AP Calculus AB Trig Unit Test Review WS #1 Name:________________________

Continuity review: 3 conditions for continuity at a point:

1) f(c) is defined
2) $\lim_{x->c} f(x)$ exists $\left(\text{This means that } \lim_{x->c^-} f(x) = \lim_{x->c^+} f(x)\right)$
3) $\lim_{x->c} f(x) = f(c)$

1. Find the constant a such that the function is continuous at the given point.

a)

$$g(x) = \begin{cases} \dfrac{7\sin(2x)}{4x}, & x > 0 \\ a - 2x, & x \le 0 \end{cases}$$

b)

$$g(x) = \begin{cases} \dfrac{1-\cos(2x)}{2x} - a, & x > 0 \\ \cos x + a, & x \le 0 \end{cases}$$

c)

$$g(x) = \begin{cases} \tan\dfrac{x}{4}, & x > \pi \\ -2\sec x + a, & x \le \pi \end{cases}$$

2. Determine whether Rolle's Theorem can be applied to $f(x) = \sec x$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. If so, find all values of c in the interval such that $f'(c) = 0$.

3. Apply the Mean Value Theorem to $f(x) = 2\sin x + \sin 2x$ on the interval $[0, \pi]$. Find all values of c in the interval $(0, \pi)$.

(2)

4. An object's position is given by the F(t) = $2\sec^3\left(\frac{t}{6}\right)$, is continuous and differentiable in domain $0 \le t < 3\pi$ seconds . F(t) is given in meters.

a. Find the average velocity (avg. rate of change) from t = 0 to t = 2π (Include Units)

b. At what point in time does the instantaneous velocity equal the average velocity from part (a)? (Set up equation but do not solve) Use MVT.

c. What is the instantaneous velocity of the object when t = π seconds? (Include Units)

d. What is the instantaneous velocity of the object when t = 2 π seconds? (Include Units)

e. Find the equation of the tangent line to the graph at t = 2 π

3

AP Calculus AB Trig Unit Test Review WS #2 Name____________________

1. Solve for x: $\cos(2x) = 1$ for $0 < x < 2\pi$

2. Solve for x: $\tan(4x) = 1$ for $0 < x < \pi$

3. Find the critical points for the function $e^{x + \sin 2x}$ in the domain $0 < x < 2\pi$

4. Find the equation of the line tangent to the graph at (3,-1) for the equation $\sin(\pi x) + \cos(\pi y) = x^2 y$

4

5. Person X and Person Y are walking on straight streets that meet at right angles. Y travels south and approaches the intersection at 2m/s. Person X travels east and moves away from the intersection at 1m/s.
   a) Find the rate at which the distance (Z) between Person X and Y is changing when Y is 10m from the intersection and X is 20 meters from the intersection.
   b) At what rate is the angle θ changing at the same moment?

6. Sketch the graph if function $f(x) = x - \sin x$ on the interval $[0, 4\pi]$. Find all ordered pairs of relative extrema, absolute extrema, and POI

a) the relative extrema min: ________________ max: ____________________

b) the point(s) of inflection: ____

c) concavity intervals up: ____

down: ____

d) Absolute extrema: Absolute min: _______________ Absolute max: __________________

d) Sketch the graph

AP Calculus AB Trig Unit Test Review WS #3 Name____________________________

1. Suppose a bird is flying horizontally 40 ft above your head at 20 ft/sec (flying away from you). How fast is the angle of elevation changing when your horizontal distance from the bird is 30 ft?

2. Find dy/dx for the equation $xe^{\sin(2y)} + 4x = \ln y$

3. Find the equation of the tangent line to function $f(x) = x\arccos(x^2)$ to the graph at the point $(0, 2\pi)$ in the given domain $[0, \pi]$

4. Find equation of the tangent line to the graph $\arctan(x + y) = y^2 + \frac{\pi}{4}$ at the point (1,0)

6

5. Sketch the graph of function $f(x) = \sin(2x)$ on interval $[0, \pi]$. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

a) the relative extrema min: ________________ max: ________________

b) the point(s) of inflection: ____

c) concavity intervals up: ____

down: ____

d) Absolute extrema: Absolute min: ______________ Absolute max: ________________

d) Sketch the graph

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6. Sketch the graph of function $f(x) = \frac{1}{2}x - cos x$ on interval $[-\pi, \pi]$. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

a) the relative extrema min: ________________ max: ________________

b) the point(s) of inflection: ____

c) concavity intervals up: ____

down: ____

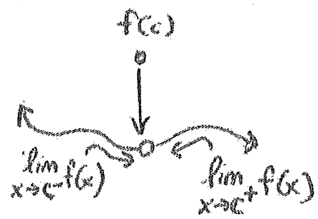
d) Absolute extrema: Absolute min: ______________ Absolute max: ________________

d) Sketch the graph

AP Calculus AB Trig Unit Test Review WS #1 Name:________________

Continuity review: 3 conditions for continuity at a point:

1) f(c) is defined
2) $\lim_{x\to c} f(x)$ exists (This means that $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$)
3) $\lim_{x\to c} f(x) = f(c)$



1. Find the constant a such that the function is continuous at the given point.

a) $g(x)=\begin{cases}\dfrac{7\sin(2x)}{4x}, & x>0\\ a-2x, & x\le 0\end{cases}$

$\lim_{x\to 0^+}\frac{7\sin(2x)}{4x} = \frac{7(2)}{4} = \frac{7}{2}$

$\lim_{x\to 0^-} a-2x = a$

$a = 7/2$

$g(x)=\begin{cases}\frac{7\sin 2x}{4}, & x>0\\ 7/2-2x, & x\le 0\end{cases}$

i) $g(0)=7/2$
ii) $\lim_{x\to 0} g(x) = 7/2$
iii) $\lim_{x\to 0} g(x) = g(0) = 7/2$ ✓

b) $g(x)=\begin{cases}\dfrac{1-\cos(2x)}{2x}-a, & x>0\\ \cos x + a, & x\le 0\end{cases}$

$\lim_{x\to 0^+}\frac{1-\cos(2x)}{2x} - a = 0-a = -a$

$\lim_{x\to 0^-}\cos x + a = \cos 0 + a = 1+a$

$1+a=-a$
$1=-2a$
$-1/2 = a$

c) $g(x)=\begin{cases}\tan\dfrac{x}{4}, & x>\pi\\ -2\sec x + a, & x\le\pi\end{cases}$

$\lim_{x\to\pi^+}\tan x/4 = \tan(\pi/4) = 1$

$\lim_{x\to\pi^-} -2\sec x + a = -2\sec\pi + a = -2(-1)+a = 2+a$

$2+a=1$
$a=-1$

2. Determine whether Rolle's Theorem can be applied to $f(x) = \sec x$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$. If so, find all values of c in the interval such that $f'(c) = 0$.

$f(a)=f(b)$?

① $f(x)$ continuous on $[-\pi/4, \pi/4]$ and ② differentiable on $(-\pi/4, \pi/4)$

③ $f(-\pi/4) = \sec(-\pi/4) = \sqrt{2}$
$f(\pi/4) = \sec(\pi/4) = \sqrt{2}$

set $f'(x)=0$ →

$f'(x) = \sec x\tan x$
$\sec x\tan x = 0$
$\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} = 0$

$\frac{\sin x}{\cos^2 x} = 0$
$\sin x = 0$
$x = 0, \pi, -\pi, 2\pi, -2\pi, \ldots$

check interval → $x=0$

$c=0$

3. Apply the Mean Value Theorem to $f(x) = 2\sin x + \sin 2x$ on the interval $[0, \pi]$. Find all values of c in the interval $(0, \pi)$.

