Unit6

Calculator Test Review

~~Ch. 4~~ Free Response WS #1

1. 1999 #1 (Calculators permitted):

A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

b) Find the acceleration of the particle at time $t = 1.5$.
Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

2. 2001 #2 (Calculators permitted):

t (days)	0	3	6	9	12	15
W(t) (°C)	20	31	28	24	22	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

a) Use data from the table to find an approximation for $W'(12)$.
Show the computations that lead to your answer. Indicate units of measure.

b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

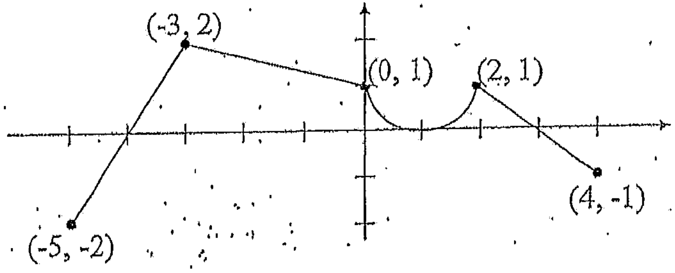
c) A student proposes the function P, given by $P(t) = 20 + 10te^{\left(-\frac{t}{3}\right)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

d) Use the function P defined in part c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

②

3. **2004 #5 (No Calculators)**

The graph of the function f shown to the right consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



a) Find g'(x) and g''(x)

b) Find g(0), g'(0), and g''(-1)

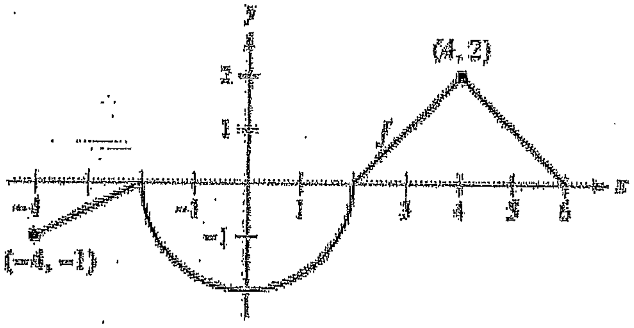
c) Find all values of x in the open interval (-5, 4) at which g attains a relative maximum. Justify your answer.

d) Find the absolute minimum value of g on the closed interval [-5, 4]. Show work and justify your answer.

e) Find all values of x in the open interval (-5, 4) at which the graph of g has a point of inflection.

Ch. 4 Test Review (Calculator Portion)

1. The graph of f consists of line segments and a semicircle, as shown: Let g(x) = $\int_{-2}^{x} f(t)\,dt$



a) Find g(2)

b) Find g(-4)

c) Find g(6)

d) Find g′(4)

e) Find g′(-2)

f) Find g″(5)

g) For what values of x is g increasing? Justify Answer

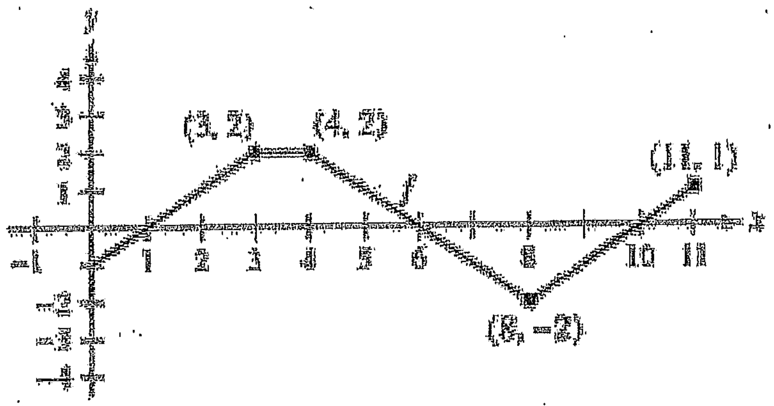
h) For what values of x is g decreasing? Justify Answer

i) Find the x-values of all points of inflection of g. Justify Answer

j) Find the absolute extrema of g on the interval [-4,6]

④

2. The graph of f consists of line segments. Let $h(x) = \int_6^x f(t)\,dt$



a) Find h(0)

b) Find h(6)

c) Find h(10)

d) Find h(11)

e) Find h'(3)

f) Find h''(9.5)

g) For what values of x is h increasing? Justify Answer

h) For what values of x is h'(x) decreasing?

i) Find the absolute extrema of g on the interval [0,11]

3. The table below shows the speed of a sprinter at the time intervals (in seconds) in the 200 meter race

time t (seconds)	0	2	5	8	11	17	20
Velocity V(t) (m/s)	5	6.5	7	8.5	9	8	7.5

a. Estimate $\int_0^{20} v(t)dt$ using the following methods

i. 6 trapezoids

ii. 3 left-handed rectangles

iii. 3 right-handed rectangles

iv. 3 middle rectangles

b. Find the average velocity on the interval [0, 20] using estimation from 6 trapezoids.

⑥

4. An object moving along a horizontal line has $v(t) = 4\sin(t^2 - 2t + 2)$ measured in meters per second from [0,4] (hint: set windows to x-values [-1, 5] and y-values [-6, 6])
*Round answers to 3 decimal places

a. Find the time(s) when the object is motionless

b. When does the object change directions in $0 < t < 4$?

c. Find the velocity of the object at $t = 3$ seconds.

d. Find the acceleration of the object at $t = 3$ seconds.

e. Is the object's speed increasing or decreasing at $t = 3$ seconds? Justify answer.

f. Find the total displacement of the object from $t = 0$ to $t = 4$ seconds (Show Integral Notation)

g. Find the total distance of the object from $t = 0$ to $t = 4$ seconds (Show Integral Notation)

h. Find the time when the object reaches minimum velocity in [0, 3]

i. Find the minimum velocity in [0, 3]

j. Given $x(0) = 2$, Find $x(4)$. (Show integral notation)

k. Find the average velocity in [0, 4]

l. Find the time(s) when object reaches average velocity.

