

40. $f(x) = \frac{1}{2}\ln x$

41. $f(x) = 3 - e^{-x}$

42. $f(x) = 4 - \ln(-x)$

In Problems 43–62, solve each equation. Verify your result using a graphing utility.

43. $4^{1-2x} = 2$

44. $8^{6+3x} = 4$

45. $3^{x^2+x} = \sqrt{3}$

46. $4^{x-x^2} = \frac{1}{2}$

47. $\log_x 64 = -3$

48. $\log_{\sqrt{2}} x = -6$

49. $5^x = 3^{x+2}$

50. $5^{x+2} = 7^{x-2}$

51. $9^{2x} = 27^{3x-4}$

52. $25^{2x} = 5^{x^2-12}$

53. $\log_3 \sqrt{x-2} = 2$

54. $2^{x+1} \cdot 8^{-x} = 4$

55. $8 = 4^{x^2} \cdot 2^{5x}$

56. $2^x \cdot 5 = 10^x$

57. $\log_6(x + 3) + \log_6(x + 4) = 1$

58. $\log_{10}(7x - 12) = 2\log_{10} x$

59. $e^{1-x} = 5$

60. $e^{1-2x} = 4$

61. $2^{3x} = 3^{2x+1}$

62. $2^{x^3} = 3^{x^2}$

In Problems 63 and 64, use the following result: If x is the atmospheric pressure (measured in millimeters of mercury), then the formula for the altitude $h(x)$ (measured in meters above sea level) is

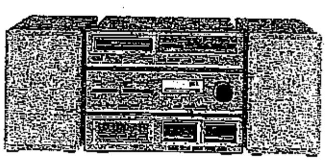
$$h(x) = (30T + 8000)\log\left(\frac{P_0}{x}\right)$$

where T is the temperature (in degrees Celsius) and P_0 is the atmospheric pressure at sea level, which is approximately 760 millimeters of mercury.

63. Finding the Altitude of an Airplane At what height is a Piper Cub whose instruments record an outside temperature of 0°C and a barometric pressure of 300 millimeters of mercury?

64. Finding the Height of a Mountain How high is a mountain if instruments placed on its peak record a temperature of 5°C and a barometric pressure of 500 millimeters of mercury?

65. Amplifying Sound An amplifier's power output P (in watts) is related to its decibel voltage gain d by the formula $P = 25e^{0.1d}$.



(a) Find the power output for a decibel voltage gain of 4 decibels.

(b) For a power output of 50 watts, what is the decibel voltage gain?

66. Limiting Magnitude of a Telescope A telescope is limited in its usefulness by the brightness of the star it is aimed at and by the diameter of its lens. One measure of a star's brightness is its *magnitude*: the dimmer the star, the larger its magnitude. A formula for the limiting magnitude L of a telescope, that is, the magnitude of the dimmest star that it can be used to view, is given by

$$L = 9 + 5.1\log d$$

where d is the diameter (in inches) of the lens.

(a) What is the limiting magnitude of a 3.5-inch telescope?

(b) What diameter is required to view a star of magnitude 14?

67. Salvage Value The number of years n for a piece of machinery to depreciate to a known salvage value can be found using the formula

$$n = \frac{\log s - \log i}{\log(1 - d)}$$

where s is the salvage value of the machinery, i is its initial value, and d is the annual rate of depreciation.

(a) How many years will it take for a piece of machinery to decline in value from \$90,000 to \$10,000 if the annual rate of depreciation is 0.20 (20%)?

(b) How many years will it take for a piece of machinery to lose half of its value if the annual rate of depreciation is 15%?

68. Funding a College Education A child's grandparents purchase a \$10,000 bond fund that matures in 18 years to be used for her college education. The bond fund pays 4% interest compounded semiannually. How much will the bond fund be worth at maturity?

69. Funding a College Education A child's grandparents wish to purchase a bond fund that matures in 18 years to be used for her college education. The bond fund pays 4% interest compounded semiannually. How much should they purchase so that the bond fund will be worth \$85,000 at maturity?

70. Funding an IRA First Colonial Bankshares Corporation advertised the following IRA investment plans.

Target IRA Plans

For each $5000 Maturity Value Desired	
Deposit:	**At a Term of:**
$620.17	20 Years
$1045.02	15 Years
$1760.92	10 Years
$2967.26	5 Years

(a) Assuming continuous compounding, what was the annual rate of interest that they offered?
(b) First Colonial Bankshares claims that $4000 invested today will have a value of over $32,000 in 20 years. Use the answer found in part (a) to find the actual value of $4000 in 20 years. Assume continuous compounding.

71. Estimating the Date that a Prehistoric Man Died The bones of a prehistoric man found in the desert of New Mexico contain approximately 5% of the original amount of carbon 14. If the half-life of carbon 14 is 5600 years, approximately how long ago did the man die?

72. Temperature of a Skillet A skillet is removed from an oven whose temperature is 450°F and placed in a room whose temperature is 70°F. After 5 minutes, the temperature of the skillet is 400°F. How long will it be until its temperature is 150°F?

73. World Population According to the U.S. Census Bureau, the growth rate of the world's population in 1997 was $k = 1.33\% = 0.0133$. The population of the world in 1997 was 5,840,445,216. Letting $t = 0$ represent 1997, use the uninhibited growth model to predict the world's population in the year 2000.

74. Radioactive Decay The half-life of radioactive cobalt is 5.27 years. If 100 grams of radioactive cobalt is present now, how much will be present in 20 years? In 40 years?

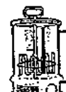
75. Logistic Growth The logistic growth model

$$P(t) = \frac{0.8}{1 + 1.67e^{-0.16t}}$$

represents the proportion of new computers sold that utilize the Microsoft Windows 98 operating system. Let $t = 0$ represent 1998, $t = 1$ represent 1999, and so on.
(a) What proportion of new computers sold in 1998 utilized Windows 98?
(b) Determine the maximum proportion of new computers sold that will utilize Windows 98.
(c) Using a graphing utility, graph $P(t)$.
(d) When will 75% of new computers sold utilize Windows 98?

76. CBL Experiment The following data were collected by placing a temperature probe in a portable heater, removing the probe, and then recording temperature over time.

According to Newton's Law of Cooling, these data should follow an exponential model.
(a) Using a graphing utility, draw a scatter diagram for the data.
(b) Using a graphing utility, fit an exponential model to the data.
(c) Graph the exponential function found in part (b) on the scatter diagram.
(d) Predict how long it will take for the probe to reach a temperature of 110°F.



Time	Temperature (F°)
0	165.07
1	164.77
2	163.99
3	163.22
4	162.82
5	161.96
6	161.20
7	160.45
8	159.35
9	158.61
10	157.89
11	156.83
12	156.11
13	155.08
14	154.40
15	153.72

77. The following data represent the per capita usage of carrots.

Year	Per Capita Usage
1985 ($t = 1$)	6.5
1990 ($t = 6$)	8.3
1991 ($t = 7$)	7.7
1992 ($t = 8$)	8.3
1993 ($t = 9$)	8.2
1994 ($t = 10$)	8.7
1995 ($t = 11$)	9.0
1996 ($t = 12$)	10.2

Source: U.S. Department of Agriculture.

(a) Using a graphing utility, draw a scatter diagram of the data using time as the independent variable and per capita consumption as the dependent variable.
(b) Use an exponential model, logarithmic model, power model, and linear model to find the "best" model to describe the relation between time and per capita consumption. Record each model's correlation coefficient.
(c) Use this model to predict the per capita consumption of carrots in 1997.

Identities (p. 403)

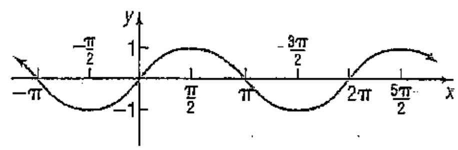
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}, \quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}, \quad \sec\theta = \dfrac{1}{\cos\theta}, \quad \csc\theta = \dfrac{1}{\sin\theta}$

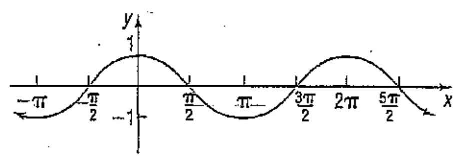
$\sin^2\theta + \cos^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta, \quad 1 + \cot^2\theta = \csc^2\theta$

Properties of the Trigonometric Functions

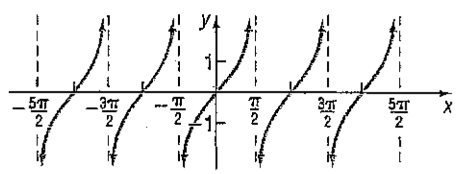
$y = \sin x$ (p. 409)
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Periodic: period $= 2\pi(360°)$
Odd function



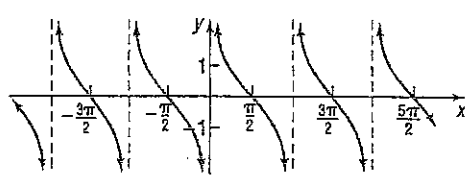
$y = \cos x$ (p. 411)
Domain: $-\infty < x < \infty$
Range: $-1 \le y \le 1$
Periodic: period $= 2\pi(360°)$
Even function



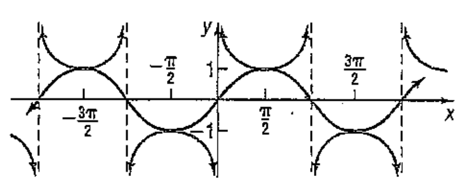
$y = \tan x$ (p. 415)
Domain: $-\infty < x < \infty$, except odd multiples of $\pi/2(90°)$
Range: $-\infty < y < \infty$
Periodic: period $= \pi(180°)$
Odd function



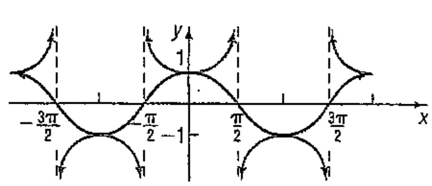
$y = \cot x$ (p. 416)
Domain: $-\infty < x < \infty$, except integral multiples of $\pi(180°)$
Range: $-\infty < y < \infty$
Periodic: period $= \pi(180°)$
Odd function



$y = \csc x$ (p. 417)
Domain: $-\infty < x < \infty$, except integral multiples of $\pi(180°)$
Range: $|y| \ge 1$
Periodic: period $= 2\pi(360°)$
Odd function



$y = \sec x$ (p. 418)
Domain: $-\infty < x < \infty$, except odd multiples of $\pi/2(90°)$
Range: $|y| \ge 1$
Periodic: period $= 2\pi(360°)$
Even function



Sinusoidal graphs

$y = A\sin(\omega x), \quad \omega > 0$

$y = A\cos(\omega x), \quad \omega > 0$

$y = A\sin(\omega x - \phi) = A\sin[\omega(x - \phi/\omega)]$

$y = A\cos(\omega x - \phi) = A\cos[\omega(x - \phi/\omega)]$

Period $= 2\pi/\omega$ (p. 421)

Amplitude $= |A|$ (p. 421)

Phase shift $= \phi/\omega$ (p. 425)

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–4, convert each angle in degrees to radians. Express your answer as a multiple of π.

1. $135°$ 2. $210°$ 3. $18°$ 4. $15°$

In Problems 5–8, convert each angle in radians to degrees.

5. $3\pi/4$ 6. $2\pi/3$ 7. $-5\pi/2$ 8. $-3\pi/2$

In Problems 9–26, find the exact value of each expression. Do not use a calculator.

9. $\tan\frac{\pi}{4} - \sin\frac{\pi}{6}$
10. $\cos\frac{\pi}{3} + \sin\frac{\pi}{2}$
11. $3\sin 45° - 4\tan\frac{\pi}{6}$
12. $4\cos 60° + 3\tan\frac{\pi}{3}$
13. $6\cos\frac{3\pi}{4} + 2\tan\left(-\frac{\pi}{3}\right)$
14. $3\sin\frac{2\pi}{3} - 4\cos\frac{5\pi}{2}$
15. $\sec\left(-\frac{\pi}{3}\right) - \cot\left(-\frac{5\pi}{4}\right)$
16. $4\csc\frac{3\pi}{4} - \cot\left(-\frac{\pi}{4}\right)$
17. $\tan\pi + \sin\pi$
18. $\cos\frac{\pi}{2} - \csc\left(-\frac{\pi}{2}\right)$
19. $\cos 180° - \tan(-45°)$
20. $\sin 270° + \cos(-180°)$
21. $\sin^2 20° + \frac{1}{\sec^2 20°}$
22. $\frac{1}{\cos^2 40°} - \frac{1}{\cot^2 40°}$
23. $\sec 50° \cos 50°$
24. $\tan 10° \cot 10°$
25. $\frac{\cos 400°}{\cos(-40°)}$
26. $\frac{\tan(-20°)}{\tan 200°}$

In Problems 27–42, find the exact value of each of the remaining trigonometric functions.