① $f(x)$ continuous on $[0,\pi]$ and ② differentiable on $(0,\pi)$

set $f'(x) = \frac{f(b)-f(a)}{b-a}$

$f(0) = 2\sin(0)+\sin(2(0)) = 0$
$f(\pi) = 2\sin(\pi)+\sin 2\pi = 0$
$\frac{f(\pi)-f(0)}{\pi-0} = \frac{0-0}{\pi} = 0$

$f'(x) = 2\cos x + \cos(2x)\cdot 2$
$2\cos x + 2\cos(2x) = 0$
$2\cos x + 2[2\cos^2 x - 1] = 0$
$4\cos^2 x + 2\cos x - 2 = 0$
$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$
$\cos x = 1/2$ | $\cos x = -1$
$x = \pi/3, \cancel{5\pi/3}$ | $x = \cancel{\pi}$

$x = \pi/3$

$c = \pi/3$

⑧

4. An object's position is given by the $F(t) = 2\sec^3\left(\frac{t}{6}\right)$, is continuous and differentiable in domain

$0 \le t < 3\pi$ seconds . F(t) is given in meters.

a. Find the average velocity (avg. rate of change) from $t = 0$ to $t = 2\pi$ (Include Units)

* This is simply finding slope between endpoints. No derivatives involved.

$f(0) = 2\sec^3(0) = 2(1) = 2$

$f(2\pi) = 2\sec^3(2\pi/6) = 2[\sec(\pi/3)]^3 = 2[2]^3 = 16$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(2\pi)-f(0)}{2\pi-0} = \frac{16-2}{2\pi-0} = \frac{14}{2\pi} = \boxed{\frac{7}{\pi}\ m/s}$$

b. At what point in time does the instantaneous velocity equal the average velocity from part (a)? (Set up equation but do not solve) Use MVT.

set $f'(x) = \frac{f(b)-f(a)}{b-a}$

$f(x) = 2[\sec(t/6)]^3$

$f'(x) = 2 \cdot 3[\sec(t/6)]^2 \cdot \sec(t/6)\tan(t/6) \cdot \frac{1}{6}$

$= \sec^3(t/6)\tan(t/6)$

$$\boxed{\sec^3(t/6)\tan(t/6) = \frac{7}{\pi}}$$

c. What is the instantaneous velocity of the object when $t = \pi$ seconds? (Include Units)

$f'(x) = \sec^3(t/6)\tan(t/6) = \frac{1}{\cos^3(t/6)} \cdot \frac{\sin(t/6)}{\cos(t/6)}$

$f'(x) = \frac{\sin(t/6)}{\cos^4(t/6)}$

$$f'(\pi) = \frac{\sin(\pi/6)}{\cos^4(\pi/6)} = \frac{(1/2)}{(\sqrt{3}/2)^4} = \frac{\frac{1}{2}}{\frac{9}{16}} = \frac{1}{2} \cdot \frac{16}{9} = \boxed{8/9\ m/s}$$

d. What is the instantaneous velocity of the object when $t = 2\pi$ seconds? (Include Units)

$$f'(2\pi) = \frac{\sin(2\pi/6)}{\cos^4(2\pi/6)} = \frac{\sin(\pi/3)}{\cos^4(\pi/3)} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{(1/2)^4} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{16}} = \frac{\sqrt{3}}{2} \cdot \frac{16}{1} = \boxed{8\sqrt{3}\ m/s}$$

e. Find the equation of the tangent line to the graph at $t = 2\pi$

$$f(2\pi) = 2\sec^3\left(\frac{2\pi}{6}\right) = 2\sec^3(\pi/3) = \frac{2}{\cos^3(\pi/3)} = \frac{2}{(1/2)^3} = \frac{2}{1/8} = 2 \cdot 8 = 16$$

$f'(2\pi) = 8\sqrt{3}$

point: $(2\pi, 16)$

slope: $m = 8\sqrt{3}$

$y - y_1 = m(x - x_1)$

$$\boxed{y - 16 = 8\sqrt{3}(x - 2\pi)}$$

AP Calculus AB Trig Unit Test Review WS #2 Name________________

1. Solve for x: $\cos(2x) = 1$ for $0 < x < 2\pi$

$(2x) = \cos^{-1}(1)$

$2x = 0, 2\pi, 4\pi, 6\pi, 8\pi, \ldots$

$x = \frac{0}{2}, \frac{2\pi}{2}, \frac{4\pi}{2}, \frac{6\pi}{2}, \frac{8\pi}{2}$

$x = \cancel{0}, \pi, \cancel{2\pi}, \cancel{3\pi}$

$\boxed{x = \pi}$

2. Solve for x: $\tan(4x) = 1$ for $0 < x < \pi$

$(4x) = \tan^{-1}(1)$

$= \pi/4, 5\pi/4, 9\pi/4, 13\pi/4, 17\pi/4$

$x = \frac{\pi}{4}\cdot\frac{1}{4}, \frac{5\pi}{4}\cdot\frac{1}{4}, \frac{9\pi}{4}\cdot\frac{1}{4}, \frac{13\pi}{4}\cdot\frac{1}{4}, \frac{17\pi}{4}\cdot\frac{1}{4}$

$x = \frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \cancel{\frac{17\pi}{16}}$

$\boxed{x = \pi/16, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}}$

3. Find the critical points for the function $e^{x+\sin 2x}$ in the domain $0 < x < 2\pi$

$y' = e^{x+\sin 2x}\cdot(1 + 2\cos(2x))$

$e^{x+\sin 2x} \neq 0$

$1 + 2\cos(2x) = 0$

$2\cos(2x) = -1$

$\cos(2x) = -1/2$

$2x = \cos^{-1}(-1/2)$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}$

$\boxed{x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3}$

OR

$1 + 2\cos(2x) = 0$

$1 + 2(2\cos^2 x - 1) = 0$

$1 + 4\cos^2 x - 2 = 0$

$4\cos^2 x = 1$

$\cos^2 x = 1/4$

$\cos x = \pm 1/2$

$\boxed{x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3}$

4. Find the equation of the line tangent to the graph at (3,-1) for the equation $\sin(\pi x) + \cos(\pi y) = x^2 y$

* Apply implicit differentiation, chain, trig, product rules.

$\cos(\pi x)\cdot\pi - \sin(\pi y)\cdot\pi\left(\frac{dy}{dx}\right) = 2xy + x^2\left(\frac{dy}{dx}\right)$

$\pi\cos(\pi x) - \pi\sin(\pi y)\frac{dy}{dx} = 2xy + x^2\left(\frac{dy}{dx}\right)$

plug in (3,-1) to find $\frac{dy}{dx}$

$\pi\cos(3\pi) - \pi\sin(-\pi)\frac{dy}{dx} = 2(3)(-1) + 3^2\frac{dy}{dx}$

$\pi(-1) - \cancel{\pi(0)\frac{dy}{dx}} = -6 + 9\frac{dy}{dx}$

$-\pi = -6 + 9\frac{dy}{dx}$

$\frac{-\pi+6}{9} = \frac{dy}{dx}$

point: (3,-1)

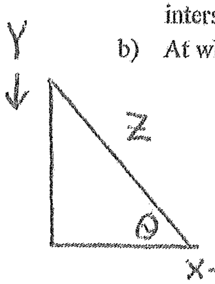
slope: $m = \frac{-\pi+6}{9}$

$y - y_1 = m(x - x_1)$

$\boxed{y + 1 = \frac{6-\pi}{9}(x - 3)}$

(10)

5. Person X and Person Y are walking on straight streets that meet at right angles. Y travels south and approaches the intersection at 2m/s. Person X travels east and moves away from the intersection at 1m/s.
   a) Find the rate at which the distance (Z) between Person X and Y is changing when Y is 10m from the intersection and X is 20 meters from the intersection.
   b) At what rate is the angle θ changing at the same moment?



