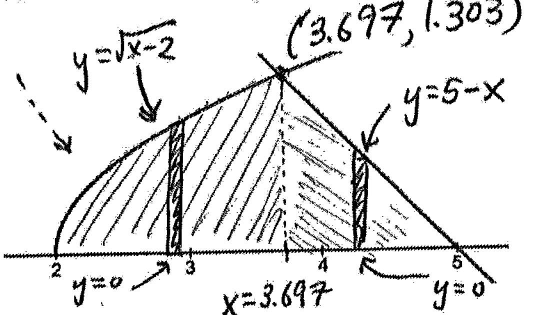
Key

A.P. Calculus AB **Chapter 7.-7.2 Area & Volume Unit Review WS #1**

1) Given the region below enclosed by $f(x) = \sqrt{x-2}$, $g(x) = 5 - x$, and the x-axis.

a) Find the area of the below region. (Write the integral notation(s) as well as the numeric approximation rounded to 3 decimal places)

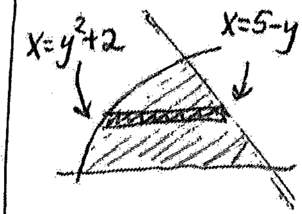


Method 2: Right-Left

$y = \sqrt{x-2}$ | $y = 5-x$

$y^2 = x-2$ | $x = 5-y$

$y^2 + 2 = x$

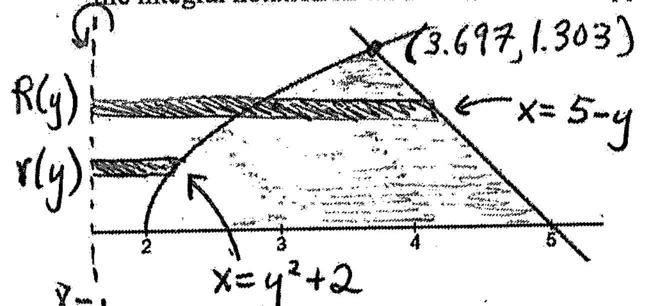


$$\text{Area} = \int_0^{1.303} 5-y-(y^2+2)\,dy = \boxed{2.323}$$

Method 1: Top-Bottom, split into 2 regions:

$$\text{Area} = \int_2^{3.697} \sqrt{x-2} - 0\,dx + \int_{3.697}^{5} 5-x-0\,dx = \boxed{2.323}$$

b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 1 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



* Washer Method, Right-Left

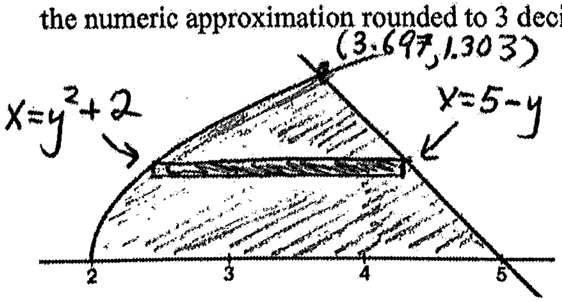
$R(y) = 5-y-(1) = 4-y$

$r(y) = y^2+2-(1) = y^2+1$

$$V = \pi \int_0^{1.303} [4-y]^2 - [y^2+1]^2\,dy$$

$$\boxed{V = 11.265\pi \text{ units}^3}$$

c) The enclosed region is the base of a solid. The cross section of the solid taken <u>perpendicular to the y-axis</u> is an equilateral triangle. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



$\text{base} = 5-y-(y^2+2) = 5-y-y^2-2$

$= 3-y^2-y$

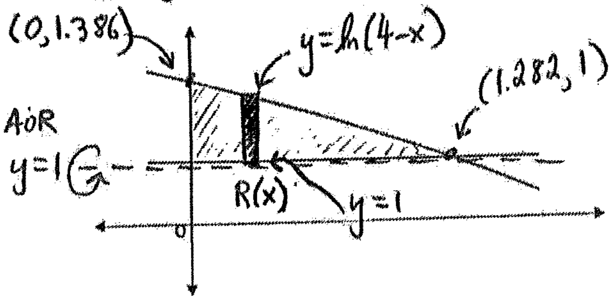
$\text{Area} = \frac{\sqrt{3}}{4}(\text{base})^2 \rightarrow \frac{\sqrt{3}}{4}(3-y^2-y)^2$

$$V = \int_0^{1.303} \frac{\sqrt{3}}{4}(3-y^2-y)^2\,dy = \boxed{2.225 \text{ units}^3}$$