27. $\sin\theta = -\frac{4}{5}$, $\cos\theta > 0$
28. $\cos\theta = -\frac{3}{5}$, $\sin\theta < 0$
29. $\tan\theta = \frac{12}{5}$, $\sin\theta < 0$
30. $\cot\theta = \frac{12}{5}$, $\cos\theta < 0$
31. $\sec\theta = -\frac{5}{4}$, $\tan\theta < 0$
32. $\csc\theta = -\frac{5}{3}$, $\cot\theta < 0$
33. $\sin\theta = \frac{12}{13}$, θ in quadrant II
34. $\cos\theta = -\frac{3}{5}$, θ in quadrant III
35. $\sin\theta = -\frac{5}{13}$, $3\pi/2 < \theta < 2\pi$
36. $\cos\theta = \frac{12}{13}$, $3\pi/2 < \theta < 2\pi$
37. $\tan\theta = \frac{1}{3}$, $180° < \theta < 270°$
38. $\tan\theta = -\frac{2}{3}$, $90° < \theta < 180°$
39. $\sec\theta = 3$, $3\pi/2 < \theta < 2\pi$
40. $\csc\theta = -4$, $\pi < \theta < 3\pi/2$
41. $\cot\theta = -2$, $\pi/2 < \theta < \pi$
42. $\tan\theta = -2$, $3\pi/2 < \theta < 2\pi$

In Problems 43–54, graph each function by hand. Each graph should contain at least one period. Verify your results using a graphing utility.

43. $y = 2\sin(4x)$
44. $y = -3\cos(2x)$
45. $y = -2\cos\left(x + \frac{\pi}{2}\right)$
46. $y = 3\sin(x - \pi)$
47. $y = \tan(x + \pi)$
48. $y = -\tan\left(x - \frac{\pi}{2}\right)$
49. $y = -2\tan(3x)$
50. $y = 4\tan(2x)$
51. $y = \cot\left(x + \frac{\pi}{8}\right)$
52. $y = -4\cot(2x)$
53. $y = \sec\left(x - \frac{\pi}{4}\right)$
54. $y = \csc\left(x + \frac{\pi}{4}\right)$

In Problems 55–58, determine the amplitude and period of each function without graphing.

55. $y = 4\cos x$
56. $y = \sin(2x)$
57. $y = -8\sin\left(\frac{\pi}{2}x\right)$
58. $y = -2\cos(3\pi x)$

In Problems 59–66, find the amplitude, period, and phase shift of each function. Graph each function by hand. Show at least one period.

59. $y = 4\sin(3x)$

60. $y = 2\cos\left(\frac{1}{3}x\right)$

61. $y = -2\sin\left(\frac{\pi}{2}x + \frac{1}{2}\right)$

62. $y = -6\sin(2\pi x - 2)$

63. $y = \frac{1}{2}\sin\left(\frac{3}{2}x - \pi\right)$

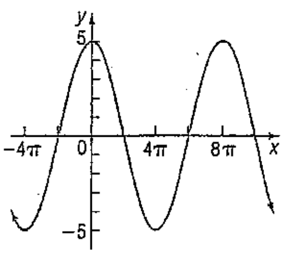
64. $y = \frac{3}{2}\cos(6x + 3\pi)$

65. $y = -\frac{2}{3}\cos(\pi x - 6) + \frac{2}{3}$

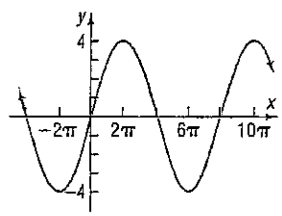
66. $y = -7\sin\left(\frac{\pi}{3}x + \frac{4}{3}\right) - 2$

In Problems 67–70, find a function for the given graph.

67.



68.



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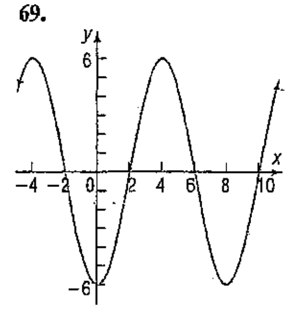
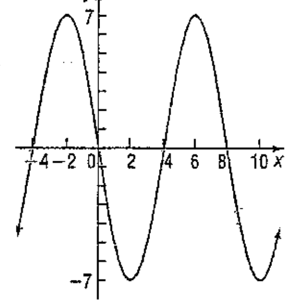


70.



71. Find the length of arc subtended by a central angle of 30° on a circle of radius 2 feet.

72. The minute hand of a clock is 8 inches long. How far does the tip of the minute hand move in 30 minutes? How far does it move in 20 minutes?

73. Angular Speed of a Race Car A race car is driven around a circular track at a constant speed of 180 miles per hour. If the diameter of the track is $\frac{1}{2}$ mile, what is the angular speed of the car? Express your answer in revolutions per hour (which is equivalent to laps per hour).

74. Merry-Go-Rounds A neighborhood carnival has a merry-go-round whose radius is 25 feet. If the time for one revolution is 30 seconds, how fast is the merry-go-round going?

75. Lighthouse Beacons The Montauk Point Lighthouse on Long Island has dual beams (two light sources opposite each other). Ships at sea observe a blinking light every 5 seconds. What angular speed is required to do this?

76. Spin Balancing Tires The radius of each wheel of a car is 16 inches. At how many revolutions per minute should a spin balancer be set to balance the tires at a speed of 90 miles per hour? Is the setting different for a wheel of radius 14 inches? What is this setting?

77. Alternating Voltage The electromotive force E, in volts, in a certain ac circuit obeys the equation

$$E(t) = 120\sin(120\pi t), \qquad t \geq 0$$

where t is measured in seconds.

(a) What is the maximum value of E?
(b) What is the period?
(c) Graph the function over two periods.

78. Alternating Current The current I, in amperes, flowing through an ac (alternating current) circuit at time t is

$$I(t) = 220\sin\left(30\pi t + \frac{\pi}{6}\right), \qquad t \geq 0$$

(a) What is the period?
(b) What is the amplitude?
(c) What is the phase shift?
(d) Graph this function over two periods.

79. Monthly Temperature The following data represent the average monthly temperatures for Phoenix, Arizona.

(a) Using a graphing utility, draw a scatter diagram of the data for one period.
(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.
(c) Draw the sinusoidal function found in part (b) on the scatter diagram.
(d) Use a graphing utility to find the sinusoidal function of best fit.
(e) Draw the sinusoidal function of best fit on the scatter diagram.

Month, m	Average Monthly Temperature, T
January, 1	51
February, 2	55
March, 3	63
April, 4	67
May, 5	77
June, 6	86
July, 7	90
August, 8	90
September, 9	84
October, 10	71
November, 11	59
December, 12	52

Source: U.S. National Oceanic and Atmospheric Administration.

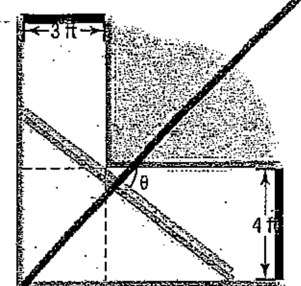
(a) In calculus, you will be asked to find the angle θ that maximizes R by solving the equation

$$\sin(2\theta) + \cos(2\theta) = 0$$

Solve this equation for θ using the method of Example 7.
(b) Solve this equation for θ by dividing each side by $\cos(2\theta)$.
(c) What is the maximum distance R if $v_0 = 32$ feet per second?
(d) Graph R, $45° \le \theta \le 90°$, and find the angle θ that maximizes the distance R. Also find the maximum distance. Use $v_0 = 32$ feet per second. Compare the results with the answers found earlier.

53. **Heat Transfer** In the study of heat transfer, the equation $x + \tan x = 0$ occurs. Graph $Y_1 = -x$ and $Y_2 = \tan x$ for $x \ge 0$. Conclude that there are an infinite number of points of intersection of these two graphs. Now find the first two positive solutions of $x + \tan x = 0$ rounded to two decimal places.

54. **Carrying a Ladder around a Corner** A ladder of length L is carried horizontally around a corner from a hall 3 feet wide into a hall 4 feet wide. See the illustration.



(a) Express L as a function of θ.
(b) In calculus, you will be asked to find the length of the longest ladder that can turn the corner by solving the equation

$$3\sec\theta\tan\theta - 4\csc\theta\cot\theta = 0, \quad 0° < \theta < 90°$$

Solve this equation for θ.
(c) What is the length of the longest ladder that can be carried around the corner?
(d) Graph L, $0° \le \theta \le 90°$, and find the angle θ that maximizes the length L. Also find the maximum length. Compare the results with the ones found in parts (b) and (c).

55. **Projectile Motion** The horizontal distance that a projectile will travel in the air is given by the equation

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

where v_0 is the initial velocity of the projectile, θ is the angle of elevation, and g is acceleration due to gravity (9.8 meters per second-squared).
(a) If you can throw a baseball with an initial speed of 34.8 meters per second, at what angle of elevation θ should you direct the throw so that the ball travels a distance of 107 meters before striking the ground?
(b) Determine the maximum distance that you can throw the ball.
(c) Graph R, with $v_0 = 34.8$ meters per second.
(d) Verify the results obtained in parts (a) and (b) using ZERO or ROOT.

56. **Projectile Motion** Refer to Problem 55.
(a) If you can throw a baseball with an initial speed of 40 meters per second, at what angle of elevation θ should you direct the throw so that the ball travels a distance of 110 meters before striking the ground?
(b) Determine the maximum distance that you can throw the ball.
(c) Graph R, with $v_0 = 40$ meters per second.
(d) Verify the results obtained in parts (a) and (b) using ZERO or ROOT.

CHAPTER REVIEW

Things To Know

Sum and Difference Formulas (pp. 452, 455, and 458)

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-Angle Formulas (pp. 462 and 463)

$\sin(2\theta) = 2\sin\theta\cos\theta$ $\quad$ $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\quad$ $\cos(2\theta) = 1 - 2\sin^2\theta$

$$\cos(2\theta) = 2\cos^2\theta - 1 \qquad \tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulas (pp. 465 and 467)

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} \qquad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} \qquad \tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} \qquad \tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

Product-to-Sum Formulas (p. 470)

$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Sum-to-Product Formulas (p. 471)

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \qquad \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Definitions of the six inverse trigonometric functions

$y = \sin^{-1}x$	means	$x = \sin y$	where	$-1 \le x \le 1,\ -\pi/2 \le y \le \pi/2$	(p. 474)
$y = \cos^{-1}x$	means	$x = \cos y$	where	$-1 \le x \le 1,\ 0 \le y \le \pi$	(p. 478)
$y = \tan^{-1}x$	means	$x = \tan y$	where	$-\infty < x < \infty,\ -\pi/2 < y < \pi/2$	(p. 480)
$y = \cot^{-1}x$	means	$x = \cot y$	where	$-\infty < x < \infty,\ 0 < y < \pi$	(p. 482)
$y = \sec^{-1}x$	means	$x = \sec y$	where	$\lvert x\rvert \ge 1,\ 0 \le y \le \pi,\ y \ne \pi/2$	(p. 482)
$y = \csc^{-1}x$	means	$x = \csc y$	where	$\lvert x\rvert \ge 1,\ -\pi/2 \le y \le \pi/2,\ y \ne 0$	(p. 482)

How To

Establish identities (p. 447)
Use sum and difference formulas to find exact values (p. 453)
Use sum and difference formulas to establish identities (p. 457)
Use double-angle formulas to find exact values (p. 462)
Use double-angle and half-angle formulas to establish identities (p. 462)
Use half-angle formulas to find exact values (p. 465)
Express products as sums (p. 470)
Express sums as products (p. 471)
Find the exact value of an inverse trigonometric function (p. 476)
Find the approximate value of an inverse trigonometric function (p. 477)
Find the exact value of expressions involving inverse trigonometric functions (p. 485)
Write a trigonometric expression as an algebraic expression (p. 488)
Establish identities involving inverse trigonometric expressions (p. 488)
Solve equations involving a single trigonometric function (p. 490)
Solve trigonometric equations that are quadratic in form (p. 495)
Solve trigonometric equations using identities (p. 496)
Solve trigonometric equations linear in sine and cosine (p. 498)
Solve trigonometric equations using a graphing utility (p. 500)

Fill-in-the-Blank Items

1. Suppose that f and g are two functions with the same domain. If $f(x) = g(x)$ for every x in the domain, the equation is called a(n) ______________. Otherwise, it is called a(n) ______________ equation.
2. $\cos(\alpha+\beta) = \cos\alpha\cos\beta$ ______________ $\sin\alpha\sin\beta$.
3. $\sin(\alpha+\beta) = \sin\alpha\cos\beta$ ______________ $\cos\alpha\sin\beta$.
4. $\cos(2\theta) = \cos^2\theta -$ ______________ $=$ ______________ $-1 = 1 -$ ______________.
5. $\sin^2\dfrac{\alpha}{2} = \dfrac{\qquad}{2}$.
6. The function $y = \sin^{-1}x$ has domain ______________ and range ______________.
7. The value of $\sin^{-1}[\cos(\pi/2)]$ is ______________.

True/False Items

T F 1. $\sin(-\theta) + \sin\theta = 0$ for all θ.

T F 2. $\sin(\alpha + \beta) = \sin\alpha + \sin\beta + 2\sin\alpha\sin\beta$.

T F 3. $\cos(2\theta)$ has three equivalent forms: $\cos^2\theta - \sin^2\theta$, $1 - 2\sin^2\theta$, and $2\cos^2\theta - 1$.

T F 4. $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$, where the + or − sign depends on the angle $\alpha/2$.

T F 5. The domain of $y = \sin^{-1} x$ is $-\pi/2 \le x \le \pi/2$.

T F 6. $\cos(\sin^{-1} 0) = 1$ and $\sin(\cos^{-1} 0) = 1$.