$x = 20m$, $y = 10m$, $z = 10\sqrt{5}m$

$10^2 + 20^2 = z^2$, $z = 10\sqrt{5}$

$dx/dt = 1 m/s$, $dy/dt = -2 m/s$, $dz/dt =$ ____

$x^2 + y^2 = z^2$

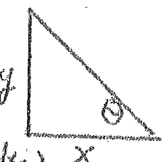
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(20)(1) + 2(20)(-2) = 2(10\sqrt{5})\frac{dz}{dt}$

$40 - 40 = 20\sqrt{5}\frac{dz}{dt}$

$\frac{0}{20\sqrt{5}} = \frac{20\sqrt{5}}{20\sqrt{5}}\frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = 0 \, m/s}$



$\tan\theta = \frac{y}{x}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{(dy/dt)(x) - y(dx/dt)}{x^2}$

$\frac{d\theta}{dt} = \frac{(dy/dt)(x) - y(\frac{dx}{dt})}{x^2 \cdot \sec^2\theta} = \frac{(-2)(20) - (10)(1)}{20^2 \cdot \left(\frac{10\sqrt{5}}{20}\right)^2}$

$= \frac{-50}{20^2 \cdot \frac{10\sqrt{5}^2}{20^2}} = \frac{-50}{(10\sqrt{5})^2} = \frac{-50}{500}$

$\boxed{\frac{d\theta}{dt} = \frac{-1}{10} \, rad/s}$

6. Sketch the graph if function f(x) = x – sinx on the interval [0, 4π]. Find all ordered pairs of absolution and relative extrema and POI

$f'(x) = 1 - \cos x$

$0 = 1 - \cos x$

$\cos x = 1$

$x = 0, 2\pi, 4\pi$

Sign chart: 0 (+ ↗) 2π (+ ↗) 4π

$f''(x) = 0 + \sin x$

$\sin x = 0$

$x = 0, \pi, 2\pi, 3\pi, 4\pi$

Sign chart: 0 (+ ∪) π (− ∩) 2π (+ ∪) 3π (− ∩) 4π

a) the relative extrema min: none max: none

$f(\pi) = \pi - \sin\pi = \pi$

$f(2\pi) = 2\pi - \sin(2\pi) = 2\pi$

$f(3\pi) = 3\pi - \sin(3\pi) = 3\pi$

$f(4\pi) = 4\pi - \sin(4\pi) = 4\pi$

b) the point(s) of inflection: $(\pi, \pi), (2\pi, 2\pi), (3\pi, 3\pi)$ b/c $f''(x)$ change signs

c) concavity intervals

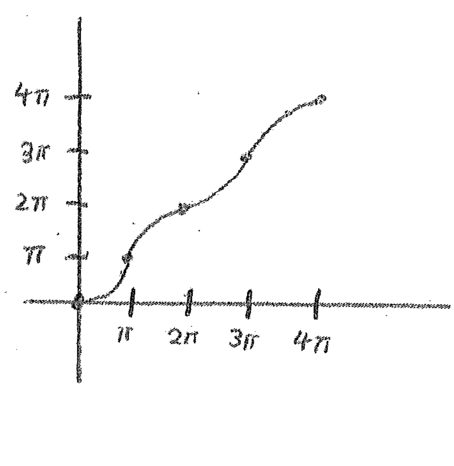
up: $(0, \pi) \cup (2\pi, 3\pi)$ b/c $f''(x) > 0$

down: $(\pi, 2\pi) \cup (3\pi, 4\pi)$ b/c $f''(x) < 0$

d) Absolute extrema: Absolute min: (0, 0) Absolute max: $(4\pi, 4\pi)$

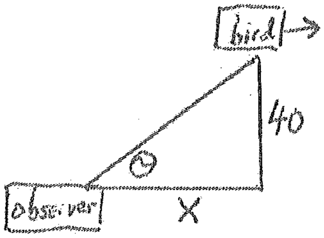
e) sketch the graph

Sign chart: 0 (∪ ↗) π (∩ ↗) 2π (∪ ↗) 3π (∩ ↗) 4π



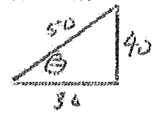
AP Calculus AB Trig Unit Test Review WS #3 Name

1. Suppose a bird is flying horizontally 40 ft above your head at 20 ft/sesc. How fast is the angle of elevation changing when your horizontal distance from the bird is 30 ft?



$x = 30$ ft $\quad \frac{dx}{dt} = 20$ ft/s
$y = 40$ ft
$z = 50$ ft

$\tan\theta = \frac{40}{x}$
$\tan\theta = 40x^{-1}$
$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -40x^{-2}\left(\frac{dx}{dt}\right)$
$\frac{d\theta}{dt} = \frac{-40}{x^2\sec^2\theta}\cdot\frac{dx}{dt}$
$= \frac{-40}{x^2}\cdot\cos^2\theta\cdot\frac{dx}{dt}$



$\cos\theta = \frac{30}{50} = \frac{3}{5}$

$\frac{d\theta}{dt} = \frac{-40}{30^2}\cdot\left(\frac{3}{5}\right)^2\cdot 20 = \frac{-40\cdot 9\cdot 20}{30\cdot 30\cdot 25}$

$\boxed{\frac{d\theta}{dt} = -8/25 \text{ rad/sec}}$

2. Find dy/dx for the equation $xe^{\sin(2y)} + 4x = \ln y$
*product rule

$1e^{\sin(2y)} + xe^{\sin(2y)}\cdot 2\cos(2y)\cdot\frac{dy}{dx} + 4 = \frac{1}{y}\frac{dy}{dx}$

$\frac{dy}{dx}\left[2xe^{\sin(2y)}\cos(2y) - \frac{1}{y}\right] = -4 - e^{\sin 2y}$

$\frac{dy}{dx} = \frac{-4 - e^{\sin 2y}}{2xe^{\sin 2y}\cos 2y - \frac{1}{y}}\cdot\frac{(y)}{(y)}$

$\boxed{= \frac{-4y - ye^{\sin 2y}}{2xye^{\sin 2y} - 1}}$

3. Find the equation of the tangent line to function $f(x) = x\arccos(x^2)$ to the graph at the point $(0, 2\pi)$ in the given domain $[0, \pi]$

*product rule $f'g + fg'$ $\quad$ * $\frac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$

$f'(x) = 1\arccos(x^2) + x\cdot\left(\frac{-2x}{\sqrt{1-x^4}}\right)$
$f'(0) = \arccos(0^2) + 0\left(\frac{-2(0)}{\sqrt{1-0^2}}\right)$
$= \arccos(0)$
$= \frac{\pi}{2}, \frac{3\pi}{2}$ (crossed out: $\frac{3\pi}{2}$)

point: $(0, 2\pi)$
slope: $m = \pi/2$

$y - y_1 = m(x - x_1)$
$\boxed{y - 2\pi = \frac{\pi}{2}(x - 0)}$

~~Find dy/dx for ln(xy) + e~~

$\tan(\arctan(x+y)) = \tan(y^2 + \pi/4)$
$x + y = \tan(y^2 + \pi/4)$
$1 + dy/dx = \sec^2(y^2 + \pi/4)\cdot 2y\left(\frac{dy}{dx}\right)$
$dy/dx = \sec^2(y^2 + \pi/4)\cdot 2y(dy/dx) - 1$

$\frac{dy}{dx}_{(1,0)} = \sec^2(\pi/4)\cdot 2(0)\left(\frac{dy}{dx}\right) - 1$
$= 0 - 1 = -1$

4. Find derivative at the point (1,0) for $\arctan(x + y) = y^2 + \frac{\pi}{4}$
find equation of tangent line

point: $(1, 0)$
slope: $m = -1$
$\boxed{y - 0 = -1(x - 1)}$

(12)

5. Sketch the graph of function $f(x) = \sin(2x)$ on interval $[0, \pi]$. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

$f'(x) = 2\cos(2x)$
$0 = 2\cos(2x)$
$\cos(2x) = 0$
$(2x) = \cos^{-1}(0)$
$2x = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$
$x = \pi/4, 3\pi/4,$ ~~$5\pi/4, 7\pi/4$~~

↗ ↘ ↗ ; + − + ; $0 \quad \pi/4 \quad 3\pi/4 \quad \pi$

$f(\pi/4) = \sin(\pi/2) = 1$
$f(3\pi/4) = \sin(3\pi/2) = -1$

$f''(x) = -4\sin(2x)$
$0 = -4\sin(2x)$
$\sin(2x) = 0$
$(2x) = \sin^{-1}(0)$
$(2x) = 0, \pi, 2\pi, 3\pi$
$x = 0, \pi/2, \pi, 3\pi/2$

∩ ∪ ; − + ; $0 \quad \pi/2 \quad \pi$

$f(\pi/2) = \sin(2(\pi/2)) = \sin(\pi) = 0$

a) the relative extrema min: $(3\pi/4, -1)$ b/c $f'(x)$ changes from − to + max: $(\pi/4, 1)$ b/c $f'(x)$ changes from + to −

b) the point(s) of inflection: $(\pi/2, 0)$ b/c $f''(x)$ change signs

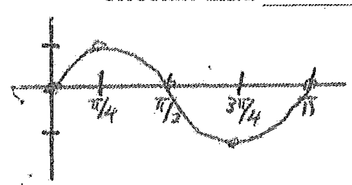
c) concavity intervals up: $(\pi/2, \pi)$ b/c $f''(x) > 0$

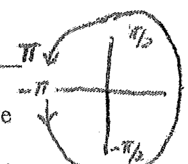
down: $(0, \pi/2)$ b/c $f''(x) < 0$

d) Absolute extrema: Absolute min: -1 at $x = 3\pi/4$ Absolute max: 1 at $x = \pi/4$

d) Sketch the graph
$f(0) = 0$
$f(\pi) = 0$

∩ ∩ ∪ ∪ ; ↗ ↘ ↘ ↗ ; $0 \quad \pi/4 \quad \pi/2 \quad 3\pi/4 \quad \pi$





6. Sketch the graph of function $f(x) = \frac{1}{2}x - \cos x$ on interval $[-\pi, \pi]$. Find all ordered pairs of absolute and relative extrema, intervals increase/decrease, intervals of concavity up/down, and POI.