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Ch. 4 Test Review WS #4 Riemann Sums Practice Worksheet

(Determining Units of Measure and Interpreting Definite Integrals!)

***Important Key Point*: When applying(or approximating) a Calculus process(derivatives or integrals), your units of measure will change!**

1)

t (minutes)	0	1	3	6	9	10
c(t) (ounces per minute)	0	5.1	4.2	3.3	1.2	2.3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The rate that water in the cup at time t is changing, $0 \leq t \leq 10$, is given by a differential function c(t), where t is measured in minutes. Select values if c(t), measured in ounces per minute are given in the table above.

a) Interpret the meaning of $c'(6)$ *and* indicate the units of measure.

b) Approximate the value of $c'(6)$ and indicate the units of measure.

c) Intepret the meaning of $\int_1^{10} c(t)dt$ and indicate the units of measure.

d) Approximate the value of $\int_1^{10} c(t)dt$ using 2 middle rectangles and indicate the units of measure.

e) Approximate the average rate of water being added on time interval [1, 10] using result from part d)

2)

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	−220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

a) Interpret the meaning of $v'(20)$ *and* indicate the units of measure.

b) Approximate the value of $v'(18)$ and indicate the units of measure.

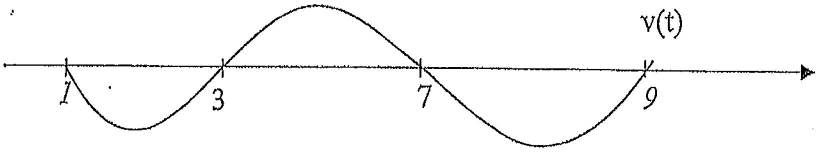
c) Intepret the meaning of $\int_{20}^{40} v(t)dt$ and indicate the units of measure.

d) Approximate the value of $\int_{20}^{40} v(t)dt$ using 2 trapezoids and indicate the units of measure.

e) Approximate Johanna's average velocity on [20, 40] using the results from part d)

AP Calculus AB (4.3-4.5)

PVA Particle Motion: Velocity Graph Practice Problem



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 9$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 1, 3, 7$ and 9 and the graph has horizontal tangents at $t = 2, 5,$ and 8.

The areas of the regions bounded are 2, 3, and 6 respectively. The position function for the particle is called x and at $t = 1$, $x(1) = 2$.

a. Create Sign lines for v(t) and a(t)

b. On what intervals (if any) is the velocity negative? Justify your answer.

c. On what intervals (if any) is the acceleration positive? Justify your answer.

d. On the interval $5 < t < 7$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

e. On the interval $7 < t < 8$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

f. Find the positions of the particle at $t = 3$, $t = 7$ and $t = 9$. (use definite integrals.)

g. State the absolute extrema and the t-values where they occur.

h. Find the total distance traveled by the particle from $t = 1$ to $t = 9$. (Use Integral Notation)

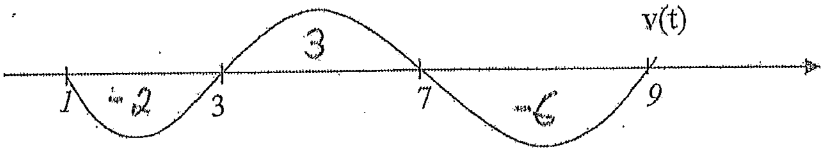
i. Find the total displacement of the particle from $t = 3$ to $t = 9$. (Use Integral Notation)

j. Sketch graph of x(t) below:

⑩

AP Calculus AB (4.3-4.5)

PVA Particle Motion: Velocity Graph Practice Problem



A particle moves horizontally so that its velocity at time t, for $1 \le t \le 9$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 1, 3, 7$ and 9 and the graph has horizontal tangents at $t = 2, 5,$ and 8.

The areas of the regions bounded are 2, 3, and 6 respectively. The position function for the particle is called x and at $t = 1$, $x(1) = 2$.

a. Create Sign lines for v(t) and a(t)

$v(t)$ | − | + | − |
1 3 7 9

$a(t)$ | − | + | − | + |
1 2 5 8 9

b. On what intervals (if any) is the velocity negative? Justify your answer.

$(1,3) \cup (7,9)$ b/c $v(t) < 0$

c. On what intervals (if any) is the acceleration positive? Justify your answer.

$(2,5) \cup (8,9)$ b/c $v'(t) > 0$

d. On the interval $5 < t < 7$, is the speed of the particle increasing or decreasing? Give a reason for your answer. Decreasing speed b/c $v(t) > 0$ and $a(t) < 0$ (opposite signs)

e. On the interval $7 < t < 8$, is the speed of the particle increasing or decreasing? Give a reason for your answer. Increasing speed b/c $v(t) < 0$ and $a(t) < 0$ (same signs)

f. Find the positions of the particle at $t = 3$, $t = 7$ and $t = 9$. (use definite integrals.)

$x(3) = x(1) + \int_1^3 v(t)\,dt = 2 + (-2) = 0$

$x(7) = x(3) + \int_3^7 v(t)\,dt = 0 + 3 = 3$

$x(9) = x(7) + \int_7^9 v(t)\,dt = 3 + (-6) = -3$

g. State the absolute extrema and the t-values where they occur.