T F 7. Most trigonometric equations have unique solutions.

T F 8. The equation $\tan\theta = \pi/2$ has no solution.

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–32, establish each identity.

1. $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$
2. $\sin\theta\csc\theta - \sin^2\theta = \cos^2\theta$
3. $\cos^2\theta(1 + \tan^2\theta) = 1$
4. $(1 - \cos^2\theta)(1 + \cot^2\theta) = 1$
5. $4\cos^2\theta + 3\sin^2\theta = 3 + \cos^2\theta$
6. $4\sin^2\theta + 2\cos^2\theta = 4 - 2\cos^2\theta$
7. $\frac{1 - \cos\theta}{\sin\theta} + \frac{\sin\theta}{1 - \cos\theta} = 2\csc\theta$
8. $\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = 2\csc\theta$
9. $\frac{\cos\theta}{\cos\theta - \sin\theta} = \frac{1}{1 - \tan\theta}$
10. $1 - \frac{\cos^2\theta}{1 + \sin\theta} = \sin\theta$
11. $\frac{\csc\theta}{1 + \csc\theta} = \frac{1 - \sin\theta}{\cos^2\theta}$
12. $\frac{1 + \sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1 - \cos\theta}$
13. $\csc\theta - \sin\theta = \cos\theta\cot\theta$
14. $\frac{\csc\theta}{1 - \cos\theta} = \frac{1 + \cos\theta}{\sin^3\theta}$
15. $\frac{1 - \sin\theta}{\sec\theta} = \frac{\cos^3\theta}{1 + \sin\theta}$
16. $\frac{1 - \cos\theta}{1 + \cos\theta} = (\csc\theta - \cot\theta)^2$
17. $\frac{1 - 2\sin^2\theta}{\sin\theta\cos\theta} = \cot\theta - \tan\theta$
18. $\frac{(2\sin^2\theta - 1)^2}{\sin^4\theta - \cos^4\theta} = 1 - 2\cos^2\theta$
19. $\frac{\cos(\alpha + \beta)}{\cos\alpha\sin\beta} = \cot\beta - \tan\alpha$
20. $\frac{\sin(\alpha - \beta)}{\sin\alpha\cos\beta} = 1 - \cot\alpha\tan\beta$
21. $\frac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} = 1 + \tan\alpha\tan\beta$
22. $\frac{\cos(\alpha + \beta)}{\sin\alpha\cos\beta} = \cot\alpha - \tan\beta$
23. $(1 + \cos\theta)\left(\tan\frac{\theta}{2}\right) = \sin\theta$
24. $\sin\theta\tan\frac{\theta}{2} = 1 - \cos\theta$
25. $2\cot\theta\cot(2\theta) = \cot^2\theta - 1$
26. $2\sin(2\theta)(1 - 2\sin^2\theta) = \sin(4\theta)$
27. $1 - 8\sin^2\theta\cos^2\theta = \cos(4\theta)$
28. $\frac{\sin(3\theta)\cos\theta - \sin\theta\cos(3\theta)}{\sin(2\theta)} = 1$
29. $\frac{\sin(2\theta) + \sin(4\theta)}{\cos(2\theta) + \cos(4\theta)} = \tan(3\theta)$
30. $\frac{\sin(2\theta) + \sin(4\theta)}{\sin(2\theta) - \sin(4\theta)} + \frac{\tan(3\theta)}{\tan\theta} = 0$
31. $\frac{\cos(2\theta) - \cos(4\theta)}{\cos(2\theta) + \cos(4\theta)} - \tan\theta\tan(3\theta) = 0$
32. $\cos(2\theta) - \cos(10\theta) = [\tan(4\theta)(\sin(2\theta) + \sin(10\theta))]$

In Problems 33–40, find the exact value of each expression.

33. $\sin 165^\circ$
34. $\tan 105^\circ$
35. $\cos\frac{5\pi}{12}$
36. $\sin\left(-\frac{\pi}{12}\right)$
37. $\cos 80^\circ\cos 20^\circ + \sin 80^\circ\sin 20^\circ$
38. $\sin 70^\circ\cos 40^\circ - \cos 70^\circ\sin 40^\circ$
39. $\tan\frac{\pi}{8}$
40. $\sin\frac{5\pi}{8}$

In Problems 41–50, use the information given about the angles α and β to find the exact value of:

(a) $\sin(\alpha + \beta)$ *(b)* $\cos(\alpha + \beta)$ *(c)* $\sin(\alpha - \beta)$ *(d)* $\tan(\alpha + \beta)$

(e) $\sin(2\alpha)$ *(f)* $\cos(2\beta)$ *(g)* $\sin\dfrac{\beta}{2}$ *(h)* $\cos\dfrac{\alpha}{2}$

41. $\sin\alpha = \frac{4}{5},\quad 0 < \alpha < \pi/2;\quad \sin\beta = \frac{5}{13},\quad \pi/2 < \beta < \pi$

42. $\cos\alpha = \frac{4}{5},\quad 0 < \alpha < \pi/2;\quad \cos\beta = \frac{5}{13},\quad -\pi/2 < \beta < 0$

43. $\sin\alpha = -\frac{3}{5},\quad \pi < \alpha < 3\pi/2;\quad \cos\beta = \frac{12}{13},\quad 3\pi/2 < \beta < 2\pi$

44. $\sin\alpha = -\frac{4}{5},\quad -\pi/2 < \alpha < 0;\quad \cos\beta = -\frac{5}{13},\quad \pi/2 < \beta < \pi$

45. $\tan\alpha = \frac{3}{4},\quad \pi < \alpha < 3\pi/2;\quad \tan\beta = \frac{12}{5},\quad 0 < \beta < \pi/2$

46. $\tan\alpha = -\frac{4}{3},\quad \pi/2 < \alpha < \pi,\quad \cot\beta = \frac{12}{5},\quad \pi < \beta < 3\pi/2$

47. $\sec\alpha = 2,\quad -\pi/2 < \alpha < 0;\quad \sec\beta = 3,\quad 3\pi/2 < \beta < 2\pi$

48. $\csc\alpha = 2,\quad \pi/2 < \alpha < \pi,\quad \sec\beta = -3,\quad \pi/2 < \beta < \pi$

49. $\sin\alpha = -\frac{2}{3},\quad \pi < \alpha < 3\pi/2;\quad \cos\beta = -\frac{2}{3},\quad \pi < \beta < 3\pi/2$

50. $\tan\alpha = -2,\quad \pi/2 < \alpha < \pi;\quad \cot\beta = -2,\quad \pi/2 < \beta < \pi$

In Problems 51–70, find the exact value of each expression. Do not use a calculator.

51. $\sin^{-1}1$ **52.** $\cos^{-1}0$ **53.** $\tan^{-1}1$ **54.** $\sin^{-1}\left(-\dfrac{1}{2}\right)$

55. $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$ **56.** $\tan^{-1}(-\sqrt{3})$ **57.** $\sin\left(\cos^{-1}\dfrac{\sqrt{2}}{2}\right)$ **58.** $\cos(\sin^{-1}0)$

59. $\tan\left[\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)\right]$ **60.** $\tan\left[\cos^{-1}\left(-\dfrac{1}{2}\right)\right]$ **61.** $\sec\left(\tan^{-1}\dfrac{\sqrt{3}}{3}\right)$ **62.** $\csc\left(\sin^{-1}\dfrac{\sqrt{3}}{2}\right)$

63. $\sin\left(\tan^{-1}\dfrac{3}{4}\right)$ **64.** $\cos\left(\sin^{-1}\dfrac{3}{5}\right)$ **65.** $\tan\left[\sin^{-1}\left(-\dfrac{4}{5}\right)\right]$ **66.** $\tan\left[\cos^{-1}\left(-\dfrac{3}{5}\right)\right]$

67. $\sin^{-1}\left(\cos\dfrac{2\pi}{3}\right)$ **68.** $\cos^{-1}\left(\tan\dfrac{3\pi}{4}\right)$ **69.** $\tan^{-1}\left(\tan\dfrac{7\pi}{4}\right)$ **70.** $\cos^{-1}\left(\cos\dfrac{7\pi}{6}\right)$

In Problems 71–76, find the exact value of each expression.

71. $\cos\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{1}{2}\right)$ **72.** $\sin\left(\cos^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5}\right)$ **73.** $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right) - \tan^{-1}\frac{3}{4}\right]$

74. $\cos\left[\tan^{-1}(-1) + \cos^{-1}\left(-\frac{4}{5}\right)\right]$ **75.** $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$ **76.** $\cos\left(2\tan^{-1}\frac{4}{3}\right)$

In Problems 77–96, solve each equation on the interval $0 \le \theta < 2\pi$. Use a graphing utility to verify your solution.

77. $\cos\theta = \frac{1}{2}$ **78.** $\sin\theta = -\sqrt{3}/2$ **79.** $2\cos\theta + \sqrt{2} = 0$

80. $\tan\theta + \sqrt{3} = 0$ **81.** $\sin(2\theta) + 1 = 0$ **82.** $\cos(2\theta) = 0$

83. $\tan(2\theta) = 0$ **84.** $\sin(3\theta) = 1$ **85.** $\sin\theta = 0.9$

86. $\tan\theta = 25$ **87.** $\sin\theta = \tan\theta$ **88.** $\cos\theta = \sec\theta$

89. $\sin\theta + \sin(2\theta) = 0$ **90.** $\cos(2\theta) = \sin\theta$ **91.** $\sin(2\theta) - \cos\theta - 2\sin\theta + 1 = 0$

92. $\sin(2\theta) - \sin\theta - 2\cos\theta + 1 = 0$ **93.** $2\sin^2\theta - 3\sin\theta + 1 = 0$ **94.** $2\cos^2\theta + \cos\theta - 1 = 0$

95. $\sin\theta - \cos\theta = 1$ **96.** $\sin\theta + 2\cos\theta = 1$

In Problems 97–102, use a graphing utility to solve each equation on the interval $0 \le x \le 2\pi$. Approximate any solutions rounded to two decimal places.

97. $2x = 5\cos x$ **98.** $2x = 5\sin x$ **99.** $2\sin x + 3\cos x = 4x$

100. $3\cos x + x = \sin x$ **101.** $\sin x = \ln x$ **102.** $\sin x = e^{-x}$

~~36. CBL Experiment~~ Pendulum motion is analyzed to estimate simple ~~harmonic motion. A~~ plot is generated with the position of the pendulum over time. ~~The graph~~ is used to find a sinusoidal curve ~~of the form~~ $y = A\cos[B(x - C)] + D$. ~~Determine~~ the amplitude, period ~~and frequency~~. (Activity 16, Real-World Math with the CBL System.)

37. **CBL Experiment** The sound from a tuning fork is collected over time. ~~The amplitude, frequency,~~ and period of ~~the graph~~ are determined. A model of the form $y = A\cos[B(x - C)]$ ~~is fitted to~~ the data. (Activity 23, Real-World Math with the CBL System.)

CHAPTER REVIEW

Things To Know

Acute angle (p. 508)	An angle θ whose measure is $0° < \theta < 90°$ (or $0 < \theta < \pi/2$)
Complementary angles (p. 510)	Two acute angles whose sum is $90°$ ($\pi/2$)
Cofunction (p. 510)	The following pairs of functions are cofunctions of each other: sine and cosine; tangent and cotangent; secant and cosecant

Formulas

Law of Sines (p. 522)	$\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$
Law of Cosines (p. 532)	$c^2 = a^2 + b^2 - 2ab\cos\gamma$ $b^2 = a^2 + c^2 - 2ac\cos\beta$ $a^2 = b^2 + c^2 - 2bc\cos\alpha$
Area of a triangle (pp. 539–540)	$A = \frac{1}{2}bh$ $A = \frac{1}{2}ab\sin\gamma$ $A = \frac{1}{2}bc\sin\alpha$ $A = \frac{1}{2}ac\sin\beta$ $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$

How To

Find the value of trigonometric functions of acute angles (p. 508)

Use the complementary angle theorem (p. 510)

Solve right triangles (p. 511)

Solve applied problems using right triangle trigonometry (p. 512)

Solve SAA or ASA triangles (p. 523)

Solve SSA triangles (p. 524)

Solve applied problems using the Law of Sines (p. 527)

Solve SAS triangles (p. 533)

Solve SSS triangles (p. 534)

Solve applied triangles using the Law of Cosines (p. 535)

Find the area of SAS triangles (p. 540)

Find the area of SSS triangles (p. 540)

Find an equation for an object in simple harmonic motion (p. 547)

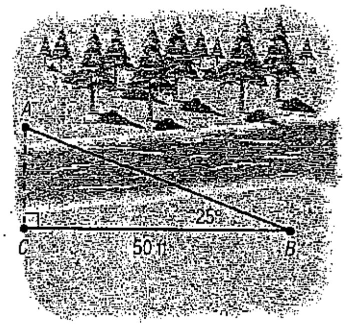
Analyze simple harmonic motion (p. 547)

Analyze an object in damped motion (p. 548)

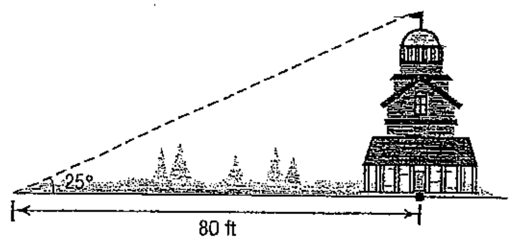
Fill-in-the-Blank Items

1. Two acute angles whose sum is a right angle are called ______________.
2. The sine and ______________ functions are cofunctions.
3. If two sides and the angle opposite one of them are known, the Law of ______________ is used to determine whether the known information results in no triangle, one triangle, or two triangles.
4. If three sides of a triangle are given, the Law of ______________ is used to solve the triangle.
5. If three sides of a triangle are given, ______________ Formula is used to find the area of the triangle.
6. The motion of an object obeys the equation $d = 4\cos(6t)$. Such motion is described as ______________ ______________.
7. The mass and damping factor of $d = 5e^{-0.5t/10}\cos\left(\sqrt{\pi^2 - \dfrac{(0.5)^2}{100}}\,t\right)$ are ______________ and ______________.