$f'(x) = \frac{1}{2} + \sin x$
$0 = \frac{1}{2} + \sin x$
$\sin x = -1/2$
$x = 7\pi/6, 11\pi/6$
→ $-\pi/6, -5\pi/6$

↗ ↘ ↗ ; + − + ; $-\pi \quad -5\pi/6 \quad -\pi/6 \quad \pi$

$f(-\frac{5\pi}{6}) = \frac{1}{2}(\frac{-5\pi}{6}) - \cos(\frac{-5\pi}{6})$
$= \frac{-5\pi}{12} - (\frac{-\sqrt{3}}{2}) \approx -1.25 + 0.75$
≈ -0.5

$f(-\pi/6) = \frac{1}{2}(-\pi/6) - \cos(-\pi/6)$
$= -\frac{\pi}{12} - \frac{\sqrt{3}}{2} \approx \frac{-1}{4} - \frac{3}{4} \approx -1$

$f''(x) = \cos x \quad \cos x = 0$
$x = \pi/2, -\pi/2$

∩ ∪ ∩ ; − + − ; $-\pi \quad -\pi/2 \quad \pi/2 \quad \pi$

$f(-\pi/2) = -\pi/4 - \cos(-\pi/2) = -\pi/4$
$f(\pi/2) = \pi/4 - \cos(\pi/2) = \pi/4$
$f(-\pi) = \frac{-\pi}{2} - \cos(-\pi) = \frac{-\pi}{2} + 1$
$f(\pi) = \pi/2 - \cos(\pi) = \pi/2 + 1$

a) the relative extrema min: $(-\pi/6, -\frac{\pi}{12} - \frac{\sqrt{3}}{2})$ max: $(-5\pi/6, -5\pi/12 + \sqrt{3}/2)$

b) the point(s) of inflection: $(-\pi/2, -\pi/4)$ and $(\pi/2, \pi/4)$ b/c $f''(x)$ change signs

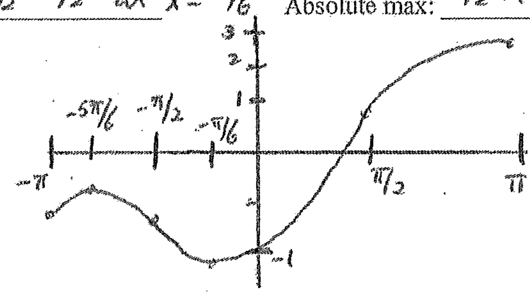
c) concavity intervals up: $(-\pi/2, \pi/2)$ b/c $f''(x) > 0$

down: $(-\pi, -\pi/2) \cup (\pi/2, \pi)$ b/c $f''(x) < 0$

d) Absolute extrema: Absolute min: $-\pi/12 - \sqrt{3}/2$ at $x = -\pi/6$ Absolute max: $\pi/2 + 1$ at $x = \pi$

d) Sketch the graph

∩ ∩ ∪ ∪ ∩ ; ↗ ↘ ↘ ↗ ↗ ; $-\pi \quad -5\pi/6 \quad -\pi/2 \quad -\pi/6 \quad \pi/2 \quad \pi$



Calculus Trig Unit 2.6 Trig Related Rates Practice Problems WS

1)

A hot air balloon rises vertically at a constant rate of 10 feet per second. An observer is lying on the ground 200 feet from the spot directly below the balloon watching the balloon rise.

a) Draw a diagram of the situation. Label all fixed quantities with their value, and label all changing distances with variables. Draw arrows to show the direction that the distances will change.

b) At 20 seconds, how high is the balloon? What is the angle of elevation from the observer to the balloon?

c) If the angle of elevation is 60 degrees, how high is the balloon? To the nearest second, how much time is needed for the balloon to reach this height?

d) If the angle of elevation from the observer to the balloon is 30 degrees, how high is the balloon? To the nearest second, in how much time did the balloon reach this height?

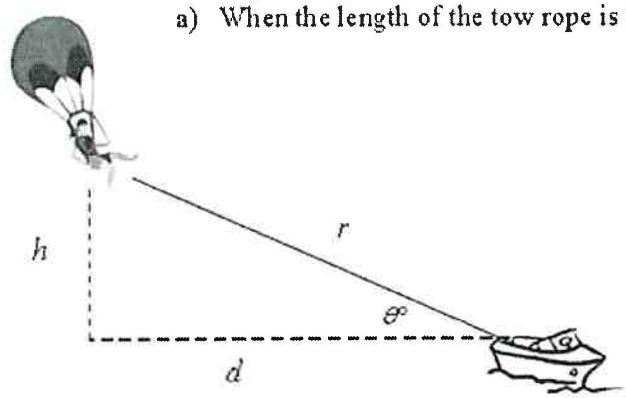
e) When the balloon is 1000 feet above the ground, what is the angle of elevation from the observer to the balloon? To the nearest second, in how much time did the balloon reach this height?

(14)

2)

A ski boat is pulling a parasailer above a large lake. The rider is attached to the boat by a tow rope, r. As the boat's speed increases, the rider's distance above the water increases. The length of the tow rope is controlled by a winch mounted on the back of the deck so that the rope forms an angle with the deck.

a) When the length of the tow rope is 200 ft and $h = 120$ feet, what is d?

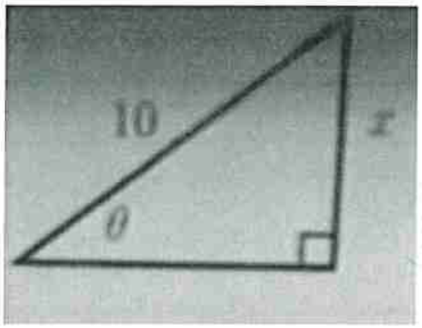


A ski boat is pulling a parasailer above a large lake. The rider is attached to the boat by a tow

b) When the length of the tow rope is 200 ft, what is the measure of the angle formed by the rope and the deck of the boat?

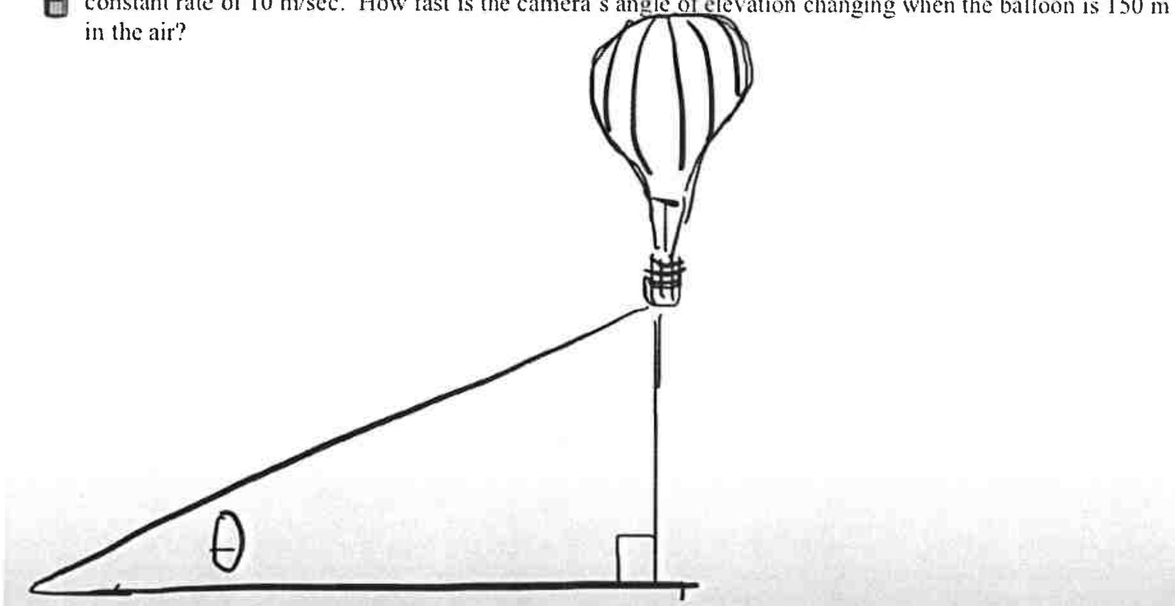
c) As more rope is released, the angle between the tow rope and the deck of the boat increases at 2 degrees per 5 feet of rope. If the total length of the tow rope increases to 240 feet, what is the measure of the angle?