Abs min at −3 where $t = 9$

Abs max at 3 where $t = 7$

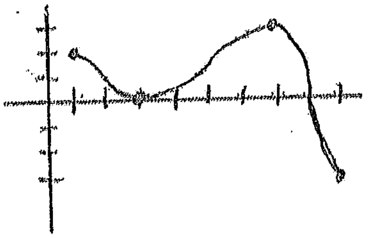
h. Find the total distance traveled by the particle from $t = 1$ to $t = 9$. (Use Integral Notation)

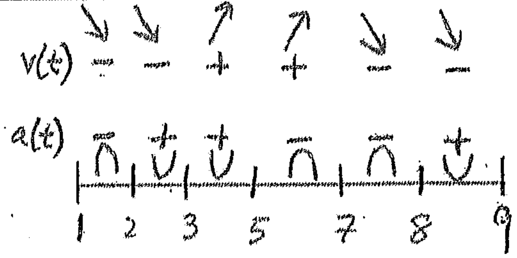
$\int_1^9 |v(t)|\,dt = 2 + 3 + 6 = \boxed{11}$

i. Find the total displacement of the particle from $t = 3$ to $t = 9$. (Use Integral Notation)

$\int_3^9 v(t)\,dt = 3 - 6 = \boxed{-3}$

j. Sketch graph of x(t) below:





AP Calculus Chapter 4 Morning Review Session (part 2: Calculator)

1. An object moving along a horizontal line has $v(t) = t\cos\left(\frac{\pi t}{6}\right)$ measured in inches per second from [0,11]

a. Create Sign line for v(t) and a(t)

b. Find the time(s) when the object is motionless

c. Find the velocity of the object at t = 4 seconds.

d. Find the acceleration of the object at t = 4 seconds.

e. Is the object's speed increasing or decreasing at t = 4 seconds? Justify answer.

f. Find the total displacement of the object from t = 0 to t = 11 seconds. (Show Integral Notation)

g. Find the total distance of the object from t = 0 to t = 11 seconds (Show Integral Notation)

h. Find the time when the object reaches minimum velocity in [0, 11]

i. Find the minimum velocity in [0, 11]

j. Given x(0) = 3, Find x(11). (Show integral notation)

k. Find the average velocity in [0, 11]

[illegible]

l. Find the time(s) when object reaches average velocity.

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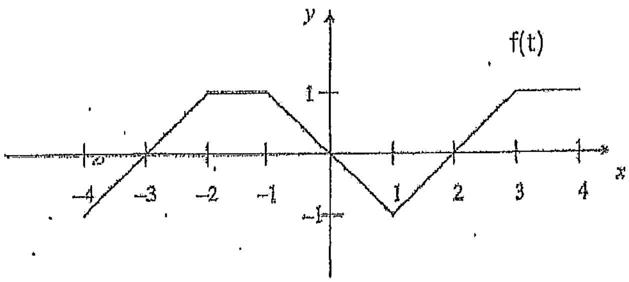
2. The graph of f consists of line segments. Let $g(x) = \int_2^x f(t)dt$

a. Find g'(x)

b. Find g''(x)



c) Find g(4)

d) Find g(-2)

e) Find g''(-3.5)

f) For what values of x is g increasing? Justify Answer

g) For what values of x is g'(x) decreasing?

h) Find the absolute extrema of g on the interval [-1,3].

3. The following table shows the size of an incoming wave headed towards shore at a given moment.

Distance from left of wave (x feet)	0	7	18	24	36	44	53
Height of wave h(x) (feet)	0	5	13	26	16	7	0

a) Use a trapezoidal sum with the six sub-intervals indicated by the data in the table to approximate the area of the face of the wave. Show correct units.

b) Estimate $\int_0^{53} h(x)dx$ using 3 middle rectangles

c) Find the average height on the interval [0, 53] using estimation from part b

KEY

Ch. 4 Free Response WS #1 $\boxed{\text{FRQ}}$

1. 1999 #1 (Calculators permitted):

calculator: Radian mode!

A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

$v(1.5) = 1.167 \quad v(1.5) > 0$ so particle is moving up.

(Math→8)

b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

calculator: nDeriv(Y_1, X, 1.5)

$v'(1.5) = -2.049 < 0$, so velocity is decreasing.

c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

$y(2) = y(0) + \int_0^2 v(t)\,dt = 3 + 0.827 = \boxed{3.827}$

(Math→9)

d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

calculator: fnInt(Y_1, X, 0, 2)

$\text{distance} = \int_0^2 |v(t)|\,dt = \boxed{1.173}$

2. 2001 #2 (Calculators permitted):

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table to the right shows the water temperature as recorded every 3 days over a 15-day period.

"x" t (days)	"y" $W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

$$W'(12) = \frac{W(15) - W(12)}{15 - 12} = \frac{21 - 22}{15 - 12} = -\frac{1}{3}\ {}^\circ\text{C/day}$$

b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

$\text{Avg. temp} = \frac{1}{b-a}\int_a^b w(t)\,dt$

$\int_0^{15} w(t)\,dt \approx \frac{w}{2}[h_1 + 2h_2 + 2h_3 + 2h_4 + \dots + h_n] = \frac{3}{2}[20 + 2(31) + 2(28) + 2(24) + 2(22) + 21]$

$= 376.5$

$= \frac{1}{15-0}\int_0^{15} w(t)\,dt = \frac{1}{15}(376.5) = \boxed{25.1^\circ\text{C}}$

c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-\frac{t}{3})}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

$P'(12) = -0.549\ {}^\circ\text{C/day}$ This is the rate of change in temperature at $t = 12$ days.

d) Use the function P defined in part c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.