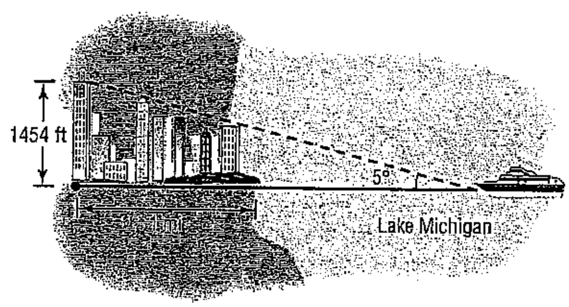
37. Finding the Width of a River Find the distance from A to C across the river illustrated in the figure.



38. Finding the Height of a Building Find the height of the building shown in the figure.

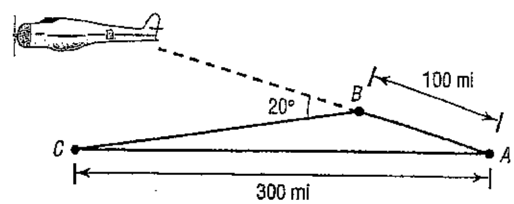


39. Finding the Distance to Shore The Sears Tower in Chicago is 1454 feet tall and is situated about 1 mile inland from the shore of Lake Michigan, as indicated in the figure. An observer in a pleasure boat on the lake directly in front of the Sears Tower looks at the top of the tower and measures the angle of elevation as 5°. How far offshore is the boat?



40. Finding the Grade of a Mountain Trail A straight trail with a uniform inclination leads from a hotel, elevation 5000 feet, to a lake in a valley, elevation 4100 feet. The length of the trail is 4100 feet. What is the inclination (grade) of the trail?

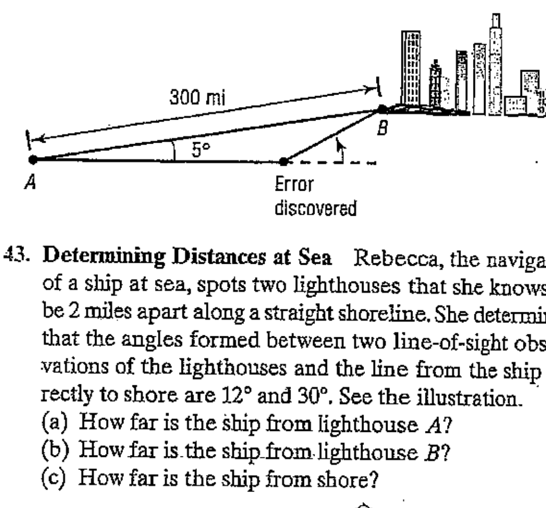
41. Navigation An airplane flies from city A to city B, a distance of 100 miles, and then turns through an angle of 20° and heads toward city C, as indicated in the figure. If the distance from A to C is 300 miles, how far is it from city B to city C?



42. Correcting a Navigation Error Two cities A and B are 300 miles apart. In flying from city A to city B, a pilot inadvertently took a course that was 5° in error.

(a) If the error was discovered after flying 10 minutes at a constant speed of 420 miles per hour, through what angle should the pilot turn to correct the course? (Consult the figure.)

(b) What new constant speed should be maintained so that no time is lost due to the error? (Assume that the speed would have been a constant 420 miles per hour if no error had occurred.)

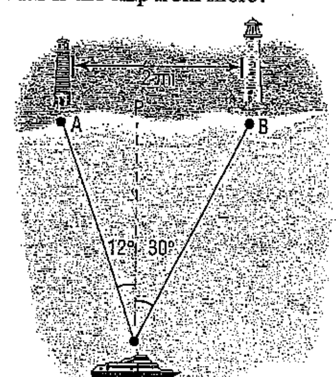


43. Determining Distances at Sea Rebecca, the navigator of a ship at sea, spots two lighthouses that she knows to be 2 miles apart along a straight shoreline. She determines that the angles formed between two line-of-sight observations of the lighthouses and the line from the ship directly to shore are 12° and 30°. See the illustration.

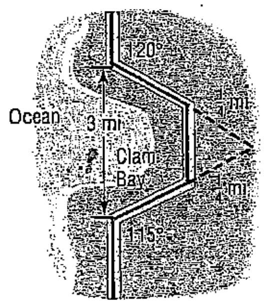
(a) How far is the ship from lighthouse A?

(b) How far is the ship from lighthouse B?

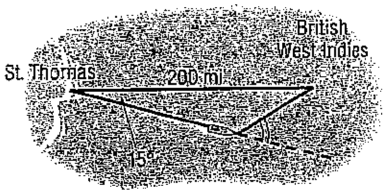
(c) How far is the ship from shore?



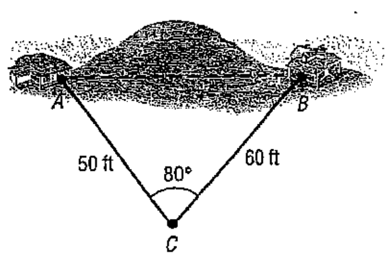
44. Constructing a Highway A highway whose primary directions are north–south is being constructed along the west coast of Florida. Near Naples, a bay obstructs the straight path of the road. Since the cost of a bridge is prohibitive, engineers decide to go around the bay. The illustration shows the path that they decide on and the measurements taken. What is the length of highway needed to go around the bay?



45. **Correcting a Navigational Error** A yacht leaves St. Thomas bound for an island in the British West Indies, 200 miles away. Maintaining a constant speed of 18 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds after 4 hours that the sailboat is off course by 15°.
(a) How far is the sailboat from the island at this time?
(b) Through what angle should the sailboat turn to correct its course?
(c) How much time has been added to the trip because of this? (Assume that the speed remains at 18 miles per hour.)

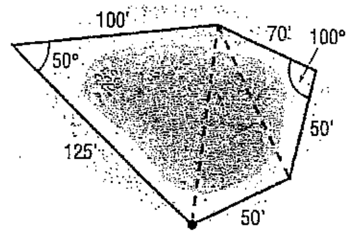


46. **Surveying** Two homes are located on opposite sides of a small hill. See the illustration. To measure the distance between them, a surveyor walks a distance of 50 feet from house A to point C, uses a transit to measure the angle ACB, which is found to be 80°, and then walks to house B, a distance of 60 feet. How far apart are the houses?

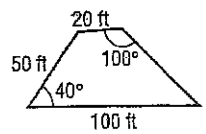


47. **Approximating the Area of a Lake** To approximate the area of a lake, Cindy walks around the perimeter of the lake, taking the measurements shown in the illustration. Using this technique, what is the approximate area of the lake?

[**Hint:** Use the Law of Cosines on the three triangles shown and then find the sum of their areas.]



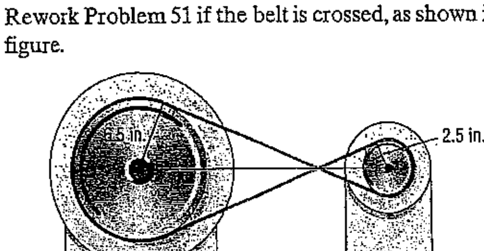
48. **Calculating the Cost of Land** The irregular parcel of land shown in the figure is being sold for $100 per square foot. What is the cost of this parcel?



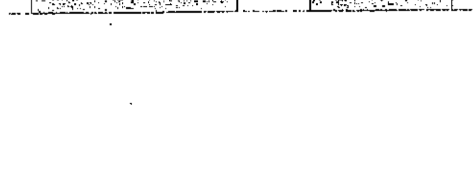
49. **Area of a Segment** Find the area of the segment of a circle whose radius is 6 inches formed by a central angle of 50°.

50. **Finding the Bearing of a Ship** The *Majesty* leaves the Port at Boston for Bermuda with a bearing of S80°E at an average speed of 10 knots. After 1 hour, the ship turns 90° toward the southwest. After 2 hours at an average speed of 20 knots, what is the bearing of the ship from Boston?

51. The drive wheel of an engine is 13 inches in diameter, and the pulley on the rotary pump is 5 inches in diameter. If the shafts of the drive wheel and the pulley are 2 feet apart, what length of belt is required to join them as shown in the figure?



52. Rework Problem 51 if the belt is crossed, as shown in the figure.



CHAPTER REVIEW

Things To Know

Relationship between polar coordinates (r, θ) and rectangular coordinates (x, y)	$x = r\cos\theta, y = r\sin\theta$ (p. 563) $x^2 + y^2 = r^2, \tan\theta = \frac{y}{x}, x \neq 0$ (p. 566)
Polar form of a complex number (p. 586)	If $z = x + yi$, then $z = r(\cos\theta + i\sin\theta)$, where $r = \lvert z\rvert = \sqrt{x^2 + y^2}, \sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}, 0 \leq \theta < 2\pi$
DeMoivre's Theorem (p. 589)	If $z = r(\cos\theta + i\sin\theta)$, then $z^n = r^n[\cos(n\theta) + i\sin(n\theta)]$, where $n \geq 1$ is a positive integer
nth root of a complex number $z = r(\cos\theta_0 + i\sin\theta_0)$ (p. 590)	$\sqrt[n]{z} = \sqrt[n]{r}\left[\cos\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right], k = 0, \ldots, n-1$
Vector (p. 594)	Quantity having magnitude and direction; equivalent to a directed line segment $\overrightarrow{PQ}$
Position vector (p. 597 and 616)	Vector whose initial point is at the origin
Unit vector (p. 597)	Vector whose magnitude is 1
Dot product (p. 606 and 618)	If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, then $\mathbf{v} \cdot \mathbf{w} = a_1a_2 + b_1b_2$. If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j} + c_1\mathbf{k}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j} + c_2\mathbf{k}$, then $\mathbf{v} \cdot \mathbf{w} = a_1a_2 + b_1b_2 + c_1c_2$.
Angle θ between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ (p. 608 and 619)	$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\lVert\mathbf{u}\rVert\,\lVert\mathbf{v}\rVert}$
Vectors in space (p. 621)	If $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, then $\mathbf{v} = \lVert\mathbf{v}\rVert[(\cos\alpha)\mathbf{i} + (\cos\beta)\mathbf{j} + (\cos\gamma)\mathbf{k}]$, where $\cos\alpha = \frac{a}{\lVert\mathbf{v}\rVert}, \quad \cos\beta = \frac{b}{\lVert\mathbf{v}\rVert}, \quad \cos\gamma = \frac{c}{\lVert\mathbf{v}\rVert}$.
Cross Product (p. 624)	If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j} + c_1\mathbf{k}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j} + c_2\mathbf{k}$, then $\mathbf{v} \times \mathbf{w} = [b_1c_2 - b_2c_1]\mathbf{i} - (a_1c_2 - a_2c_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$
(p. 627)	$\lVert\mathbf{u} \times \mathbf{v}\rVert = \lVert\mathbf{u}\rVert\,\lVert\mathbf{v}\rVert\sin\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

How To

Plot points using polar coordinates (p. 560)
Convert from polar coordinates to rectangular coordinates (p. 563)
Convert from rectangular coordinates to polar coordinates (p. 564)
Graph and identify polar equations by converting to rectangular equations (p. 569)
Graph polar equations using a graphing utility (p. 570)
Test polar equations for symmetry (p. 574)
Graph polar equations by plotting points (p. 575)
Convert a complex number from rectangular form to polar form (p. 586)
Plot points in the complex plane (p. 586)
Find products and quotients of complex numbers in polar form (p. 587)

Use DeMoivre's Theorem (p. 588)
Find complex roots (p. 590)
Graph vectors (p. 596)
Find a position vector (p. 597 and 616)
Add and subtract vectors (p. 600 and 617)
Find a scalar product and the magnitude of a vector (p. 600 and 618)
Find a unit vector (p. 600 and 618)
Find a vector from its direction and magnitude (p. 601)
Work with objects in static equilibrium (p. 602)
Find the dot product of two vectors (p. 606 and 618)
Find the angle between two vectors (p. 607 and 619)

Determine whether two vectors are parallel (p. 609)
Determine whether two vectors are orthogonal (p. 609)
Decompose a vector into two orthogonal vectors (p. 610)
Compute work (p. 611)
Find the distance between two points in space (p. 615)
Find the direction angles of a vector in space (p. 620)
Find the cross product of two vectors in space (p. 624)
Know algebraic properties of the cross product (p. 626)
Know the geometric properties of the cross product (p. 627)
Find a vector orthogonal to two given vectors (p. 627)
Find the area of a parallelogram (p. 628)

True/False Items

T F 1. $\tan 62° = \cot 38°$

T F 2. $\sin 182° = \cos 2°$

T F 3. An oblique triangle in which two sides and an angle are given always results in at least one triangle.