3) The angle θ is increasing at a constant rate of 6 radians per hour. At what rate is the side of length x increasing when x = 6 feet?



4)

A camera on the ground 200 meters away from a hot air balloon records the balloon rising into the sky at a constant rate of 10 m/sec. How fast is the camera's angle of elevation changing when the balloon is 150 m in the air?



16

5)

A kite 40 m above the ground moves horizontally at the rate of $3\,\frac{\text{m}}{\text{s}}$. At what rate is the angle between the string and the horizontal decreasing when 80 m of string has been let out?

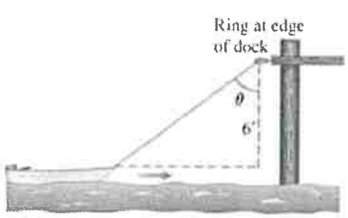
6)

A man stands 12 metres away from a flagpole. He holds onto a long rope attached to the flag. As the flag is raised at a rate of 10 metres per minute, the rope runs tautly through the man's hands (so that it is always kept straight). Find the rate of change of the angle between the rope and the flagpole, at the moment when there is 24 metres of rope between the flag and the man.

7)

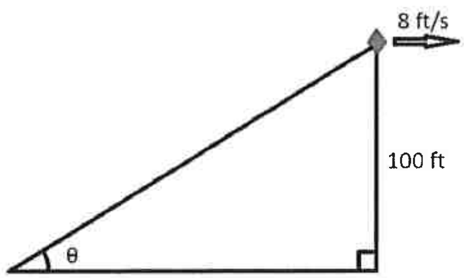
A man walks along a straight path at a speed of 4 ft/sec. A searchlight is located on the ground 20 ft from the path, and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

8) A rowboat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft. above the bow. The rope is hauled in a rate of 2 ft/s.
At what rate is the angle θ changing when 10 ft. of rope is out?



(18)

9) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?



10)

A model rocket is launched 30 feet from Maria, and is rising vertically at a constant rate of 20 ft/s when the rocket has an elevation of 40 feet. How fast is the angle of elevation from Maria to the rocket changing at that moment?

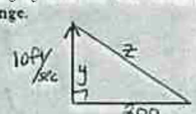
Key

Calculus Trig Unit **2.6 Trig Related Rates Practice Problems WS**

1) https://www.showme.com/sh/?h=OJRDzfs

A hot air balloon rises vertically at a constant rate of 10 feet per second. An observer is lying on the ground 200 feet from the spot directly below the balloon watching the balloon rise.

a) Draw a diagram of the situation. Label all fixed quantities with their value, and label all changing distances with variables. Draw arrows to show the direction that the distances will change.



b) At 20 seconds, how high is the balloon? What is the angle of elevation from the observer to the balloon?

$\frac{10 ft}{sec} \cdot 20 secs = $ **200 ft** off the ground

$\tan\theta = \frac{200}{200} = 1$

$\tan\theta = 1$

$\theta = 45°$

c) If the angle of elevation is 60 degrees, how high is the balloon? To the nearest second, how much time is needed for the balloon to reach this height?

$\tan 60° = \frac{y}{200}$

$200\tan 60° = y$

$y = 346.4$ ft

$y = 200\sqrt{3}$ ft

$200\sqrt{3}$ ft $\cdot \frac{1 sec}{10 ft} =$

34.64 seconds for balloon to reach height

d) If the angle of elevation from the observer to the balloon is 30 degrees, how high is the balloon? To the nearest second, in how much time did the balloon reach this height?

$y = \frac{200}{\sqrt{3}}$ ft.

$\frac{200}{\sqrt{3}}$ ft $\cdot \frac{1 sec}{10 ft} = $ 11.547 seconds

e) When the balloon is 1000 feet above the ground, what is the angle of elevation from the observer to the balloon? To the nearest second, in how much time did the balloon reach this height?

$\tan\theta = \frac{1000}{200}$

$\tan\theta = 5$

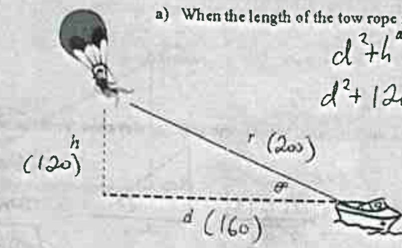
$\theta = 78.69°$

1000 ft $\cdot \frac{1 sec}{10 ft} =$

100 seconds to reach height

2) https://www.showme.com/sh/?h=FGbdclk

A ski boat is pulling a parasailer above a large lake. The rider is attached to the boat by a tow rope, *r*. As the boat's speed increases, the rider's distance above the water increases. The length of the tow rope is controlled by a winch mounted on the back of the deck so that the rope forms an angle with the deck.



a) When the length of the tow rope is 200 ft and *h* = 120 feet, what is *d*?

$d^2 + h^2 = r^2$

$d^2 + 120^2 = 200^2$

$d^2 = \sqrt{25600}$

$d = 160$ ft

A ski boat is pulling a parasailer above a large lake. The rider is attached to the boat by a tow

b) When the length of the tow rope is 200 ft, what is the measure of the angle formed by the rope and the deck of the boat?

$\sin\theta = \frac{120}{200}$

$\sin\theta = 0.6$

$\theta = \sin^{-1}(0.6)$

$\theta = 36.870°$

c) As more rope is released, the angle between the tow rope and the deck of the boat increases at 2 degrees per 5 feet of rope. If the total length of the tow rope increases to 240 feet, what is the measure of the angle?

r (240)

$\frac{2 \text{ degrees}}{5 \text{ ft}} = \frac{x \text{ degrees}}{40}$ ← 240-200 = 40

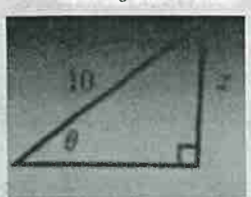
$5x = 80$

$x = 16$ degrees

original angle + newly added angle = new angle

$36.870° + 16° = $ **52.870° degrees**

3) The angle θ is increasing at a constant rate of 6 radians per hour. At what rate is the side of length x increasing when x = 6 feet?



$x = 6$, $\frac{d\theta}{dt} = 6$ | $\frac{dx}{dt} = \_\_\_ ?$

$\sin\theta = \frac{x}{10}$

$\sin\theta = \frac{1}{10}x$

$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{10}\left(\frac{dx}{dt}\right)$

$\cos\theta = \frac{8}{10} = \frac{4}{5}$

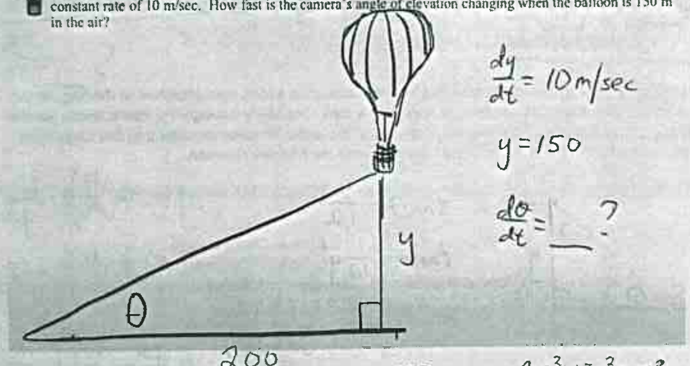
$\left(\frac{4}{5}\right)(6) = \frac{1}{10}\left(\frac{dx}{dt}\right)$

$\left(\frac{4}{5}\right)(6)\left(\frac{10}{1}\right) = \frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = 48 \text{ ft/hr.}}$

4) https://www.youtube.com/watch?v=v1Qv7Rj9L0A

A camera on the ground 200 meters away from a hot air balloon records the balloon rising into the sky at a constant rate of 10 m/sec. How fast is the camera's angle of elevation changing when the balloon is 150 m in the air?