$\text{Avg. temperature} = \frac{1}{15-0}\int_0^{15} P(t)\,dt = \frac{1}{15}(386.362) = \boxed{25.757^\circ\text{C}}$

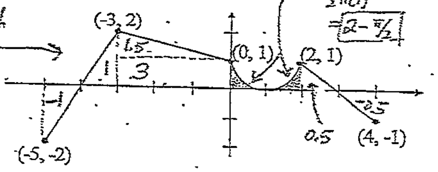
Ch. 4 Review WS #1 (continued)

3. 2004 #5 (No Calculators)

*Let f(t) graph represent velocity

The graph of the function f shown to the right consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

SFTC



Area(rectangle) − Area(semicircle)
$b(h) - \frac{1}{2}\pi r^2$
$2(1) - \frac{1}{2}\pi(1)^2$
$= \boxed{2 - \frac{\pi}{2}}$

a) Find $g'(x)$ and $g''(x)$

$g'(x) = \frac{d}{dx}\int_{-3}^{x} f(t)\,dt = f(x)\cdot(1)$

$g'(x) = f(x)$

$g''(x) = f'(x)$

b)

$g(0) = \int_{-3}^{0} f(t)\,dt = \boxed{4.5}$

$g'(0) = f(0) = \boxed{1}$

$g''(-1) = f'(-1) = \frac{2-1}{-3-0} = \frac{1}{-3} = \boxed{\frac{-1}{3}}$

slope of line segment using points (-3,2) and (0,1)

c) Find all values of x in the open interval (-5, 4) at which g attains a relative maximum. Justify your answer.

$g'(x)$: ↘ − | ↗ + | ↗ + | ↘ −

−5 −4 1 3 4

Rel. max at $x = 3$ b/c $g'(x)$ changes from + to −.

d) Find the absolute minimum value of g on the closed interval [-5, 4]. Justify your answer.

(y-value!)

EVT:
1) Test endpoints
2) Test rel. mins.

$g(-5) = \int_{-3}^{-5} f(t)\,dt = -\int_{-5}^{-3} f(t)\,dt = -(-1+1) = 0$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -\int_{-4}^{-3} f(t)\,dt = -(1) = -1$

$g(4) = \int_{-3}^{4} f(t)\,dt = 3 + 1.5 + 2 - \pi/2 + 0.5 - 0.5 = 6.5 - \frac{\pi}{2} \approx 5$

$\boxed{\text{Abs. min is } -1 \text{ at } x = -4}$

e) Find all values of x in the open interval (-5, 4) at which the graph of g has a point of inflection.

$g''(x)$: ∪ + | ∩ − | ∪ + | ∩ −

−5 −3 1 2 4

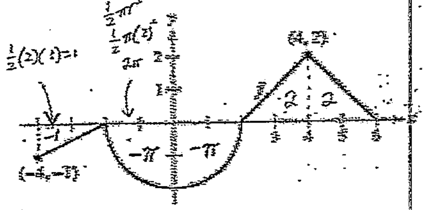
$\boxed{\text{POI occur at } x = -3, 1, 2 \text{ b/c } g''(x) \text{ changes signs}}$

(14)

Ch. 4 Test Review (Calculator Portion) WS #3

KEY

1. The graph of f consists of line segments and a semicircle, as shown: Let $g(x)=\int_{-2}^{x} f(t)\,dt$



$g'(x)=\frac{d}{dx}\int_{-2}^{x} f(t)dt = f(x)\cdot 1$

$g'(x)=f(x)$

$g''(x)=f'(x)$

a) Find g(2)

$g(2)=\int_{-2}^{2} f(t)dt = \boxed{-2\pi}$

b) Find g(-4)

$g(-4)=\int_{-2}^{-4} f(t)dt = -\int_{-4}^{-2} f(t)dt$

$=-(-1)=\boxed{1}$

c) Find g(6)

$g(6)=\int_{-2}^{6} f(t)dt = \boxed{-2\pi+4}$

d) Find g'(4)

$g'(4)=f(4)=\boxed{2}$

e) Find g'(-2)

$g'(-2)=f(-2)=\boxed{0}$

f) Find g''(5)

$g''(5)=f'(5)=\frac{2-0}{4-6}=\frac{2}{-2}=\boxed{-1}$

find slope of line segment using (4,2) and (6,0)

g) For what values of x is g increasing? Justify Answer

$g'(x)$ with sign line: $-$ $-$ $+$ at -4, -2, 2, 6

g(x) is increasing (2,6) b/c $g'(x)>0$

h) For what values of x is g decreasing? Justify Answer

g(x) is decreasing $(-4,-2)\cup(-2,2)$

b/c $g'(x)<0$

i) Find the x-values of all points of inflection of g. Justify Answer

$g''(x)$ with sign line: $\cup$ $\cap$ $\cup$ $\cap$; $+$ $-$ $+$ $-$ at -4, -2, 0, 4, 6

POI at $x=-2,0,4$ b/c $g''(x)$ change signs.

j) Find the absolute extrema of g on the interval [-4,6] (EVT) Test endpts and critical pts of g(x)

$g(-4)=1$

$g(-2)=\int_{-2}^{-2} f(t)dt=0$

$g(2)=-2\pi\approx-6$

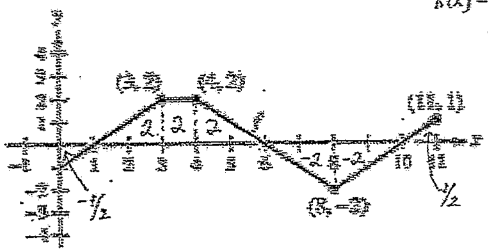
$g(6)=-2\pi+4\approx-2$

Abs min is -2π at $x=2$

Abs max is 1 at $x=-4$

Ch. 4 Test Review WS #3 (continued)