T F 4. Given three sides of a triangle, there is a formula for finding its area.

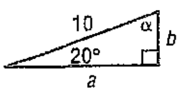
T F 5. In a right triangle, if two sides are known, we can solve the triangle.

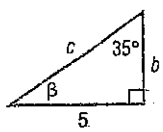
T F 6. The ambiguous case refers to the fact that, when two sides and the angle opposite one of them is known, sometimes the Law of Sines cannot be used.

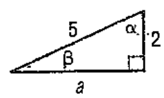
Review Exercises

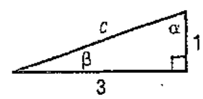
Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–4, solve each triangle.

1. 

2. 

3. 

4. 

In Problems 5–24, find the remaining angle(s) and side(s) of each triangle, if it (they) exists. If no triangle exists, say "No triangle."

~~5. $\alpha = 50°$, $\beta = 30°$, $a = 1$~~ 6. $\alpha = 10°$, $\gamma = 40°$, $c = 2$ ~~7. $\alpha = 100°$, $a = 5$, $c = 2$~~

8. $a = 2$, $c = 5$, $\alpha = 60°$ ~~9. [illegible], $c = 1$, $\gamma = 110°$~~ ~~10. $a = 3$, $c = 1$, $\gamma = 20°$~~

~~11. $a = 3$, $c = 1$, $\beta = 100°$~~ 12. $a = 3$, $b = 5$, $\beta = 80°$ 13. $a = 2$, $b = 3$, $c = 1$

~~14. [illegible] 10, [illegible] 8~~ 15. $a = 1$, $b = 3$, $\gamma = 40°$ 16. $a = 4$, $b = 1$, $\gamma = 100°$

17. $a = 5$, $b = 3$, $\alpha = 80°$ 18. $a = 2$, $b = 3$, $\alpha = 20°$ 19. $a = 1$, $b = \frac{1}{2}$, $c = \frac{4}{3}$

20. $a = 3$, $b = 2$, $c = 2$ 21. $a = 3$, $\alpha = 10°$, $b = 4$ 22. $a = 4$, $\alpha = 20°$, $\beta = 100°$

23. $c = 5$, $b = 4$, $\alpha = 70°$ 24. $a = 1$, $b = 2$, $\gamma = 60°$

In Problems 25–34, find the area of each triangle.

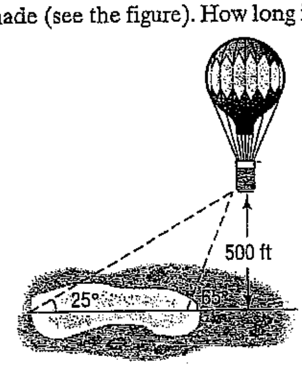
25. $a = 2$, $b = 3$, $\gamma = 40°$ 26. $b = 5$, $c = 5$, $\alpha = 20°$ 27. $b = 4$, $c = 10$, $\alpha = 70°$

28. $a = 2$, $b = 1$, $\gamma = 100°$ 29. $a = 4$, $b = 3$, $c = 5$ 30. $a = 10$, $b = 7$, $c = 8$

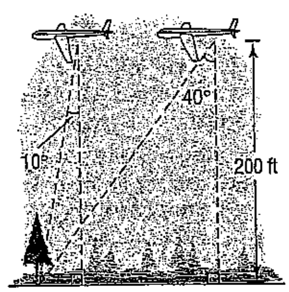
31. $a = 4$, $b = 2$, $c = 5$ 32. $a = 3$, $b = 2$, $c = 2$ ~~33. $\alpha = 50°$, $\beta = 30°$, $a = 1$~~

34. $\alpha = 10°$, $\gamma = 40°$, $c = 3$

35. **Measuring the Length of a Lake** From a stationary hot-air balloon 500 feet above the ground, two sightings of a lake are made (see the figure). How long is the lake?



36. **Finding the Speed of a Glider** From a glider 200 feet above the ground, two sightings of a stationary object directly in front are taken 1 minute apart (see the figure). What is the speed of the glider?



ill-in-the-Blank Items

1. In polar coordinates, the origin is called the _______________, and the positive x-axis is referred to as the _______________.
2. Another representation in polar coordinates for the point $(2, \pi/3)$ is (_______________, $4\pi/3$).
3. Using polar coordinates (r, θ), the circle $x^2 + y^2 = 2x$ takes the form _______________.
4. In a polar equation, replace θ by $-\theta$. If an equivalent equation results, the graph is symmetric with respect to _______________.
5. When a complex number z is written in the polar form $z = r(\cos\theta + i\sin\theta)$, the nonnegative number r is the _______________ or _______________ of z, and the angle θ, $0 \le \theta < 2\pi$, is the _______________ of z.
6. A vector whose magnitude is 1 is called a(n) _______________ vector.
7. If the angle between two vectors $\mathbf{v}$ and $\mathbf{w}$ is $\pi/2$, then the dot product $\mathbf{v} \cdot \mathbf{w}$ equals _______________.

rue/False Items

F 1. The polar coordinates of a point are unique.
F 2. The rectangular coordinates of a point are unique.
F 3. The tests for symmetry in polar coordinates are necessary, but not sufficient.
F 4. DeMoivre's Theorem is useful for raising a complex number to a positive integer power.
F 5. Vectors are quantities that have magnitude and direction.
F 6. Force is a physical example of a vector.
F 7. If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal vectors, then $\mathbf{u} \cdot \mathbf{v} = 0$.
F 8. The sum of the squares of the direction cosines of a vector in space equals 1.

eview Exercises

lue problem numbers indicate the authors' suggestions for use in a Practice Test.

n Problems 1–6, plot each point given in polar coordinates, and find its rectangular coordinates.

1. $(3, \pi/6)$ 2. $(4, 2\pi/3)$ 3. $(-2, 4\pi/3)$
4. $(-1, 5\pi/4)$ 5. $(-3, -\pi/2)$ 6. $(-4, -\pi/4)$

n Problems 7–12, the rectangular coordinates of a point are given. Find two pairs of polar coordinates (r, θ) for each point, one ith $r > 0$ and the other with $r < 0$. Express θ in radians.

7. $(-3, 3)$ 8. $(1, -1)$ 9. $(0, -2)$
0. $(2, 0)$ 11. $(3, 4)$ 12. $(-5, 12)$

n Problems 13–18, the letters x and y represent rectangular coordinates. Write each equation using polar coordinates (r, θ).

3. $3x^2 + 3y^2 = 6y$ 14. $2x^2 - 2y^2 = 5y$ 15. $2x^2 - y^2 = \dfrac{y}{x}$

6. $x^2 + 2y^2 = \dfrac{y}{x}$ 17. $x(x^2 + y^2) = 4$ 18. $y(x^2 - y^2) = 3$

n Problems 19–24, the letters r and θ represent polar coordinates. Write each polar equation as an equation in rectangular coor-inates (x, y).

9. $r = 2\sin\theta$ 20. $3r = \sin\theta$ 21. $r = 5$
2. $\theta = \pi/4$ 23. $r\cos\theta + 3r\sin\theta = 6$ 24. $r^2\tan\theta = 1$

Problems 25–30, sketch the graph of each polar equation. Be sure to test for symmetry. Verify your results using a graphing tility.

5. $r = 4\cos\theta$ 26. $r = 3\sin\theta$ 27. $r = 3 - 3\sin\theta$
8. $r = 2 + \cos\theta$ 29. $r = 4 - \cos\theta$ 30. $r = 1 - 2\sin\theta$

In Problems 31–34, write each complex number in polar form. Express each argument in degrees.

31. $-1 - i$ **32.** $-\sqrt{3} + i$ **33.** $4 - 3i$ **34.** $3 - 2i$

In Problems 35–40, write each complex number in the standard form $a + bi$.

35. $2(\cos 150° + i \sin 150°)$ **36.** $3(\cos 60° + i \sin 60°)$ **37.** $3\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

38. $4\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$ **39.** $0.1(\cos 350° + i \sin 350°)$ **40.** $0.5(\cos 160° + i \sin 160°)$

In Problems 41–46, find zw and z/w. Leave your answers in polar form.

41. $z = \cos 80° + i \sin 80°$, $w = \cos 50° + i \sin 50°$

42. $z = \cos 205° + i \sin 205°$, $w = \cos 85° + i \sin 85°$

43. $z = 3\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$, $w = 2\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$

44. $z = 2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$, $w = 3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

45. $z = 5(\cos 10° + i \sin 10°)$, $w = \cos 355° + i \sin 355°$

46. $z = 4(\cos 50° + i \sin 50°)$, $w = \cos 340° + i \sin 340°$

In Problems 47–54, write each expression in the standard form $a + bi$. Verify your results using a graphing utility.

47. $[3(\cos 20° + i \sin 20°)]^3$

48. $[2(\cos 50° + i \sin 50°)]^3$

49. $\left[\sqrt{2}\left(\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}\right)\right]^4$

50. $\left[2\left(\cos \frac{5\pi}{16} + i \sin \frac{5\pi}{16}\right)\right]^4$

51. $(1 - \sqrt{3}i)^6$ **52.** $(2 - 2i)^8$ **53.** $(3 + 4i)^4$ **54.** $(1 - 2i)^4$

55. Find all the complex cube roots of 27.

56. Find all the complex fourth roots of -16.

In Problems 57–64, the vector **v** *is represented by the directed line segment $\overrightarrow{PQ}$. Write* **v** *in the form $a\mathbf{i} + b\mathbf{j}$ or in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and find $\|\mathbf{v}\|$.*

57. $P = (1, -2)$; $Q = (3, -6)$

58. $P = (-3, 1)$; $Q = (4, -2)$

59. $P = (0, -2)$; $Q = (-1, 1)$

60. $P = (3, -4)$; $Q = (-2, 0)$

61. $P = (6, 2, 1)$; $Q = (3, 0, 2)$

62. $P = (4, 7, 0)$; $Q = (0, 5, 6)$

63. $P = (-1, 0, 1)$; $Q = (2, 0, 0)$

64. $P = (6, 2, 2)$; $Q = (2, 6, 2)$

In Problems 65–72, use the vectors $\mathbf{v} = -2\mathbf{i} + \mathbf{j}$ and $\mathbf{w} = 4\mathbf{i} - 3\mathbf{j}$.

65. Find $4\mathbf{v} - 3\mathbf{w}$. **66.** Find $-\mathbf{v} + 2\mathbf{w}$. **67.** Find $\|\mathbf{v}\|$.

68. Find $\|\mathbf{v} + \mathbf{w}\|$. **69.** Find $\|\mathbf{v}\| + \|\mathbf{w}\|$. **70.** Find $\|2\mathbf{v}\| - 3\|\mathbf{w}\|$.

71. Find a unit vector in the same direction as **v**.

72. Find a unit vector in the opposite direction of **w**.

In Problems 73–80, use the vectors $\mathbf{v} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{w} = -3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ to find each expression.

73. $4\mathbf{v} - 3\mathbf{w}$ **74.** $-\mathbf{v} + 2\mathbf{w}$ **75.** $\|\mathbf{v} - \mathbf{w}\|$ **76.** $\|\mathbf{v} + \mathbf{w}\|$

77. $\|\mathbf{v}\| - \|\mathbf{w}\|$ **78.** $\|\mathbf{v}\| + \|\mathbf{w}\|$ **79.** $\mathbf{v} \times \mathbf{w}$ **80.** $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w})$

81. Find a unit vector in the same direction as **v** and then in the opposite direction of **v**.

82. Find a unit vector orthogonal to both **v** and **w**.

In Problems 83–90, find the dot product $\mathbf{v} \cdot \mathbf{w}$ and the angle between **v** *and* **w**.

83. $\mathbf{v} = -2\mathbf{i} + \mathbf{j}$, $\mathbf{w} = 4\mathbf{i} - 3\mathbf{j}$

84. $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$, $\mathbf{w} = \mathbf{i} + \mathbf{j}$

85. $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$, $\mathbf{w} = -\mathbf{i} + \mathbf{j}$

86. $\mathbf{v} = \mathbf{i} + 4\mathbf{j}$, $\mathbf{w} = 3\mathbf{i} - 2\mathbf{j}$

87. $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{w} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

88. $\mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{w} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$

89. $\mathbf{v} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{w} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$

90. $\mathbf{v} = -\mathbf{i} - 2\mathbf{i} + 3\mathbf{k}$, $\mathbf{w} = 5\mathbf{i} + \mathbf{j} + \mathbf{k}$

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–20, solve each system of equations algebraically using the method of substitution or the method of elimination. If the system has no solution, say that it is inconsistent. Verify your result using a graphing utility.