2

2) Given the region below enclosed by $f(x) = \ln(4-x)$, the line $y = 1$, and the y-axis.

a) Find the Volume of solid generated when the enclosed region is revolved about the line $y = 1$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



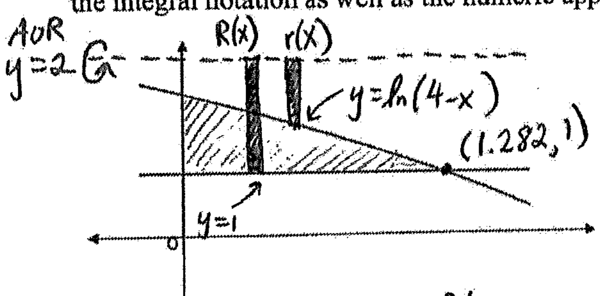
$R(x) = \ln(4-x) - (1)$

$$V = \pi \int_0^{1.282} [\ln(4-x) - 1]^2 \, dx$$

$$\boxed{V = 0.069\pi \text{ units}^3}$$

* Disc Method, Top-Bottom

b) Find the Volume of solid generated when the enclosed region is revolved about the line $y = 2$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



$R(x) = 2 - (1) = 1$

$r(x) = 2 - (\ln(4-x))$

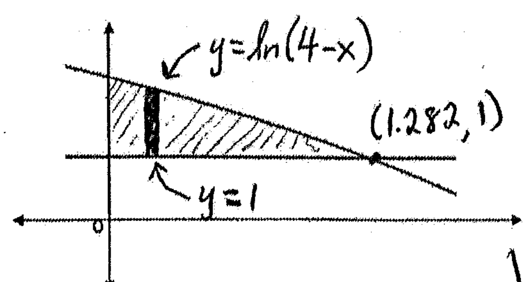
$$V = \pi \int_0^{1.282} [1]^2 - [2 - \ln(4-x)]^2 \, dx$$

$$\boxed{V = 0.457\pi \text{ units}^3}$$

*washer Method, Right-Left

c) The enclosed region is the base of a solid. The cross section of the solid taken <u>perpendicular to the x-axis</u> is a rectangle whose height is twice the base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)

height $= 2[\ln(4-x) - 1]$



base $= \ln(4-x) - (1)$

Area $=$ (base)(height)

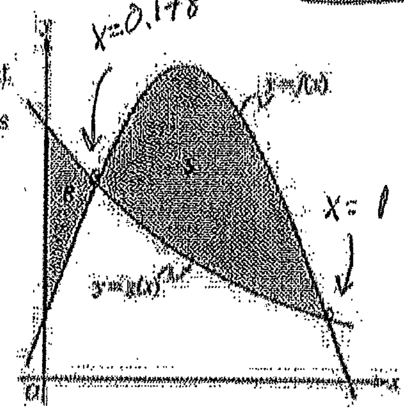
Area $= 2[\ln(4-x) - 1]^2$

$$V = \int_0^{1.282} 2[\ln(4-x) - 1]^2 \, dx$$

$$\boxed{V = 0.139 \text{ units}^3}$$

A.P. Calculus AB **Chapter 7.-7.2 Area & Volume Unit Review WS #2**

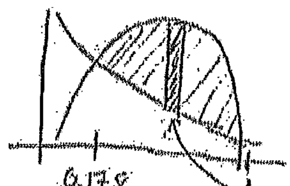
1)



Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

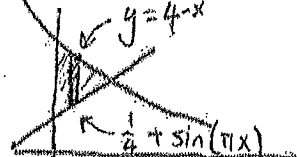
a) Find the area of S

$$A = \int_{0.178}^{1} \frac{1}{4} + \sin(\pi x) - \left(4^{-x}\right) dx$$



$$\boxed{A = 0.410} \text{ units}^2$$

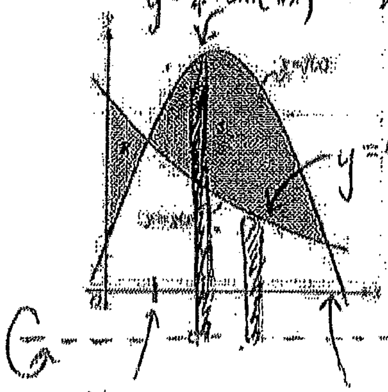
b) Find the area of R



$$\text{Area} = \int_0^{0.178} 4^{-x} - \left(\frac{1}{4} + \sin(\pi x)\right) dx = \boxed{0.0648 \text{ units}^2}$$

c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

$y = \frac{1}{4} + \sin(\pi x)$ *Washer Method



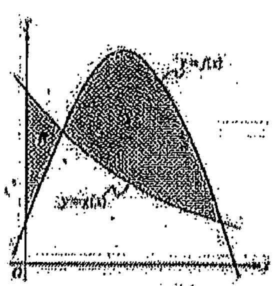
$$R(x) = \frac{1}{4} + \sin(\pi x) - (-1) = \frac{5}{4} + \sin(\pi x)$$