2. The graph of f consists of line segments. Let $h(x)=\int_{6}^{x} f(t)dt$



$h'(x)=\frac{d}{dx}\int_{6}^{x} f(t)dt = f(x)\cdot 1 = f(x)$

$h'(x)=f(x)$

$h''(x)=f'(x)$

a) Find h(0)

$h(0)=\int_{6}^{0} f(t)dt = -\int_{0}^{6} f(t)dt$

$=-\left(-\frac{1}{2}+2+2+2\right)$

$=-(5.5)=\boxed{-5.5 \text{ or } -11/2}$

b) Find h(6)

$h(6)=\int_{6}^{6} f(t)dt = \boxed{0}$

c) Find h(10)

$h(10)=\int_{6}^{10} f(t)dt = \boxed{-4}$

d) Find h(11)

$h(11)=\int_{6}^{11} f(t)dt = -2-2+\frac{1}{2}$

$=\boxed{-3.5 \text{ or } -7/2}$

e) Find h'(3)

$h'(3)=f(3)=\boxed{2}$

f) Find h''(9.5)

$h''(9.5)=f'(9.5)=\frac{-2-1}{8-11}$

$=\frac{-3}{-3}=\boxed{1}$

find slope of line segment using (8,-2) and (11,1)

g) For what values of x is h increasing? Justify Answer

$h'(x)$ with sign line: $-$ $+$ $-$ $+$ at 0, 1, 6, 10, 11

h(x) is increasing $(1,6)\cup(10,11)$

b/c $h'(x)>0$

h) For what values of x is h'(x) decreasing?

h'(x) decreasing (4,8) b/c $h''(x)<0$

Test endpts and critical pts.

$h(0)=-5.5$

$h(1)=\int_{6}^{1} f(t)dt = -\int_{1}^{6} f(t)dt = -(6)=-6$

$h(6)=0$

$h(10)=-4$

$h(11)=-3.5$

i) Find the absolute extrema of g on the interval [0,11]

Abs. max is 0 at $x=6$

Abs. min is -6 at $x=1$

Ch. 4 Test Review WS #3 (continued)

3. The table below shows the speed of a sprinter at the time intervals (in seconds) in the 200 meter race

time t (seconds)	0	2	5	8	11	17	20
Velocity (m/s) $v(t)$	5	6.5	7	8.5	9	8	7.5

a. Estimate $\int_0^{20} v(t)dt$ using the following methods

i. 6 trapezoids $A = \frac{w}{2}[h_1+h_2]$

$A \approx \frac{2}{2}[5+6.5]+\frac{3}{2}[6.5+7]+\frac{3}{2}[7+8.5]$
$+\frac{3}{2}[8.5+9]+\frac{6}{2}[9+8]+\frac{3}{2}[8+7.5]$

$= 11.5+20.25+23.25+26.25$
$+ 51 + 23.25$
$= \boxed{155.5}\ m$

ii. 3 left-handed rectangles $A = b \cdot h$

$= (5)\cdot v(0) + 6\cdot v(5) + 9\cdot v(11)$

$= 5(5) + 6(7) + 9(9)$

$= \boxed{148}\ m$

iii. 3 right-handed rectangles

$= 5\cdot v(5) + 6\cdot v(11) + 9\cdot v(20)$
$= 5(7) + 6(9) + 9(7.5)$

$= \boxed{156.5\ m}$

iv. 3 middle rectangles

$= 5\cdot v(2) + 6\cdot v(8) + 9\cdot v(17)$

$= 5(6.5) + 6(8.5) + 9(8)$
$32.5 + 51 + 72$

$= \boxed{155.5\ m}$

b. Find the average velocity on the interval [0, 20] using estimation from 6 trapezoids.

$\text{Avg. velocity} = \frac{1}{b-a}\int_a^b v(t)dt = \frac{1}{20-0}\int_0^{20} v(t)dt$

$\text{Avg. velocity} = \frac{1}{20}(155.5) = \boxed{7.775\ m/s}$

Ch. 4 Test Review WS #3 (continued)

4. An object moving along a horizontal line has $v(t) = 4\sin(t^2-2t+2)$ measured in meters per second from [0,4] (hint: set windows to x-values [-1, 5] and y-values [-6, 6])
*Round answers to 3 decimal places

a. Find the time(s) when the object is motionless

$v(t)$ + − + − on the line 0, 2.463, 3.299, 3.903, 4

object motionless at $t = 2.463, 3.299, 3.903$ secs b/c $v(t)=0$

b. When does the object change directions in $0<t<4$?

object changes direction at $t = 2.463$, 3.299, and 3.903 seconds b/c $v(t)$ change signs.

c. Find the velocity of the object at $t=3$ seconds.

$v(3) = -3.836\ m/s$

d. Find the acceleration of the object at $t=3$ seconds.

$v'(3) = 4.539\ m/s^2$ or $a(3) = 4.539\ m/s^2$

Calculator syntax: nderiv(Y₁,X,3)

e. Is the object's speed increasing or decreasing at $t=3$ seconds? Justify answer.