1. $\begin{cases} 2x - y = 5 \\ 5x + 2y = 8 \end{cases}$

2. $\begin{cases} 2x + 3y = 2 \\ 7x - y = 3 \end{cases}$

3. $\begin{cases} 3x - 4y = 4 \\ x - 3y = \frac{1}{2} \end{cases}$

4. $\begin{cases} 2x + y = 0 \\ 5x - 4y = -\frac{13}{2} \end{cases}$

5. $\begin{cases} x - 2y - 4 = 0 \\ 3x + 2y - 4 = 0 \end{cases}$

6. $\begin{cases} x - 3y + 5 = 0 \\ 2x + 3y - 5 = 0 \end{cases}$

7. $\begin{cases} y = 2x - 5 \\ x = 3y + 4 \end{cases}$

8. $\begin{cases} x = 5y + 2 \\ y = 5x + 2 \end{cases}$

9. $\begin{cases} x - y + 4 = 0 \\ \frac{1}{2}x + \frac{1}{6}y + \frac{2}{3} = 0 \end{cases}$

10. $\begin{cases} x + \frac{1}{4}y = 2 \\ y + 4x + 2 = 0 \end{cases}$

11. $\begin{cases} x - 2y - 8 = 0 \\ 2x + 2y - 10 = 0 \end{cases}$

12. $\begin{cases} x - 3y + \frac{7}{2} = 0 \\ \frac{1}{2}x + 3y - 5 = 0 \end{cases}$

13. $\begin{cases} y - 2x = 11 \\ 2y - 3x = 18 \end{cases}$

14. $\begin{cases} 3x - 4y - 12 = 0 \\ 5x + 2y + 6 = 0 \end{cases}$

15. $\begin{cases} 2x + 3y - 13 = 0 \\ 3x - 2y = 0 \end{cases}$

16. $\begin{cases} 4x + 5y = 21 \\ 5x + 6y = 42 \end{cases}$

17. $\begin{cases} 3x - 2y = 8 \\ x - \frac{2}{3}y = 12 \end{cases}$

18. $\begin{cases} 2x + 5y = 10 \\ 4x + 10y = 15 \end{cases}$

19. $\begin{cases} x + 2y - z = 6 \\ 2x - y + 3z = -13 \\ 3x - 2y + 3z = -16 \end{cases}$

20. $\begin{cases} x + 5y - z = 2 \\ 2x + y + z = 7 \\ x - y + 2z = 11 \end{cases}$

In Problems 21–28, use the following matrices to compute each expression. Verify your result using a graphing utility.

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -4 \\ 1 & 5 \\ 5 & -2 \end{bmatrix}$$

21. $A + C$ **22.** $A - C$ **23.** $6A$ **24.** $-4B$

25. AB **26.** BA **27.** CB **28.** BC

In Problems 29–34, find the inverse of each matrix algebraically, if there is one. If there is not an inverse, say that the matrix is singular. Verify your result using a graphing utility.

29. $\begin{bmatrix} 4 & 6 \\ 1 & 3 \end{bmatrix}$

30. $\begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix}$

31. $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

32. $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

33. $\begin{bmatrix} 4 & -8 \\ -1 & 2 \end{bmatrix}$

34. $\begin{bmatrix} -3 & 1 \\ -6 & 2 \end{bmatrix}$

In Problems 35–44, solve each system of equations algebraically using matrices. If the system has no solution, say that it is inconsistent. Verify your result using a graphing utility.

35. $\begin{cases} 3x - 2y = 1 \\ 10x + 10y = 5 \end{cases}$

36. $\begin{cases} 3x + 2y = 6 \\ x - y = -\frac{1}{2} \end{cases}$

37. $\begin{cases} 5x + 6y - 3z = 6 \\ 4x - 7y - 2z = -3 \\ 3x + y - 7z = 1 \end{cases}$

38. $\begin{cases} 2x + y + z = 5 \\ 4x - y - 3z = 1 \\ 8x + y - z = 5 \end{cases}$

39. $\begin{cases} x - 2z = 1 \\ 2x + 3y = -3 \\ 4x - 3y - 4z = 3 \end{cases}$

40. $\begin{cases} x + 2y - z = 2 \\ 2x - 2y + z = -1 \\ 6x + 4y + 3z = 5 \end{cases}$

41. $\begin{cases} x - y + z = 0 \\ x - y - 5z - 6 = 0 \\ 2x - 2y + z - 1 = 0 \end{cases}$

42. $\begin{cases} 4x - 3y + 5z = 0 \\ 2x + 4y - 3z = 0 \\ 6x + 2y + z = 0 \end{cases}$

43. $\begin{cases} x - y - z - t = 1 \\ 2x + y - z + 2t = 3 \\ x - 2y - 2z - 3t = 0 \\ 3x - 4y + z + 5t = -3 \end{cases}$

44. $\begin{cases} x - 3y + 3z - t = 4 \\ x + 2y - z = -3 \\ x + 3z + 2t = 3 \\ x + y + 5z = 6 \end{cases}$

In Problems 45–50, find the value of each determinant algebraically. Verify your result using a graphing utility.

45. $\begin{vmatrix} 3 & 4 \\ 1 & 3 \end{vmatrix}$

46. $\begin{vmatrix} -4 & 0 \\ 1 & 3 \end{vmatrix}$

47. $\begin{vmatrix} 1 & 4 & 0 \\ -1 & 2 & 6 \\ 4 & 1 & 3 \end{vmatrix}$

48. $\begin{vmatrix} 2 & 3 & 10 \\ 0 & 1 & 5 \\ -1 & 2 & 3 \end{vmatrix}$

49. $\begin{vmatrix} 2 & 1 & -3 \\ 5 & 0 & 1 \\ 2 & 6 & 0 \end{vmatrix}$

50. $\begin{vmatrix} -2 & 1 & 0 \\ 1 & 2 & 3 \\ -1 & 4 & 2 \end{vmatrix}$

In Problems 51–56, use Cramer's Rule, if applicable, to solve each system.

51. $\begin{cases} x - 2y = 4 \\ 3x + 2y = 4 \end{cases}$

52. $\begin{cases} x - 3y = -5 \\ 2x + 3y = 5 \end{cases}$

53. $\begin{cases} 2x + 3y - 13 = 0 \\ 3x - 2y = 0 \end{cases}$

54. $\begin{cases} 3x - 4y - 12 = 0 \\ 5x + 2y + 6 = 0 \end{cases}$

55. $\begin{cases} [illegible] = 6 \\ 2x - y + 3z = -13 \\ 3x - 2y + 3z = -16 \end{cases}$

56. $\begin{cases} x - y + z = 8 \\ 2x + 3y - z = -2 \\ 3x - y - 9z = 9 \end{cases}$

In Problems 57–66, write the partial decomposition of each rational expression.

57. $\dfrac{6}{x(x-4)}$

58. $\dfrac{x}{(x+2)(x-3)}$

59. $\dfrac{x-4}{x^2(x-1)}$

60. $\dfrac{2x-6}{(x-2)^2(x-1)}$

61. $\dfrac{x}{(x^2+9)(x+1)}$

62. $\dfrac{3x}{(x-2)(x^2+1)}$

63. $\dfrac{x^3}{(x^2+4)^2}$

64. $\dfrac{x^3+1}{(x^2+16)^2}$

65. $\dfrac{x^2}{(x^2+1)(x^2-1)}$

66. $\dfrac{4}{(x^2+4)(x^2-1)}$

In Problems 67–76, solve each system of equations algebraically. Verify your result using a graphing utility.

67. $\begin{cases} 2x + y + 3 = 0 \\ x^2 + y^2 = 5 \end{cases}$

68. $\begin{cases} x^2 + y^2 = 16 \\ 2x - y^2 = -8 \end{cases}$

69. $\begin{cases} 2xy + y^2 = 10 \\ 3y^2 - xy = 2 \end{cases}$

70. $\begin{cases} 3x^2 - y^2 = 1 \\ 7x^2 - 2y^2 - 5 = 0 \end{cases}$

71. $\begin{cases} x^2 + y^2 = 6y \\ x^2 = 3y \end{cases}$

72. $\begin{cases} 2x^2 + y^2 = 9 \\ x^2 + y^2 = 9 \end{cases}$

73. $\begin{cases} 3x^2 + 4xy + 5y^2 = 8 \\ x^2 + 3xy + 2y^2 = 0 \end{cases}$

74. $\begin{cases} 3x^2 + 2xy - 2y^2 = 6 \\ xy - 2y^2 + 4 = 0 \end{cases}$

75. $\begin{cases} x^2 - 3x + y^2 + y = -2 \\ \dfrac{x^2 - x}{y} + y + 1 = 0 \end{cases}$

76. $\begin{cases} x^2 + x + y^2 = y + 2 \\ x + 1 = \dfrac{2-y}{x} \end{cases}$

In Problems 77–82, graph each system of inequalities. Tell whether the graph is bounded or unbounded, and label the corner points.

77. $\begin{cases} -2x + y \le 2 \\ x + y \ge 2 \end{cases}$

78. $\begin{cases} x - 2y \le 6 \\ 2x + y \ge 2 \end{cases}$

79. $\begin{cases} x \ge 0 \\ y \ge 0 \\ x + y \le 4 \\ 2x + 3y \le 6 \end{cases}$

80. $\begin{cases} x \ge 0 \\ y \ge 0 \\ 3x + y \ge 6 \\ 2x + y \ge 2 \end{cases}$

81. $\begin{cases} x \ge 0 \\ y \ge 0 \\ 2x + y \le 8 \\ x + 2y \ge 2 \end{cases}$

82. $\begin{cases} x \ge 0 \\ y \ge 0 \\ 3x + y \le 9 \\ 2x + 3y \ge 6 \end{cases}$

In Problems 83–88, solve each linear programming problem.

83. Maximize $z = 3x + 4y$ subject to $x \ge 0$, $y \ge 0$, $3x + 2y \ge 6$, $x + y \le 8$
84. Maximize $z = 2x + 4y$ subject to $x \ge 0$, $y \ge 0$, $x + y \le 6$, $x \ge 2$
85. Minimize $z = 3x + 5y$ subject to $x \ge 0$, $y \ge 0$, $x + y \ge 1$, $3x + 2y \le 12$, $x + 3y \le 12$
86. Minimize $z = 3x + y$ subject to $x \ge 0$, $y \ge 0$, $x \le 8$, $y \le 6$, $2x + y \ge 4$
87. Maximize $z = 5x + 4y$ subject to $x \ge 0$, $y \ge 0$, $x + 2y \ge 2$, $3x + 4y \le 12$, $y \ge x$
88. Maximize $z = 4x + 5y$ subject to $x \ge 0$, $y \ge 0$, $2x + 3y \ge 6$, $x \ge 4$, $2x + y \le 12$

89. Find A such that the system of equations has infinitely many solutions.

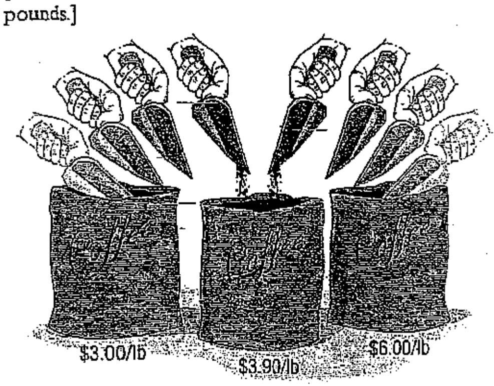
$$\begin{cases} 2x + 5y = 5 \\ 4x + 10y = A \end{cases}$$

90. Find A such that the system in Problem 89 is inconsistent.

91. **Curve Fitting** Find the quadratic function $y = ax^2 + bx + c$ that passes through the three points $(0, 1)$, $(1, 0)$, and $(-2, 1)$.

92. **Curve Fitting** Find the general equation of the circle that passes through the three points $(0, 1)$, $(1, 0)$, and $(-2, 1)$.
[**Hint:** The general equation of a circle is $x^2 + y^2 + Dx + Ey + F = 0$.]

93. **Blending Coffee** A coffee distributor is blending a new coffee that will cost \$3.90 per pound. It will consist of a blend of \$3.00 per pound coffee and \$6.00 per pound coffee. What amounts of each type of coffee should be mixed to achieve the desired blend?
[**Hint:** Assume that the weight of the blended coffee is 100 pounds.]



94. **Farming** A 1000-acre farm in Illinois is used to raise corn and soy beans. The cost per acre for raising corn is \$65 and the cost per acre for soy beans is \$45. If \$54,325 has been budgeted for costs and all the acreage is to be used, how many acres should be allocated for each crop?

95. **Cookie Orders** A cookie company makes three kinds of cookies, oatmeal raisin, chocolate chip, and shortbread, packaged in small, medium, and large boxes. The small box contains 1 dozen oatmeal raisin and 1 dozen chocolate chip; the medium box has 2 dozen oatmeal raisin, 1 dozen chocolate chip, and 1 dozen shortbread; the large box contains 2 dozen oatmeal raisin, 2 dozen chocolate chip, and 3 dozen shortbread. If you require exactly 15 dozen oatmeal raisin, 10 dozen chocolate chip, and 11 dozen shortbread, how many of each size box should you buy?

96. **Mixed Nuts** A store that specializes in selling nuts has 72 pounds of cashews and 120 pounds of peanuts available. These are to be mixed in 12-ounce packages as follows: a lower-priced package containing 8 ounces of peanuts and 4 ounces of cashews and a quality package containing 6 ounces of peanuts and 6 ounces of cashews.
(a) Use x to denote the number of lower-priced packages, and use y to denote the number of quality packages. Write a system of linear inequalities that describes the possible number of each kind of package.
(b) Graph the system and label the corner points.

97. A small rectangular lot has a perimeter of 68 feet. If its diagonal is 26 feet, what are the dimensions of the lot?

98. The area of a rectangular window is 4 square feet. If the diagonal measures $2\sqrt{2}$ feet, what are the dimensions of the window?

99. **Geometry** A certain right triangle has a perimeter of 14 inches. If the hypotenuse is 6 inches long, what are the lengths of the legs?