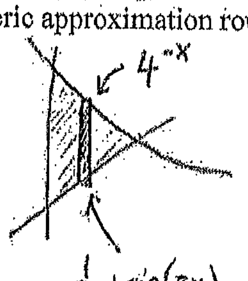
$$r(x) = 4^{-x} - (-1) = 4^{-x} + 1$$

$$V = \pi \int_{x_1}^{x_2} R(x)^2 - r(x)^2 \, dx$$

$$V = \pi \int_{0.178}^{1} \left[\frac{5}{4} + \sin(\pi x)\right]^2 - \left[4^{-x} + 1\right]^2 dx = \boxed{1.451\pi \text{ units}^3}$$

d) The enclosed region R is the base of a solid. The cross section of the solid taken parallel to the y-axis is a isosceles right triangle with leg on base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)





$$\text{base} = 4^{-x} - \left(\frac{1}{4} + \sin(\pi x)\right)$$

$$\text{base} = 4^{-x} - \frac{1}{4} - \sin(\pi x)$$

$$\text{Area} = \frac{1}{2}(\text{base})^2 \rightarrow \frac{1}{2}\left[4^{-x} - \frac{1}{4} - \sin(\pi x)\right]^2$$

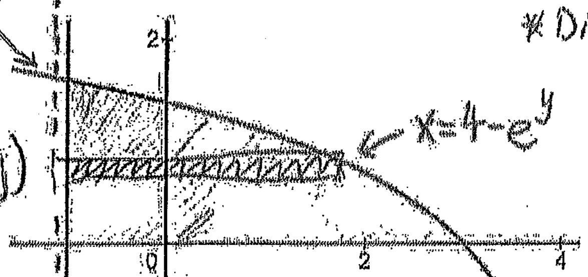
$$V = \int_{x_1}^{x_2} [\text{Area}] \, dx \rightarrow V = \int_0^{0.178} \frac{1}{2}\left[4^{-x} - \frac{1}{4} - \sin(\pi x)\right]^2 dx = \boxed{0.016 \text{ units}^3}$$

(4)

2) Given the region below enclosed by $f(x) = \ln(4-x)$, the line x = -1, and the x-axis.

✓ AOR $x=-1$

a) Find the Volume of solid generated when the enclosed region is revolved about the line x = -1 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



x=-1
AOR

* Disc Method

$y = \ln(4-x)$

$e^y = e^{\ln(4-x)}$

$e^y = (4-x)$

$e^y = 4-x$

$x = 4-e^y$

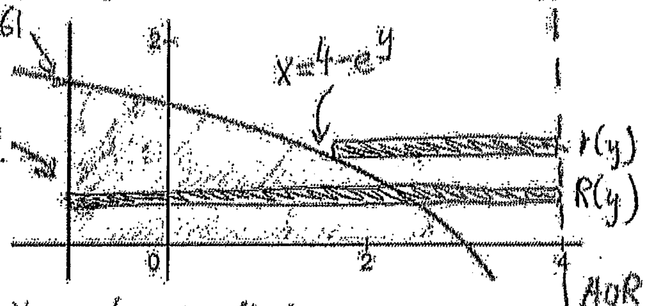
$R(y) = 4 - e^y - (-1)$

$R(y) = 5 - e^y$

$V = \pi \int_{y_1}^{y_2} R[y]^2 \, dy$

$V = \pi \int_0^{1.61} [5-e^y]^2 \, dy = \boxed{12.236\pi \text{ units}^3}$

b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 4 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