Speed is decreasing b/c $v(3) < 0$ and $a(3) > 0$ (opposite signs)

f. Find the total displacement of the object from $t=0$ to $t=4$ seconds (show integral setup)

$\int_0^4 v(t)dt = \boxed{7.753\ m}$

fnInt(Y₁,X,0,4)

g. Find the total distance of the object from $t=0$ to $t=4$ seconds (show integral setup)

$\int_0^4 |v(t)|dt = \boxed{12178\ m}$

h. Find the time when the object reaches minimum velocity in [0, 3]

$\boxed{t = 2927 \text{ seconds}}$

i. Find the minimum velocity in [0, 3]

$v(2927) = \boxed{-4\ m/s...}$

j. Given $x(0) = 2$, Find $x(4)$. (Show integral notation)

$x(4) = x(0) + \int_0^4 v(t)dt$

$= 2 + 7.753 = \boxed{9.753}$

k. Find the average velocity in [0, 4]

$\text{Avg. velocity} = \frac{1}{b-a}\int_a^b v(t)dt = \frac{1}{4-0}\int_0^4 v(t)dt$
$= \frac{1}{4}(7.753) = \boxed{1.938}\ m/s$

l. Find the time(s) when object reaches average velocity.

Set $4\sin(t^2-2t+2) = 1.938$

$4\sin(t^2-2t+2) - 1.938 = 0$

$\boxed{t = 2.279, 3.406, 3.814 \text{ seconds}}$

Ch. 4 Test Review WS #4 Riemann Sums Practice Worksheet

Key

(Determining Units of Measure and Interpreting Definite Integrals!)

***Important Key Point*: When applying(or approximating) a Calculus process(derivatives or integrals), your units of measure will change!**

1)

t (minutes)	0	1	3	6	9	10
c(t) (ounces per minute)	0	5.1	4.2	3.3	1.2	2.3

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The rate that water in the cup at time t is changing, $0 \le t \le 10$, is given by a differential function c(t), where t is measured in minutes. Select values if c(t), measured in ounces per minute are given in the table above.

a) Interpret the meaning of $c'(6)$ *and* indicate the units of measure.

c′(6) tells us how fast the rate of water added to the cup is changing. (units is ounces/min²)

b) Approximate the value of c′(6) and indicate the units of measure.

$$c'(6) \approx \frac{1.2-3.3}{9-6} = -0.7 \text{ ounces/min}^2$$

← *choosing any ordered pairs close to t=6 and finding slope would be acceptable

c) Intepret the meaning of $\int_1^{10} c(t)dt$ and indicate the units of measure.

*using 1st Theorem, $\int_1^{10} c(t)dt = C(10) - C(1)$. This represents the change in the amount of Coffee in the cup between the 1st minute and the 10th minute (units is ounces)

d) Approximate the value of $\int_1^{10} c(t)dt$ using 2 middle rectangles and indicate the units of measure.

$$\int_1^{10} c(t)dt \approx 5(4.2) + 4(1.2) = 21 + 4.8 = \boxed{25.8 \text{ ounces}}$$

e) Approximate the average rate of water being added on time interval [1, 10] using result from part d)

*Avg. value theorem $\frac{1}{b-a}\int_a^b f(x)dx$

$$\frac{1}{10-1}\int_1^{10} c(t)dt = \frac{1}{9}(25.8) = \boxed{2.867 \text{ ounces/minute}}$$

---

2)

t (minutes)	0	12	20	24	40
v(t) (meters per minute)	0	200	240	−220	150

Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differentiable function v. Selected values of v(t), where t is measured in minutes and v(t) is measured in meters per minute, are given in the table above.

a) Interpret the meaning of $v'(20)$ *and* indicate the units of measure.

$v'(20) = a(20)$ is the rate of change of velocity at t=20 (or acceleration) units is meters/min².

b) Approximate the value of $v'(18)$ and indicate the units of measure.

$$v'(18) = a(18) = \frac{v(20)-v(12)}{20-12} = \frac{240-200}{20-12} = \frac{40}{8} = \boxed{5 \text{ meters/min}^2}$$

c) Intepret the meaning of $\int_{20}^{40} v(t)dt$ and indicate the units of measure.

*FFTC: $\int_a^b f(x) = F(b) - F(a)$

$\int_{20}^{40} v(t)dt = x(40) - x(20)$ is the change in distance between 20 and 40 minutes (or displacement). Units is meters

d) Approximate the value of $\int_{20}^{40} v(t)dt$ using 2 trapezoids and indicate the units of measure.

$$\int_{20}^{40} v(t)dt = \frac{1}{2}(4)[240+-220] + \frac{1}{2}(16)[-220+150] = 40-560 = \boxed{-520 \text{ meters}}$$

↖ Trapezoid Area is $\frac{1}{2}w[h_1+h_2]$

e) Approximate Johanna's average velocity on [20, 40] using the results from part d)

*Avg. value theorem: $\frac{1}{b-a}\int_a^b f(x)dx$

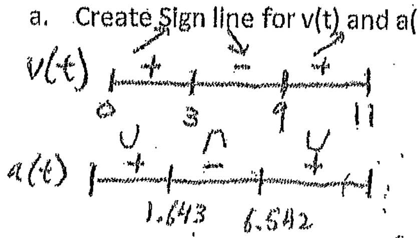
$$\frac{1}{40-20}\int_{20}^{40} v(t)dt = \frac{1}{20}\int_{20}^{40} v(t)dt$$

$$= \frac{1}{20}[-520] = \boxed{-26 \text{ meters/minute}}$$

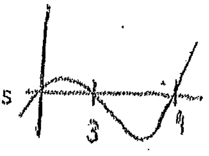
AP Calculus Chapter 4 Morning Review Session (part 2: Calculator) * Make sure you are in Radian Mode!