100. **Geometry** A certain isosceles triangle has a perimeter of 18 inches. If the altitude is 6 inches, what is the length of the base?

101. **Building a Fence** How much fence is required to enclose 5000 square feet by two squares whose sides are in the ratio of 1:2?

102. **Mixing Acids** A chemistry laboratory has three containers of hydrochloric acid, HCl. One container holds a solution with a concentration of 10% HCl, the second holds 25% HCl, and the third holds 40% HCl. How many liters of each should be mixed to obtain 100 liters of a solution with a concentration of 30% HCl? Construct a table showing some of the possible combinations.

103. **Calculating Allowances** Katy, Mike, Danny, and Colleen agreed to do yard work at home for \$45 to be split among them. After they finished, their father determined that Mike deserves twice what Katy gets, Katy and Colleen deserve the same amount, and Danny deserves half of what Katy gets. How much does each receive?

104. **Finding the Speed of the Jet Stream** On a flight between Midway Airport in Chicago and Ft. Lauderdale, Florida, a Boeing 737 jet maintains an airspeed of 475 miles per hour. If the trip from Chicago to Ft. Lauderdale takes 2 hours, 30 minutes and the return flight takes 2 hours, 50 minutes, what is the speed of the jet stream? (Assume that the speed of the jet stream remains constant at the various altitudes of the plane and that the plane flies with the jet stream one way and against it the other way.)

105. **Constant Rate Jobs** If Bruce and Bryce work together for 1 hour and 20 minutes, they will finish a certain job. If Bryce and Marty work together for 1 hour and 36 minutes, the same job can be finished. If Marty and Bruce work together, they can complete this job in 2 hours and 40 minutes. How long will it take each of them working alone to finish the job?

106. **Maximizing Profit on Figurines** A factory manufactures two kinds of ceramic figurines: a dancing girl and a mermaid. Each requires three processes: molding, painting, and glazing. The daily labor available for molding is no more than 90 work-hours, labor available for painting does not exceed 120 work-hours, and labor available for glazing is no more than 60 work-hours. The dancing girl requires 3 work-hours for molding, 6 work-hours for painting, and 2 work-hours for glazing. The mermaid requires 3 work-hours for molding, 4 work-hours for painting, and 3 work-hours for glazing. If the profit on each figurine is \$25 for dancing girls and \$30 for mermaids, how many of each should be produced each day to maximize profit? If management decides to produce the number of each figurine that maximizes profit, determine which of these processes has excess work-hours assigned to it.

CHAPTER REVIEW

Things To Know

Sequence (p. 810)	A function whose domain is the set of positive integers.
Factorials (p. 814)	$0! = 1, 1! = 1, n! = n(n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$ if $n \geq 2$
Amount of an annuity (p. 820)	$A_0 = M, A_n = \left(1 + \frac{r}{N}\right)A_{n-1} + P$
Arithmetic sequence (p. 825)	$a_1 = a, a_n = a_{n-1} + d$, where a = first term, d = common difference, $a_n = a + (n-1)d$
Sum of the first n terms of an arithmetic sequence (p. 827)	$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + a_n)$
Geometric sequence (p. 831)	$a_1 = a, \quad a_n = ra_{n-1}$, where a = first term, r = common ratio, $a_n = ar^{n-1}, \quad r \neq 0$
Sum of the first n terms of a geometric sequence (p. 833)	$S_n = a\frac{1-r^n}{1-r}, \quad r \neq 0, 1$
Infinite geometric series (p. 834)	$a + ar + \cdots + ar^{n-1} + \cdots = \sum_{k=1}^{\infty} ar^{k-1}$
Sum of an infinite geometric series (p. 835)	$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}, \quad \lvert r \rvert < 1$
Principle of Mathematical Induction (p. 841)	Condition I: The statement is true for the natural number 1. Condition II: If the statement is true for some natural number k, it is also true for $k + 1$. Then the statement is true for all natural numbers.
Binomial coefficient (p. 845)	$\binom{n}{j} = \frac{n!}{j!(n-j)!}$
Pascal triangle (p. 846)	See Figure 18.
Binomial Theorem (p. 847)	$(x + a)^n = \binom{n}{0}x^n + \binom{n}{1}ax^{n-1} + \cdots + \binom{n}{j}a^jx^{n-j} + \cdots + \binom{n}{n}a^n$

How To

Write the first several terms of a sequence (p. 811)

Write the terms of a sequence defined by a recursion formula (p. 814)

Write a sequence in summation notation (p. 816)

Find the sum of a sequence by hand and by using a graphing utility (p. 817)

Solve annuity and amortization problems (p. 819)

Determine if a sequence is arithmetic (p. 825)

Find a formula for an arithmetic sequence (p. 826)

Find the sum of an arithmetic sequence (p. 827)

Determine if a sequence is geometric (p. 831)

Find a formula for a geometric sequence (p. 832)

Find the sum of a geometric sequence (p. 833)

Find the sum of a geometric series (p. 834)

Prove statements using mathematical induction (p. 840)

Evaluate a binomial coefficient (p. 845)

Expand a binomial (p. 847)

Fill-in-the-Blank Items

1. A(n) ______________ is a function whose domain is the set of positive integers.
2. In a(n) ______________ sequence, the difference between successive terms is always the same number.
3. In a(n) ______________ sequence, the ratio of successive terms is always the same number.
4. The ______________ ______________ is a triangular display of the binomial coefficients.
5. $\binom{6}{2} =$ ______________.

True/False Items

T F 1. A sequence is a function.

T F 2. For arithmetic sequences, the difference of successive terms is always the same number.

T F 3. For geometric sequences, the ratio of successive terms is always the same number.

T F 4. Mathematical induction can sometimes be used to prove theorems that involve natural numbers.

T F 5. $\binom{n}{j} = \dfrac{j!}{n!(n-j)!}$

T F 6. The expansion of $(x + a)^n$ contains n terms.

T F 7. $\sum_{i=1}^{n+1} i = 1 + 2 + 3 + \cdots + n$

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

In Problems 1–8, write down the first five terms of each sequence.

1. $\left\{(-1)^n\left(\dfrac{n+3}{n+2}\right)\right\}$ **2.** $\{(-1)^{n+1}(2n+3)\}$ **3.** $\left\{\dfrac{2^n}{n^2}\right\}$ **4.** $\left\{\dfrac{e^n}{n}\right\}$

5. $a_1 = 3;\quad a_n = \frac{2}{3}a_{n-1}$ **6.** $a_1 = 4;\quad a_n = -\frac{1}{4}a_{n-1}$ **7.** $a_1 = 2;\quad a_n = 2 - a_{n-1}$ **8.** $a_1 = -3;\quad a_n = 4 + a_{n-1}$

In Problems 9–20, determine whether the given sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference and the sum of the first n terms. If the sequence is geometric, find the common ratio and the sum of the first n terms.

9. $\{n+5\}$ **10.** $\{4n+3\}$ **11.** $\{2n^3\}$ **12.** $\{2n^2-1\}$

13. $\{2^{3n}\}$ **14.** $\{3^{2n}\}$ **15.** $0, 4, 8, 12, \ldots$ **16.** $1, -3, -7, -11, \ldots$

17. $3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \ldots$ **18.** $5, -\frac{5}{3}, \frac{5}{9}, -\frac{5}{27}, \frac{5}{81}, \ldots$ **19.** $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$ **20.** $\frac{3}{2}, \frac{5}{4}, \frac{7}{6}, \frac{9}{8}, \frac{11}{10}, \ldots$

In Problems 21–26, evaluate each sum. Verify your results using a graphing utility.

21. $\sum_{k=1}^{5}(k^2+12)$ **22.** $\sum_{k=1}^{3}(k+2)^2$ **23.** $\sum_{k=1}^{10}(3k-9)$ **24.** $\sum_{k=1}^{9}(-2k+8)$

25. $\sum_{k=1}^{7}\left(\frac{1}{3}\right)^k$ **26.** $\sum_{k=1}^{10}(-2)^k$

In Problems 27–32, find the indicated term in each sequence (a) by hand and (b) using a graphing utility.

27. 9th term of $3, 7, 11, 15, \ldots$
28. 8th term of $1, -1, -3, -5, \ldots$
29. 11th term of $1, \frac{1}{10}, \frac{1}{100}, \ldots$
30. 11th term of $1, 2, 4, 8, \ldots$
31. 9th term of $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, \ldots$
32. 9th term of $\sqrt{2}, 2, 2^{3/2}, \ldots$

In Problems 33–36, find a general formula for each arithmetic sequence.

33. 7th term is 31; 20th term is 96
34. 8th term is -20; 17th term is -47
35. 10th term is 0; 18th term is 8
36. 12th term is 30; 22nd term is 50

In Problems 37–42, find the sum of each infinite geometric series.

37. $3 + 1 + \frac{1}{3} + \frac{1}{9} + \cdots$
38. $2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$
39. $2 - 1 + \frac{1}{2} - \frac{1}{4} + \cdots$
40. $6 - 4 + \frac{8}{3} - \frac{16}{9} + \cdots$
41. $\displaystyle\sum_{k=1}^{\infty} 4\left(\frac{1}{2}\right)^{k-1}$
42. $\displaystyle\sum_{k=1}^{\infty} 3\left(-\frac{3}{4}\right)^{k-1}$

In Problems 43–48, use the Principle of Mathematical Induction to show that the given statement is true for all natural numbers.

43. $3 + 6 + 9 + \cdots + 3n = \dfrac{3n}{2}(n + 1)$
44. $2 + 6 + 10 + \cdots + (4n - 2) = 2n^2$
45. $2 + 6 + 18 + \cdots + 2 \cdot 3^{n-1} = 3^n - 1$
46. $3 + 6 + 12 + \cdots + 3 \cdot 2^{n-1} = 3(2^n - 1)$
47. $1^2 + 4^2 + 7^2 + \cdots + (3n - 2)^2 = \frac{1}{2}n(6n^2 - 3n - 1)$
48. $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n + 2) = \dfrac{n}{6}(n + 1)(2n + 7)$

In Problems 49–52, expand each expression using the Binomial Theorem.

49. $(x + 2)^5$
50. $(x - 3)^4$
51. $(2x + 3)^5$
52. $(3x - 4)^4$

53. Find the coefficient of x^7 in the expansion of $(x + 2)^9$.
54. Find the coefficient of x^3 in the expansion of $(x - 3)^8$.
55. Find the coefficient of x^2 in the expansion of $(2x + 1)^7$.
56. Find the coefficient of x^6 in the expansion of $(2x + 1)^8$.
57. **Constructing a Brick Staircase** A brick staircase has a total of 25 steps. The bottom step requires 80 bricks. Each successive step requires three less bricks than the prior step.
    (a) How many bricks are required for the top step?
    (b) How many bricks are required to build the staircase?
58. **Creating a Floor Design** A mosaic tile floor is designed in the shape of a trapezoid 30 feet wide at the base and 15 feet wide at the top. The tiles, 12 inches by 12 inches, are to be placed so that each successive row contains one less tile than the row below. How many tiles will be required? [**Hint:** Refer to Figure 13].
59. **Retirement Planning** Chris gets paid once a month and contributes \$200 each pay period into his 401(k). If Chris plans on retiring in 20 years, what will the value of his 401(k) be if the per annum rate of return of the 401(k) is 10% compounded monthly?
60. **Retirement Planning** Jacky contributes \$500 every quarter to an IRA. If Jacky plans on retiring in 30 years, what will the value of the IRA be if the per annum rate of return of the IRA is 8% compounded quarterly?
61. **Bouncing Balls** A ball is dropped from a height of 20 feet. Each time it strikes the ground, it bounces up to three-quarters of the previous height.
    (a) What height will the ball bounce up to after it strikes the ground for the third time?
    (b) What is its height after it strikes the ground for the nth time?
    (c) How many times does the ball need to strike the ground before its height is less than 6 inches?
    (d) What total distance does the ball travel before it stops bouncing?
62. **Salary Increases** Your friend has just been hired at an annual salary of \$20,000. If she expects to receive annual increases of 4%, what will her salary be as she begins her fifth year?
63. **Home Loan** Mike and Yola borrowed \$190,000 at 6.75% per annum compounded monthly for 30 years to purchase a home. Their monthly payment is determined to be \$1232.34.
    (a) Find a recursive formula for their balance after each monthly payment has been made.
    (b) Determine Mike and Yola's balance after the first payment.
    (c) Using a graphing utility, create a table showing Mike and Yola's balance after each monthly payment.
    (d) Using a graphing utility, determine when Mike and Yola's balance will be below \$100,000.
    (e) Using a graphing utility, determine when Mike and Yola will pay off the balance.
    (f) Determine Mike and Yola's interest expense when the loan is paid.
    (g) Suppose that Mike and Yola decide to pay an additional \$100 each month on their loan. Answer parts (a) to (f) under this scenario.

CHAPTER REVIEW

Things To Know

Limit (p. 902)

$\lim_{x \to c} f(x) = N$ — As x gets closer to c, $x \neq c$, the values of f get closer to N.

Limit Formulas (p. 908)

$\lim_{x \to c} A = A$ — The limit of a constant is the constant.

$\lim_{x \to c} x = c$ — The limit of x as x approaches c is c.

Limit Properties

$\lim_{x \to c}[f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$ (p. 909) — The limit of a sum equals the sum of the limits.

