5. A sphere of diameter 4 in. is inside a cylinder with radius of 12 in and constant height of 8 in. How fast is the volume between the sphere and cylinder changing if the diameter of the sphere is increasing at a rate of 2 in/min and the radius of the cylinder is decreasing at a rate of 4 in/min? dV

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$V_{Between} = V_{cylinder} - V_{sphere}$

$\frac{dV}{dt_B} = \frac{dV}{dt_c} - \frac{dV}{dt_s}$

Sphere $V = \frac{4}{3}\pi r^3$

$r = 2$

$\frac{dr}{dt} = 1$ in/min

$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$

$\frac{dV}{dt_s} = 4\pi(2)^2(1)$

$= 16\pi$ in^3/min

Cylinder: $V = \pi r^2 h$

$h = 8$ (constant)

$r = 12$ in.

$\frac{dr}{dt} = -4$ in/min

$V = \pi r^2 (8)$

$V = 8\pi r^2$

$\frac{dV}{dt} = 16\pi r \left(\frac{dr}{dt}\right)$

$\frac{dV}{dt_c} = 16\pi(12)(-4)$

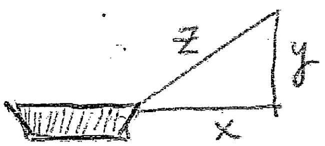
$\frac{dV}{dt_c} = -768\pi$ in^3/min

$\frac{dV}{dt_B} = \frac{dV}{dt_c} - \frac{dV}{dt_s}$

$= -768\pi - 16\pi$

$= \boxed{-784\pi \text{ in}^3/\text{min}}$

6. A boat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at a rate of 2 ft/sec



$\frac{dz}{dt} = -2$ ft/s

$y = 6$

$z = 10$

$x = 8$

$\frac{dz}{dt} = -2$ ft/s

$\frac{dy}{dt} = 0$

a) How fast is the boat approaching the dock when 10 ft of rope are out?

Find $\frac{dx}{dt}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(8)\left(\frac{dx}{dt}\right) + 2(10)(0) = 2(10)(-2)$

$16\frac{dx}{dt} = -40$

$\boxed{\frac{dx}{dt} = -2.5 \text{ ft/s}}$

b) At what rate is area of triangle changing at that moment?

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\frac{dy}{dt}$

$= \frac{1}{2}(-2.5)(6) + \frac{1}{2}(8)(0)$

$\boxed{\frac{dA}{dt} = -7.5 \text{ ft}^2/\text{s}}$