*Washer Method

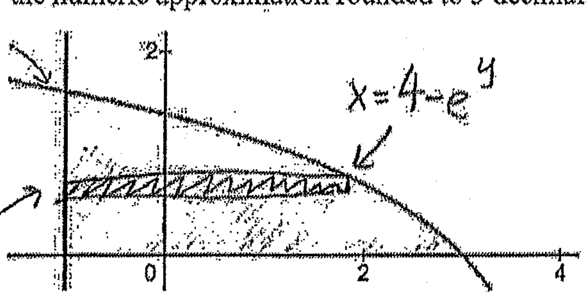
$R(y) = 4-(-1) = 5$

$r(y) = 4-(4-e^y) = e^y$

$V = \pi \int_{y_1}^{y_2} R(y)^2 - r(y)^2 \, dy$

$V = \pi \int_0^{1.61} [5]^2 - [e^y]^2 \, dy = \boxed{28.236\pi \text{ units}^3}$

c) The enclosed region is the base of a solid. The cross section of the solid taken parallel to the x-axis is a rectangle whose height is 4. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



$\text{base} = 4 - e^y - (-1) = 5 - e^y$

$\text{Area} = (\text{base})(\text{height})$

height = 4

$A = (5-e^y)(4)$

$V = \int_{y_1}^{y_2} [\text{Area}] \, dy \rightarrow \int_0^{1.61} 4(5-e^y) \, dy = \boxed{16.189 \text{ units}^3}$

Key

Ch. 7 Area/Volume AB FRQ Problems Worksheet #3

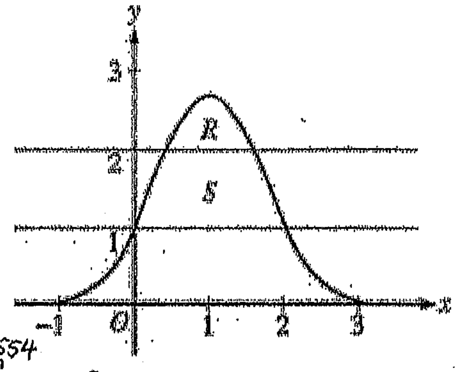
1)

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

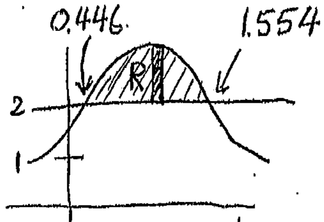
(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



a)



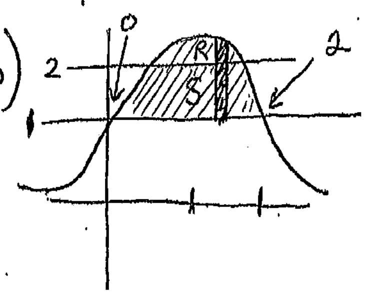
Top/bottom

$y = \underline{e^{2x-x^2}}$

$y = \underline{2}$

$$\text{Area} = \int_{0.446}^{1.554} e^{2x-x^2} - 2\,dx = \boxed{0.514}$$

b)



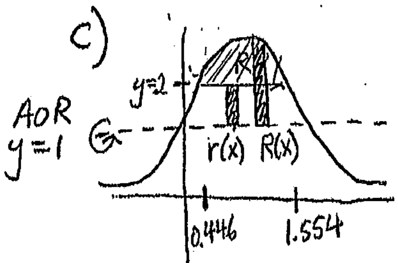
Area of S = Area of R+S − Area of R

$$\text{Area}(R+S) = \int_0^2 e^{2x-x^2} - 1\,dx = 2.06016$$

(Top/bottom)

$$\text{Area of } S = 2.06016 - 0.514 = \boxed{1.546}$$

c)



$R(x) = e^{2x-x^2} - 1$

$r(x) = 2 - 1 = 1$

$$V = \pi \int_{x_1}^{x_2} R(x)^2 - r(x)^2\,dx$$

$$\boxed{V = \pi \int_{0.446}^{1.554} \left[e^{2x-x^2} - 1\right]^2 - \left[1\right]^2 dx}$$

Washer Method

Top/bottom

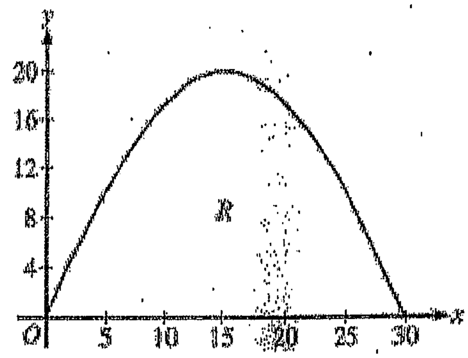
$y = \underline{e^{2x-x^2}}$

$y = \underline{2}$

(6)

2)

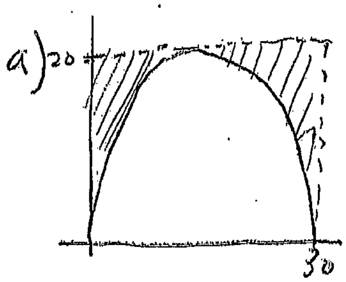
A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

a)