$\frac{dy}{dt} = 10 \text{ m/sec}$

$y = 150$

$\frac{d\theta}{dt} = \_\_\_ ?$

$200^2 + 150^2 = z^2$

$z = 250$

$\sec\theta = \frac{hyp}{adj} = \frac{250}{200} = \frac{5}{4}$

$\tan\theta = \frac{y}{200}$

$\tan\theta = \frac{1}{200}(y)$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{200}\left(\frac{dy}{dt}\right)$

$\left(\frac{5}{4}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{200}(10)$

$\frac{d\theta}{dt} = \left(\frac{4}{5}\right)^2\left(\frac{10}{200}\right) = 0.032$

$\boxed{\frac{d\theta}{dt} = 0.032 \text{ rad/sec}}$

5) https://www.youtube.com/watch?v=kXLKmovhSP8

A kite 40 m above the ground moves horizontally at the rate of $3\frac{m}{s}$. At what rate is the angle between the string and the horizontal decreasing when 80 m of string has been let out?

$\frac{dx}{dt} = 3 \text{ m/s}$

$x = 80$

$\frac{d\theta}{dt} = \_\_\_ ?$

$\tan\theta = \frac{40}{x}$

$\tan\theta = 40x^{-1}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -40x^{-2}\left(\frac{dx}{dt}\right)$

$x^2 + 40^2 = 80^2$, $x = 40\sqrt{3}$

$\sec\theta = \frac{80}{40\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{-40}{(40\sqrt{3})^2}(3)$

$\frac{d\theta}{dt} = \left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{-40}{\quad}\right)(3)$

$\frac{d\theta}{dt} = \frac{3}{4} \cdot \frac{-40}{4800} \cdot 3$

$\boxed{\frac{d\theta}{dt} = \frac{-3}{160} \text{ rad/sec}}$

6) https://www.youtube.com/watch?v=mbmaW7X54To

A man stands 12 metres away from a flagpole. He holds onto a long rope attached to the flag. As the flag is raised at a rate of 10 metres per minute, the rope runs tautly through the man's hands (so that it is always kept straight). Find the rate of change of the angle between the rope and the flagpole, at the moment when there is 24 metres of rope between the flag and the man.

$\frac{d\theta}{dt} = \_\_\_ ?$

$\frac{dy}{dt} = 10 \text{ m/min}$

$z = 24$

$\tan\theta = \frac{12}{y}$

$\tan\theta = 12y^{-1}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -12y^{-2}\left(\frac{dy}{dt}\right)$

$12^2 + y^2 = 24^2$

$y = \sqrt{432}$

$y = 12\sqrt{3}$

$\sec\theta = \frac{24}{12\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{-12}{y^2}\left(\frac{dy}{dt}\right)$

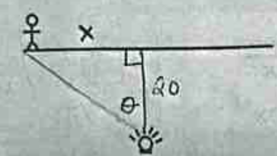
$\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{-12}{(12\sqrt{3})^2}(10)$

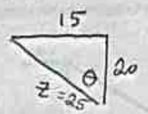
$\frac{d\theta}{dt} = \left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{-12}{432}\right)(10)$

$\boxed{\frac{d\theta}{dt} = -\frac{5}{24} \text{ rad/min.}}$

7) https://www.youtube.com/watch?v=LWJYL513Rm8

A man walks along a straight path at a speed of 4 ft/sec. A searchlight is located on the ground 20 ft from the path, and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?





$15^2+20^2=z^2$

$z=25$

$\sec\theta = \frac{25}{20} = \frac{5}{4}$

$\frac{dx}{dt} = -4$ ft/s

$x = 15$

$\frac{d\theta}{dt} = $ __ ?

$\tan\theta = \frac{x}{20}$

$\tan\theta = \frac{1}{20}x$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{20}\left(\frac{dx}{dt}\right)$

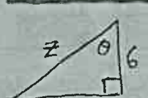
$\left(\frac{5}{4}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{20}(-4)$

$\frac{d\theta}{dt} = \left(\frac{4}{5}\right)^2\left(\frac{1}{20}\right)(-4)$

$\frac{d\theta}{dt} = \frac{-16}{125}$ rad/sec

https://math.stackexchange.com/questions/1826233/at-what-rate-is-the-angle-theta-changing-when-10-ft-of-rope-is-out

8) A rowboat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft. above the bow. The rope is hauled in a rate of 2 ft/s.
At what rate is the angle θ changing when 10 ft. of rope is out?



$\frac{dz}{dt} = -2$

$z = 10$

$\frac{d\theta}{dt} = $ __ ?

$\cos\theta = \frac{6}{z}$

$\cos\theta = 6z^{-1}$

$-\sin\theta\left(\frac{d\theta}{dt}\right) = -6z^{-2}\left(\frac{dz}{dt}\right)$

$-\sin\theta\left(\frac{d\theta}{dt}\right) - \frac{6}{z^2}\left(\frac{dz}{dt}\right)$

$6^2+x^2=10^2$

$x=8$

$\sin\theta = \frac{8}{10}$

$\sin\theta = \frac{4}{5}$

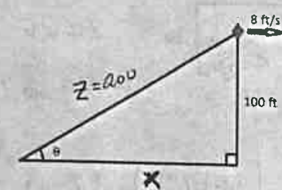
$-\left(\frac{4}{5}\right)\left(\frac{d\theta}{dt}\right) = \frac{-6}{10^2}(-2)$

$\frac{d\theta}{dt} = \frac{-5}{4}\cdot\frac{6}{100}\cdot 2$

$\frac{d\theta}{dt} = \frac{-15}{100} = \frac{-3}{20}$ or -0.15 rad/sec

https://jakesmathlessons.com/derivatives/solution-a-kite-100-ft-above-the-ground-moves-horizontally-at-a-speed-of-8-ft-s-at-what-rate-is-the-angle-between-the-string-and-the-horizontal-decreasing-when-200-ft-of-string-has-been-let-out/

9) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?



$z = 200$ ft

$\frac{dx}{dt} = 8$ ft/s

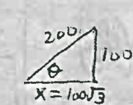
$\frac{d\theta}{dt} = $ __ ?

$\tan\theta = \frac{100}{x}$

$\tan\theta = 100x^{-1}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = -100x^{-2}\left(\frac{dx}{dt}\right)$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{-100}{x^2}\left(\frac{dx}{dt}\right)$



$x^2+100^2=200^2$ $x = 100\sqrt{3}$

$\sec\theta = \frac{200}{100\sqrt{3}} = \frac{2}{\sqrt{3}}$

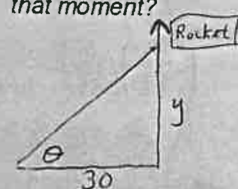
$\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{-100}{(100\sqrt{3})^2}(8)$

$\frac{d\theta}{dt} = \left(\frac{\sqrt{3}}{2}\right)^2\cdot\frac{-100}{30{,}000}\cdot 8$

$\frac{d\theta}{dt} = \frac{-1}{50}$ rad/sec

10) https://www.youtube.com/watch?v=o_k-I7pJgRA

A model rocket is launched 30 feet from Maria, and is rising vertically at a constant rate of 20 ft/s when the rocket has an elevation of 40 feet. How fast is the angle of elevation from Maria to the rocket changing at that moment?



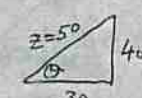
$\frac{dy}{dt} = 20$ ft/s

$y = 40$ ft

$\frac{d\theta}{dt} = $ __ ?