1. An object moving along a horizontal line has $v(t) = t\cos\left(\frac{\pi t}{6}\right)$ measured in inches per second from [0,11]

a. Create Sign line for v(t) and a(t)



b. Find the time(s) when the object is motionless



$t = 0, 3, 9$ seconds

c. Find the velocity of the object at t = 4 seconds.

$v(4) = -2$ in./sec.

calculator: $Y_1(4)$

d. Find the acceleration of the object at t = 4 seconds.

$a(4) = -2.314$ in./s²

calculator: nDeriv$(Y_1, X, 4)$

e. Is the object's speed increasing or decreasing at t = 4 seconds? Justify answer.

Speed is increasing b/c $v(4) < 0$ and $a(4) < 0$ (same signs)

f. Find the total displacement of the object from t = 0 to t = 11 seconds (Show Integral Notation)

$\int_0^{11} v(t)\,dt = -10.993$ in.

calculator: fnInt$(Y_1, X, 0, 11)$

g. Find the total distance of the object from t = 0 to t = 11 seconds (Show Integral Notation)

$\int_0^{11} |v(t)|\,dt = 34.844$ in.

calculator: fnInt(Abs(Y_1), X, 0, 11)

h. Find the time when the object reaches minimum velocity in [0, 11]

$t = 6.542$ when $a(t)$ changes from − to +

i. Find the minimum velocity in [0, 11]

$v(6.542) = -6.28$ in./sec.

j. Given x(0) = 3, Find x(11). (Show integral notation)

$x(11) = x(0) + \int_0^{11} v(t)\,dt$

$= 3 + (-10.993) = -7.993$

$\boxed{x(11) = -7.993}$

k. Find the average velocity in [0, 11]

Avg. velocity $= \frac{1}{11-0}\int_0^{11} v(t)\,dt = \frac{1}{11}(-10.993)$

$\boxed{\text{Avg. velocity} = -0.999 \text{ in/s}}$

l. Find the time(s) when object reaches average velocity.

set $v(t) = -0.999$

$t\cos(\pi t/6) = -0.999$

$t\cos(\pi t/6) + 0.999 = 0$

$\boxed{t = 3.546 \text{ sec and } t = 8.782 \text{ sec.}}$

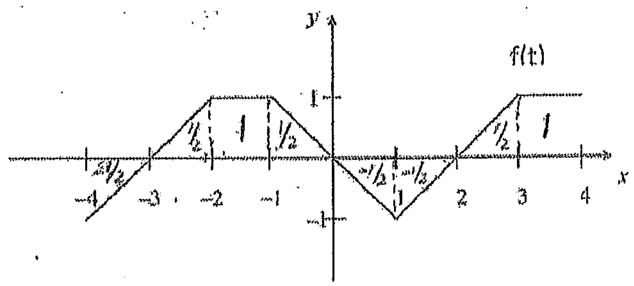
⑱

2. The graph of f consists of line segments. Let $g(x) = \int_2^x f(t)\,dt$

a. Find $g'(x) = f(x) \cdot 1 = f(x)$

$g'(x) = f(x)$

b. Find $g''(x) = f'(x)$



c) Find g(4)

$h(4) = \int_2^4 f(t)\,dt = \boxed{1.5}$

d) Find g(-2)

$h(-2) = \int_2^{-2} f(t)\,dt = -\int_{-2}^{2} f(t)\,dt$

$= -(1/2) = \boxed{-1/2}$

e) Find g''(-3.5) $= f'(-3.5)$

Find slope b/t (-4,-1) and (-3,0)

$g''(-3.5) = \frac{-1-0}{-4+3} = \frac{-1}{-1} = \boxed{1}$

f) For what values of x is g increasing? Justify Answer

$g'(x)$: $-$ $+$ $-$ $+$ at -4, -3, 0, 2, 4

$g(x)$ is increasing on $(-3,0) \cup (2,4)$ b/c $g'(x) > 0$

g) For what values of x is g'(x) decreasing?

$g'(x)$ is decreasing on $(-1,1)$ b/c $g''(x) < 0$

h) Find the absolute extrema of g on the interval [-1, 3]
Show Work

*Test endpts and critical pts.

$g(-1) = \int_2^{-1} f(t)\,dt = -\int_{-1}^{2} f(t)\,dt = -(-1/2) = \frac{1}{2}$

$g(0) = \int_2^0 f(t)\,dt = -\int_0^2 f(t)\,dt = -(-1) = 1$

$g(2) = \int_2^2 f(t)\,dt = 0$

$g(3) = \int_2^3 f(t)\,dt = 1/2$

Abs. max is 1 at x = 0

Abs. min is 0 at x = 2

3. The following table shows the size of an incoming wave headed towards shore at a given moment.

Distance from left of wave (feet) x	0	7	18	24	36	44	53
Height of wave H(x) (feet)	0	5	13	26	16	7	0

a) Use a trapezoidal sum with the six sub-intervals indicated by the data in the table to approximate the area of the face of the wave. Show correct units. $\frac{W}{2}[h_1+h_2]$

$$A \approx \frac{7}{2}[0+5] + \frac{11}{2}[5+13] + \frac{6}{2}[13+26] + \frac{12}{2}[26+16] + \frac{8}{2}[16+7] + \frac{9}{2}[7+0] = 609$$

$$17.5 + 99 + 117 + 252 + 92 + 31.5 = \boxed{609}\ ft^2$$

b) Estimate $\int_0^{53} h(x)dx$ using 3 middle rectangles

$$\int_0^{53} h(x)dx \approx 18 \cdot h(7) + 18 \cdot h(24) + 17 \cdot h(44)$$

$$= 18(5) + 18(26) + 17(7) = \boxed{677}\ ft^2$$

c) Find the average height on the interval [0, 53] using estimation from part b

$$\text{Avg. height} = \frac{1}{53-0}\int_0^{53} h(x)dx$$

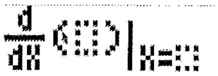
$$= \frac{1}{53}(677) = \boxed{12.774\ ft.}$$