$\lim_{x \to c}[f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$ (p. 909) — The limit of a difference equals the difference of the limits.

$\lim_{x \to c}[f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$ (p. 910) — The limit of a product equals the product of the limits.

$\lim_{x \to c}[f(x)/g(x)] = [\lim_{x \to c} f(x)]/[\lim_{x \to c} g(x)]$ (p. 912) provided $\lim_{x \to c} g(x) \neq 0$ — The limit of a quotient equals the quotient of the limits, provided that the limit of the denominator is not zero.

Limit of a Polynomial (p. 911)

$\lim_{x \to c} P(x) = P(c)$, where P is a polynomial

Continuous Function (p. 918)

$\lim_{x \to c} f(x) = f(c)$

Derivative of a Function (p. 923)

$f'(c) = \lim_{x \to c} \dfrac{f(x) - f(c)}{x - c}$, provided that the limit exists

Area Under a Graph (p. 934)

$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(u_i)\,\Delta x$, provided the limit exists

How To

Find a limit using a table (p. 902)

Find a limit using a graph (p. 904)

Find the limit of a sum, a difference, a product, and a quotient (p. 909)

Find the limit of polynomial (p. 911)

Find the limit of a power or a root (p. 912)

Find the limit of an average rate of change (p. 914)

Find the one-sided limits of a function (p. 915)

Determine whether a function is continuous (p. 917)

Find an equation of the tangent line to the graph of a function (p. 922)

Find the derivative of a function (p. 923)

Find instantaneous rates of change (p. 925)

Find the speed of a particle (p. 925)

Approximate the area under the graph of a function (p. 930)

Approximate integrals using a graphing utility (p. 935)

Fill-in-the-Blank Items

1. The notation ______________ may be described by saying, "For x approximately equal to c, but $x \neq c$, the value $f(x)$ is approximately equal to N."
2. If $\lim_{x \to c} f(x) = N$ and f is continuous at c, then $f(c)$ ______________ N.
3. If there is no single number that the value of f approaches when x is close to c, then $\lim_{x \to c} f(x)$ does ______________ ______________.
4. When $\lim_{x \to c} f(x) = f(c)$, we say that f is ______________ at c.
5. $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, provided that $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ each exist and $\lim_{x \to c} g(x)$ ______________ 0.

6. If $\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = R$, then $\lim_{x \to c} f(x)$ exists provided that L ______________ R.
7. The derivative of f at c equals the slope of the ______________ line to the graph of f at $(c, f(c))$.
8. The area under the graph of $f(x) = \sqrt{x^2 + 1}$ from 0 to 2 may be symbolized by the integral ______________.

True/False Items

T F 1. The limit of the sum of two functions equals the sum of their limits, provided that each limit exists.

T F 2. The limit of a function f as x approaches c always equals $f(c)$.

T F 3. $\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = 8$

T F 4. The function $f(x) = \frac{5x^2}{x^2 + 4}$ is continuous at $x = -2$.

T F 5. The limit of a quotient of two functions equals the quotient of their limits, provided that each limit exists and the limit of the denominator is not zero.

T F 6. The derivative of a function is the limit of an average rate of change.

T F 7. The area under the graph of $f(x) = x^4$ from 0 to 2 equals $\int_0^2 x^4\,dx$.

Review Exercises

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

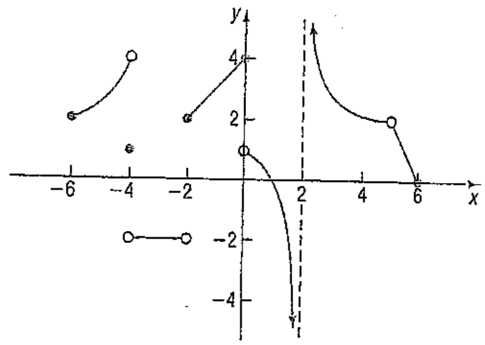
In Problems 1–22, find the limit.

1. $\lim_{x \to 2}(3x^2 - 2x + 1)$
2. $\lim_{x \to 1}(-2x^3 + x + 4)$
3. $\lim_{x \to -2}(x^2 + 1)^2$
4. $\lim_{x \to -2}(x^3 + 1)^2$
5. $\lim_{x \to 3}\sqrt{x^2 + 7}$
6. $\lim_{x \to -2}\sqrt[3]{x + 10}$
7. $\lim_{x \to 1^-}\sqrt{1 - x^2}$
8. $\lim_{x \to 2^+}\sqrt{3x - 2}$
9. $\lim_{x \to 2}(5x + 6)^{3/2}$
10. $\lim_{x \to -3}(15 - 3x)^{-3/2}$
11. $\lim_{x \to -1}\frac{x^2 + x + 2}{x^2 - 9}$
12. $\lim_{x \to 3}\frac{3x + 4}{x^2 + 1}$
13. $\lim_{x \to 1}\frac{x - 1}{x^3 - 1}$
14. $\lim_{x \to -1}\frac{x^2 - 1}{x^2 + x}$
15. $\lim_{x \to -3}\frac{x^2 - 9}{x^2 - x - 12}$
16. $\lim_{x \to -3}\frac{x^2 + 2x - 3}{x^2 - 9}$
17. $\lim_{x \to -1^-}\frac{x^2 - 1}{x^3 - 1}$
18. $\lim_{x \to 2^+}\frac{x^2 - 4}{x^3 - 8}$
19. $\lim_{x \to 2}\frac{x^3 - 8}{x^3 - 2x^2 + 4x - 8}$
20. $\lim_{x \to 1}\frac{x^3 - 1}{x^3 - x^2 + 3x - 3}$
21. $\lim_{x \to 3}\frac{x^4 - 3x^3 + x - 3}{x^3 - 3x^2 + 2x - 6}$
22. $\lim_{x \to -1}\frac{x^4 + x^3 + 2x + 2}{x^3 + x^2}$

In Problems 23–30, determine whether f is continuous at c.

23. $f(x) = 3x^4 - x^2 + 2, \quad c = 5$
24. $f(x) = \frac{x^2 - 9}{x + 10}, \quad c = 2$
25. $f(x) = \frac{x^2 - 4}{x + 2}, \quad c = -2$
26. $f(x) = \frac{x^2 + 6x}{x^2 - 6x}, \quad c = 0$
27. $f(x) = \begin{cases} \frac{x^2 - 4}{x + 2} & \text{if } x \neq -2 \\ 4 & \text{if } x = -2 \end{cases}, \quad c = -2$
28. $f(x) = \begin{cases} \frac{x^2 + 6x}{x^2 - 6x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}, \quad c = 0$
29. $f(x) = \begin{cases} \frac{x^2 - 4}{x + 2} & \text{if } x \neq -2 \\ -4 & \text{if } x = -2 \end{cases}, \quad c = -2$
30. $f(x) = \begin{cases} \frac{x^2 + 6x}{x^2 - 6x} & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases}, \quad c = 0$

In Problems 31–50, use the accompanying graph of $y = f(x)$.



31. What is the domain of f?

32. What is the range of f?

33. Find the x-intercept(s), if any, of f.

34. Find the y-intercept(s), if any, of f.

35. Find $f(-6)$ and $f(-4)$.

36. Find $f(-2)$ and $f(6)$.

37. Find $\lim_{x\to -4^-} f(x)$.

38. Find $\lim_{x\to -4^+} f(x)$.

39. Find $\lim_{x\to -2^-} f(x)$.

40. Find $\lim_{x\to -2^+} f(x)$.

41. Find $\lim_{x\to 2^-} f(x)$.

42. Find $\lim_{x\to 2^+} f(x)$.

43. Does $\lim_{x\to 0} f(x)$ exist? If it does, what is it?

44. Does $\lim_{x\to 2} f(x)$ exist? If it does, what is it?

45. Is f continuous at -2?

46. Is f continuous at -4?

47. Is f continuous at 0?

48. Is f continuous at 2?

49. Is f continuous at 4?

50. Is f continuous at 5?

In Problems 51 and 52, discuss whether R is continuous at c. Use the one-sided limits of R at c to analyze the graph of R. Graph R.

51. $R(x) = \dfrac{x+4}{x^2-16}$ at $c = -4$ and $c = 4$

52. $R(x) = \dfrac{3x^2+6x}{x^2-4}$ at $c = -2$ and $c = 2$

In Problems 53 and 54, determine where each rational function is undefined. Determine whether an asymptote or a hole appears at such numbers. Graph R using a graphing utility to verify your answers.

53. $R(x) = \dfrac{x^3-2x^2+4x-8}{x^2-11x+18}$

54. $R(x) = \dfrac{x^3+3x^2-2x-6}{x^2+x-6}$

In Problems 55–60, find the slope of the tangent line to the graph of f at P. Graph f and the tangent line.

55. $f(x) = 2x^2 + 8x$ at $(1, 10)$

56. $f(x) = 3x^2 - 6x$ at $(0, 0)$

57. $f(x) = x^2 + 2x - 3$ at $(-1, -4)$

58. $f(x) = 2x^2 + 5x - 3$ at $(1, 4)$

59. $f(x) = x^3 + x^2$ at $(2, 12)$

60. $f(x) = x^3 - x^2$ at $(1, 0)$

In Problems 61–66, find the derivative of each function at the number indicated.

61. $f(x) = -4x^2 + 5$ at 3

62. $f(x) = -4 + 3x^2$ at 1

63. $f(x) = x^2 - 3x$ at 0

64. $f(x) = 2x^2 + 4x$ at -1

65. $f(x) = 2x^2 + 3x + 2$ at 1

66. $f(x) = 3x^2 - 4x + 1$ at 2

In Problems 67–70, find the derivative of each function at c using a graphing utility.

67. $f(x) = 4x^4 - 3x^3 + 6x - 9$ at -2

68. $f(x) = \dfrac{-6x^3+9x-2}{8x^2+6x-1}$ at 5

69. $f(x) = x^3 \tan x$ at $\dfrac{\pi}{6}$

70. $f(x) = x \sec x$ at $\dfrac{\pi}{6}$

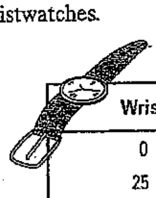
71. **Instantaneous Speed of a Ball** In physics it is shown that the height s of a ball thrown straight up with an initial speed of 96 ft/sec from a rooftop 112 feet high is

$$s = s(t) = -16t^2 + 96t + 112$$

where t is the elapsed time that the ball is in the air. The ball misses the rooftop on its way down and eventually strikes the ground.
(a) When does the ball strike the ground? That is, how long is the ball in the air?
(b) At what time t will the ball pass the rooftop on its way down?
(c) What is the average speed of the ball between $t = 0$ and $t = 2$?
(d) What is the instantaneous speed of the ball at time t?
(e) What is the instantaneous speed of the ball at $t = 2$?
(f) When is the instantaneous speed of the ball equal to zero?
(g) What is the instantaneous speed of the ball as it passes the rooftop on the way down?
(h) What is the instantaneous speed of the ball when it strikes the ground?

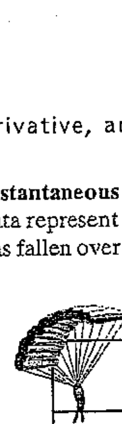
72. **Finding an Instantaneous Rate of Change** The area A of a circle is πr^2. Find the instantaneous rate of change of area with respect to r at $r = 2$ feet. What is the average rate of change between 2 and 3? What is the average rate of change between 2 and 2.5? Between 2 and 2.1?

73. **Instantaneous Rate of Change** The following data represent the revenue, R (in dollars), received from selling x wristwatches at Wilk's Watch Shop.
(a) Find the average rate of change of revenue from 25 to 130 wristwatches.
(b) Find the average rate of change of revenue from 25 to 90 wristwatches.
(c) Find the average rate of change of revenue from 25 to 50 wristwatches.
(d) Using a graphing utility, find the quadratic function of best fit.
(e) Using the function found in part (d), determine the instantaneous rate of change of revenue at $x = 25$ wristwatches.



Wristwatches, x	Revenue, R
0	0
25	2340
40	3600
50	4375
90	6975
130	8775
160	9600
200	10,000
220	9900
250	9375

74. **Instantaneous Speed of a Parachutist** The following data represent the distances s (in feet) that a parachutist has fallen over time t (in seconds).



Time, t (in Seconds)	Distance, s (in Feet)
1	16
2	64
3	144
4	256
5	400

(a) Find the average rate of change of distance from 1 to 4 seconds.
(b) Find the average rate of change of distance from 1 to 3 seconds.
(c) Find the average rate of change of distance from 1 to 2 seconds.
(d) Using a graphing utility, find the power function of best fit.
(e) Using the function found in part (d), determine the instantaneous speed at $t = 1$ second.

75. The function $f(x) = 2x + 3$ is defined on the interval $[0, 4]$.
(a) Graph f
In (b)–(e), approximate the area A under f from 0 to 4 as follows:
(b) By partitioning $[0, 4]$ into four subintervals of equal length and choosing u as the left endpoint of each subinterval.
(c) By partitioning $[0, 4]$ into four subintervals of equal length and choosing u as the right endpoint of each subinterval.
(d) By partitioning $[0, 4]$ into eight subintervals of equal length and choosing u as the left endpoint of each subinterval.
(e) By partitioning $[0, 4]$ into eight subintervals of equal length and choosing u as the right endpoint of each subinterval.
(f) What is the actual area A?

76. Repeat Problem 75 for $f(x) = -2x + 8$.