$\tan\theta = \frac{y}{30} = \frac{1}{30}(y)$

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{30}\left(\frac{dy}{dt}\right)$



$\sec\theta = \frac{50}{30} = \frac{5}{3}$

$\left(\frac{5}{3}\right)^2\left(\frac{d\theta}{dt}\right) = \frac{1}{30}(20)$

$\frac{d\theta}{dt} = \left(\frac{3}{5}\right)^2\left(\frac{1}{30}\right)(20)$

$\frac{d\theta}{dt} = \frac{9}{25}\cdot\frac{2}{3}$

$\frac{d\theta}{dt} = \frac{6}{25}$ rad/sec

22

where t is the time in seconds.

(a) Find the time of one complete cycle of the rod.

(b) What is the lowest point reached by the end of the rod on the y-axis?

(c) Find the speed of the y-axis endpoint when the x-axis endpoint is $\left(\frac{1}{4}, 0\right)$.

32. **Machine Design** Repeat Exercise 31 for a position function of $x(t) = \frac{3}{5}\sin \pi t$. Use the point $\left(\frac{3}{10}, 0\right)$ for part (c).

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38. **Angle of Elevation** A balloon rises at a rate of 4 meters per second from a point on the ground 50 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 50 meters above the ground.

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43 . (Calculator)

A missile rises vertically from a point on the ground75,000 feet from a radar station. If the missile is rising at the rate of 40,000 feet per minute at the instant when it is 100,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

(A) $\frac{18}{25}$ (B) $\frac{8}{15}$ (C) $\frac{24}{125}$ (D) $\frac{18}{125}$ (E) $\frac{8}{25}$

where t is the time in seconds.

(a) Find the time of one complete cycle of the rod.

(b) What is the lowest point reached by the end of the rod on the y-axis?

(c) Find the speed of the y-axis endpoint when the x-axis endpoint is $\left(\frac{1}{4}, 0\right)$.

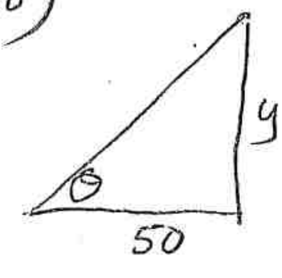
32. **Machine Design** Repeat Exercise 31 for a position function of $x(t) = \frac{3}{5}\sin \pi t$. Use the point $\left(\frac{3}{10}, 0\right)$ for part (c).

[redacted]

38. **Angle of Elevation** A balloon rises at a rate of 4 meters per second from a point on the ground 50 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 50 meters above the ground.

Key

38)



$\frac{dy}{dt} = 4\,m/s$

$\frac{d\theta}{dt} = \_\_\_\_$

$y = 50$

$\tan\theta = \frac{y}{50}$

$\tan\theta = \frac{1}{50}y$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{50}\cdot\frac{dy}{dt}$

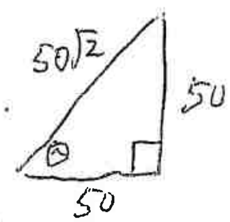
$\frac{1}{\sec^2\theta}\left(\sec^2\theta \cdot \frac{d\theta}{dt}\right) = \left(\frac{1}{50}\cdot\frac{dy}{dt}\right)\frac{1}{\sec^2\theta}$

$\frac{d\theta}{dt} = \frac{1}{50}\cdot\frac{dy}{dt}\cdot\cos^2\theta$

$\frac{d\theta}{dt} = \frac{1}{50}\cdot(4)\cdot\left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{4}{50}\cdot\frac{1}{2} = \frac{4}{100}$

$\boxed{\frac{d\theta}{dt} = \frac{1}{25}\ rad/s}$



$\cos\theta = \frac{50}{50\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$

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Key

The function f is given by $f(x)=x^4+x^2-2$. On which of the following intervals is f increasing?

* 1st derivative test, use sign line

(A) $\left(-\frac{1}{\sqrt{2}},\infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

[✓ (C)] $(0,\infty)$

(D) $(-\infty,0)$

(E) $\left(-\infty,-\frac{1}{\sqrt{2}}\right)$

$f'(x)=4x^3+2x$

$f'(x)=x(4x^2+2)$

$0=x(4x^2+2)$

$x=0$ | $4x^2+2=0$

$\sqrt{x^2}=\sqrt{\frac{-2}{4}}$

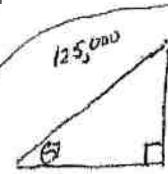
No critical pts.

$f'(x)$ — + ; 0

$f(x)$ increasing on $(0,\infty)$ b/c $f'(x)>0$

43) (calculator)

43) A missile rises vertically from a point on the ground75,000 feet from a radar station. If the missile is rising at the rate of 40,000 feet per minute at the instant when it is 100,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?



$\frac{dy}{dt}=40{,}000\text{ ft/min}$

$y=100{,}000$

$x^2+y^2=z^2$

$75{,}000^2+100{,}000^2=z^2$, $z=125{,}000$

$\tan\theta=\frac{y}{75{,}000}$

$\sec^2\theta\left(\frac{d\theta}{dt}\right)=\frac{1}{75{,}000}\left(\frac{dy}{dt}\right)$

(A) $\frac{18}{25}$ (B) $\frac{8}{15}$ [✓ (C)] $\frac{24}{125}$ (D) $\frac{18}{125}$ (E) $\frac{8}{25}$

$\frac{d\theta}{dt}=\frac{1}{\sec^2\theta}\cdot\frac{1}{75{,}000}\cdot\frac{dy}{dt}$

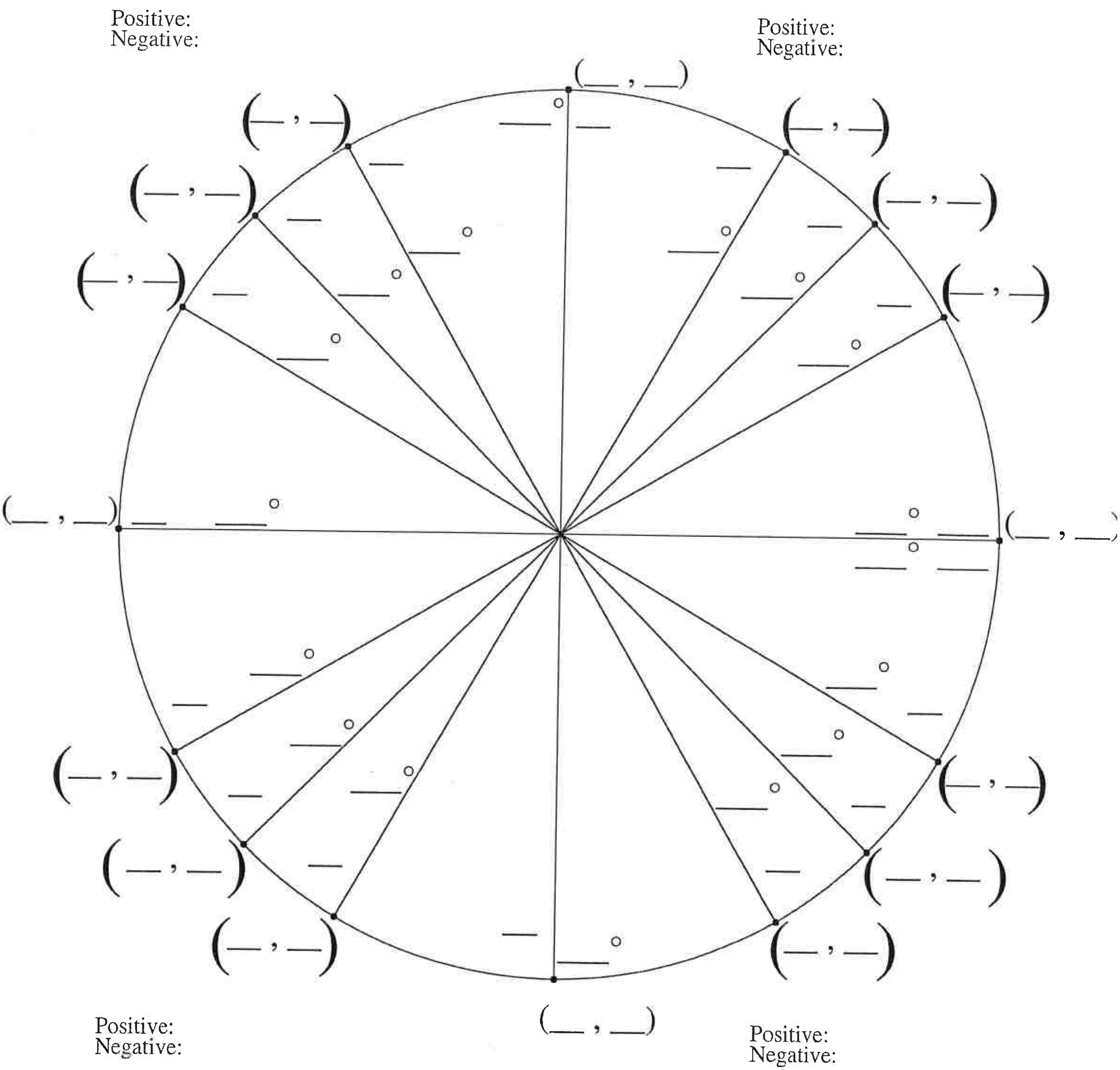
$\frac{d\theta}{dt}=(\cos\theta)^2\left(\frac{1}{75{,}000}\right)\left(\frac{dy}{dt}\right)$