Calculus on the TI-83/84 Graphing Calculators

*** * * Remember – Calculator <u>ALWAYS in RADIAN</u> mode * * ***

<u>Evaluating derivatives:</u> Example: Evaluate $f'(7)$ if $f(x) = 2x^3 - 5x^2 + \sqrt{7x}$

1. Enter $y_1 = 2x^3 - 5x^2 + \sqrt{7x}$
2. *Math* 8 →



$\frac{d}{dx}(Y_1) | \, x = 7$ Answer: 224.5

3. (If no MathPrint, then Math 8 → nDeriv$(y_1, x, 7)$

[to enter y_1 go to VARS / Y-VARS / Function / 1 }

<u>Evaluating values:</u> Example: Evaluate $f(7)$ if $f(x) = 2x^3 - 5x^2 + \sqrt{7x}$

1. Enter $y_1 = 2x^3 - 5x^2 + \sqrt{7x}$
2. Return to Home Screen: 2nd → Mode (Quit)
3. **go to VARS / Y-VARS / Function / 1**
4. $Y_1(7)$ press enter
5. Answer: (448)

<u>Finding Values on a Graph:</u> Example: $Y_1 = 0.5x^3 + x^2 - 2x - 1$

1) Set Window:
   a. Standard Window: **Xmin**: -10 **Xmax**: 10 **Ymin**: -10 **Ymax** 10
   (You will adjust window values on a case-by-case basis)
   b. Or enter Zoom → 6 (ZStandard)

2) Finding x-Intercepts:
   a. 2nd → Trace → 2:Zero
   b. Left Bound? (Scroll cursor to a point LEFT of **x-intercept** on graph then press ENTER)
   c. Right Bound? (Scroll cursor to a point RIGHT of **x-intercept** on graph then press ENTER)
   d. Guess? (Press ENTER)
   e. (Answer: x = -3.082, x = -0.428, x = 1.514)

3) Finding Relative Minimum on Graph:
   a. 2nd → Trace → 3: Minimum
   b. Left Bound? (Scroll cursor to a point LEFT of **rel. minimum** on graph then press ENTER)
   c. Right Bound? (Scroll cursor to a point RIGHT of **rel. minimum** on graph then press ENTER)
   d. Guess? (Press ENTER)
   e. (Answer: x = 0.666 y = -1.7407)

4) Finding Relative Maximum on Graph:
   a. 2nd → Trace → 4: Maximum
   b. Left Bound? (Scroll cursor to a point LEFT of **rel. maximum** on graph then press ENTER)
   c. Right Bound? (Scroll cursor to a point RIGHT of **rel. maximum** on graph then press ENTER)
   d. Guess? (Press ENTER)
   e. (Answer: x = -2 y = 3)

2nd Semester TI-83/84 Integral Steps:

Evaluating Definite Integrals

e.g. Evaluate $\int_{-2}^{11}\left(2x^3-5x^2+\sqrt{7x}\right)dx$

1. Enter $y_1=2x^3-5x^2+\sqrt{7x}$
2. **go to VARS / Y-VARS / Function / 1: Y_1**
3. Math 9 → $\text{FnInt}\left(y_1,x,-2,11\right)$

Evaluating Total Distance

e.g. Evaluate $\int_{0}^{5}\left|3x^2+11x+4\right|dx$

1. Enter $y_1=3x^2+11x+4$
2. Math 9 → $\text{fnInt}\left(Abs\left(y_1\right),x,0,5\right)$
3. The absolute value feature (Abs) is found in Math → Num → Abs

---

Evaluating Value of a function

e.g. Evaluate $f(7)$ if $f(x)=3e^{2x}-\ln(x^2)$

1. Enter $y_1=3e^{2x}-\ln(x^2)$
2. $y_1(7)$

TI –36x Pro

Finding Values on a Graph:

1) Table → 2:Edit function
2) f(x) = $0.5x^3 + x^2 - 2x - 1$ (Press Enter)
3) Start = 0 (Press Enter) * This value you will adjust on a case-by-case basis
   Step = **0.5** (Press Enter) *This value you will adjust on a case-by-case basis
   Auto (Press Enter)
   CALC (Press Enter)

4) Estimate x-intercepts:
   a. Scroll through table and estimate where f(x) column (y-values) changes signs from – to + or from + to -
   b. Gain increased accuracy by adjusting step to smaller increments (ex. 0.1)
   c. Change your "start =" value to reduce the amount of scrolling
   d. (Answer: x = -3.082, x = -0.428, x = 1.514)

5) Estimate relative minimum:
   a. Scroll through table and estimate where f(x) column (y-values) changes from decreasing to increasing values
   b. Gain increased accuracy by adjusting step to smaller increments (ex. 0.1)
   c. Change your "start =" value to reduce the amount of scrolling
   d. (Answer: x = 0.666 y = -1.7407)

6) Estimate x-intercepts:
   a. Scroll through table and estimate where f(x) column (y-values) changes from increasing to decreasing values
   b. Gain increased accuracy by adjusting step to smaller increments (ex. 0.1)
   c. Change your "start =" value to reduce the amount of scrolling
   d. (Answer: x = -2 y = 3)