$\frac{d\theta}{dt}=\left(\frac{75{,}000}{125{,}000}\right)^2\left(\frac{1}{75{,}000}\right)(40{,}000)=0.192=\frac{192}{1000}=\frac{24}{125}$

$\cos\theta=\frac{adj}{hyp}$

$\boxed{\frac{d\theta}{dt}=\frac{24}{125}\text{ rad/min.}}$

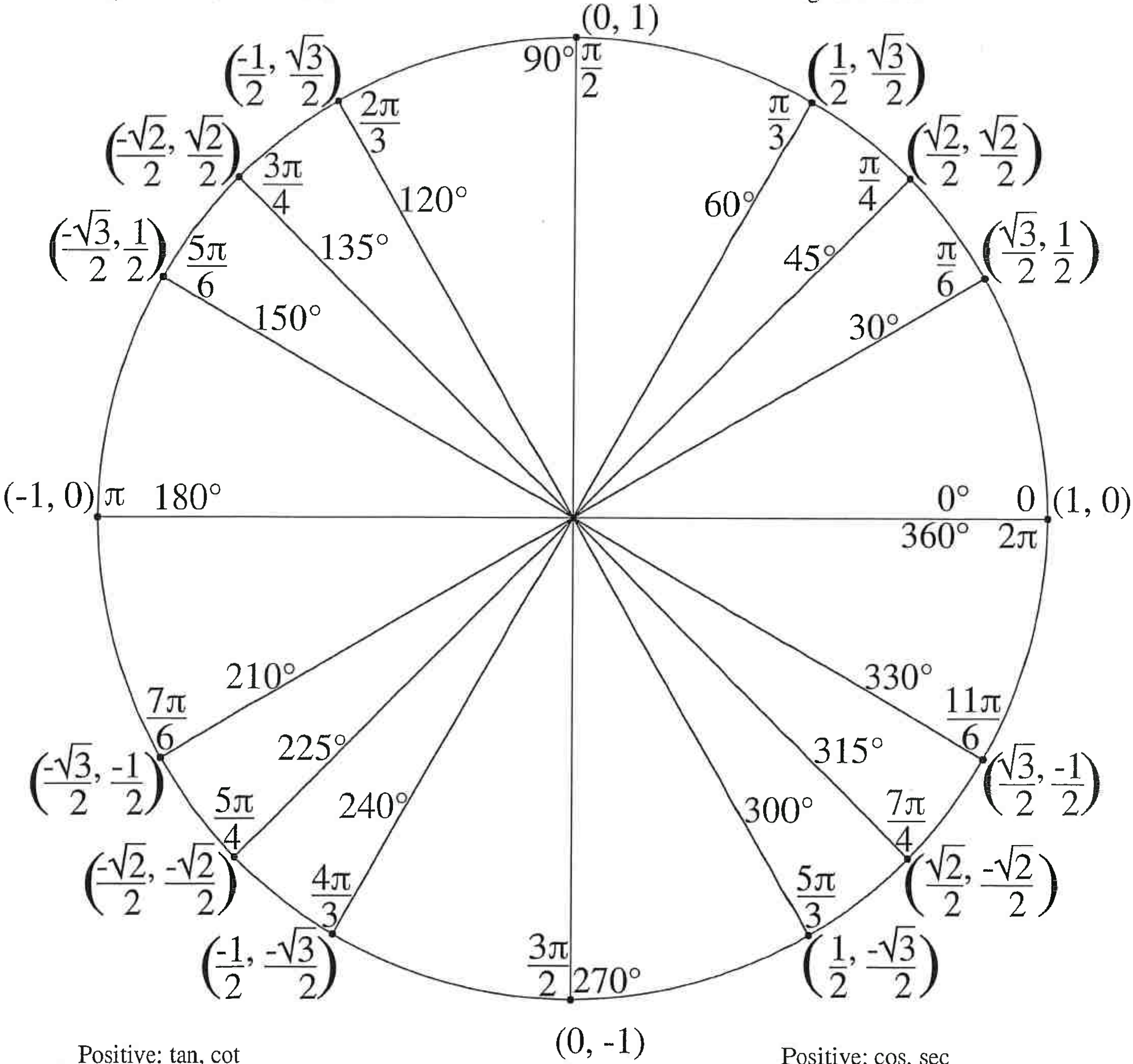
Fill in The Unit Circle



The Unit Circle

Positive: sin, csc
Negative: cos, tan, sec, cot

Positive: sin, cos, tan, sec, csc, cot
Negative: none



Positive: tan, cot
Negative: sin, cos, sec, csc

Positive: cos, sec
Negative: sin, tan, csc, cot

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Top 5 (Calculus AB) Trig Unit Mistakes

Quiz 1 Trig Limits

1. Solving trig equations using factoring correctly
2. Applying trig limits incorrectly
3. Not testing out various trig limits by using trig identities to find limits

Quiz 2 Trig Derivatives

1. Forgetting to apply (nested) chain rules to these types of problems:
   a. $sin^4(x^2 - \pi x) \neq 4\cos(2x - \pi)$
2. Forgetting trig derivatives: Memorize your trig derivatives!
3. Not knowing the difference in trig exponents: $sin^3(3x) \neq \sin(3x)^3$
4. Not recognizing when to apply implicit differentiation, quotient, product rule (or a combination of these rules)
5. Forgetting to rewrite trig exponents and put outside of brackets before taking derivative:
   a. $Rewrite\ cos^4(5x)\ as\ \ [\cos(5x)]^4\ before\ using\ chain\ rule\ to\ find\ derivative$
6. If equation is in the form of y=$\sin(3x)^3$, $go\ ahead\ and\ rewrite\ as$ $y = \sin(27x^3)$ before finding the derivative. Chain rule mistakes/confusion can be avoided this way.

Quiz 3 Trig Curve Sketching

1. Forgetting details involving Mean Value Theorem
2. Misusing Rolle's Theorem
3. Forgetting to write because statements for curve sketching (especially for relative max, relative min, and POI's !)

Trig Unit Test

1. Mistakes with understanding limit problems involving knowledge of 3 Continuity Conditions
2. Mistakes with ArcTrig derivative rules. Know and memorize ArcTrig and all other derivative rules.
3. Mistakes with one of the number of steps with Trig Related Rates problem.
4. Mistakes with curve sketching
   a. 1st derivative test, slope sign line, testing intervals, finding relative extrema
   b. Concavity test, concavity sign line, testing intervals, finding POI
   c. Forgetting to include **because** statements (for relative max, relative min, intervals increasing, decreasing ,POI, intervals concave up, concave down)
   d. Know how to find absolute Max and Min (EVT: Consider endpoints and critical pts)
   e. Mistakes with sketching graph of f(x)

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AP Calculus AB: Trig Unit Test Topics

1) Inverse Trig Derivatives

2) Implicit Differentiation involving trig (tangent line equation)

3) Particle motion problem

4) Related Rates (trig)

5) MVT, Trig Derivatives, Avg Rate of Change

6) Trig Curve Sketching problem

*Make sure to Know: Unit Circle Values, derivative rules for ln(u) , e^u, power rule, trig derivatives, inverse trig derivatives

*No Calculators!