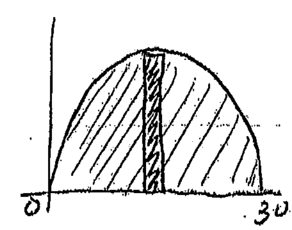


Area = Area of Box − Area under parabola (Top-bottom) $y = \underline{20\sin\left(\frac{\pi x}{30}\right)}$, $y = \underline{0}$

$$= 30(20) - \int_0^{30} 20\sin\left(\frac{\pi x}{30}\right) - 0\,dx$$

$$= 600 - 381.972 = \boxed{218.028\ cm^2}$$

b)



Top/bottom
$y = \underline{20\sin\left(\frac{\pi x}{30}\right)}$
$y = \underline{0}$

base $= 20\sin\left(\frac{\pi x}{30}\right) - 0$

base $= 20\sin\left(\frac{\pi x}{30}\right)$

Area (semicircle) $= \frac{\pi}{8}[\text{base}]^2$

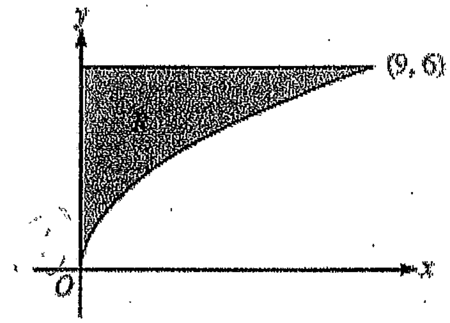
$$= \frac{\pi}{8}\left[20\sin\left(\frac{\pi x}{30}\right)\right]^2$$

$$\text{Volume} = \int_0^{30} \frac{\pi}{8}\left[20\sin\left(\frac{\pi x}{30}\right)\right]^2 dx$$

$V = 2356.194\ cm^3$

$$\frac{0.05 \text{ grams}}{1\ \cancel{cm^3}} \cdot 2356.194\ \cancel{cm^3} = \boxed{117.809 \text{ grams (of chocolate)}}$$

3) (Non-calculator)



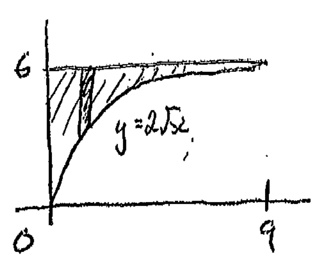
Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

a)



Top-bottom
$y = 2\sqrt{x}$
$y = 6$

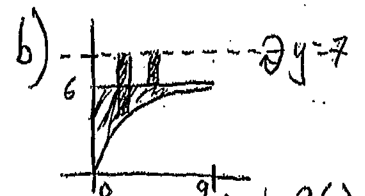
$$\text{Area} = \int_0^9 6 - 2\sqrt{x}\,dx$$

$$= \int_0^9 6 - 2x^{1/2}\,dx = 6x - \frac{2x^{3/2}}{3/2}\Big]_0^9$$

$$6x - \frac{4}{3}x^{3/2}\Big]_0^9 = 6(9) - \frac{4}{3}(9)^{3/2} - (0 - 0)$$

$$= 54 - \frac{4}{3}(3)^3 = 54 - 4(9) = \boxed{18}$$

b)



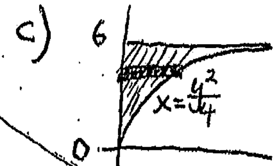
washer method

Top/bottom
$y = 6$
$y = 2\sqrt{x}$

$R(x) = 7 - 2\sqrt{x}$
$r(x) = 7 - 6 = 1$

$$\boxed{V = \pi \int_0^9 [7 - 2\sqrt{x}]^2 - [1]^2\,dx}$$

c)



$y = 2\sqrt{x} \rightarrow \frac{y}{2} = \sqrt{x}$
$\left(\frac{y}{2}\right)^2 = x$

Right/Left
$x = y^2/4$
$x = 0$

$\text{base} = \frac{y^2}{4} - 0 = \frac{y^2}{4}$
$\text{height} = 3(\text{base}) = 3\left(\frac{y^2}{4}\right)$
$\text{Area} = \text{base} \times \text{height}$
$= \left(\frac{y^2}{4}\right) \times 3\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$

$$\boxed{V = \int_0^6 \frac{3}{16}y^4\,dy}$$

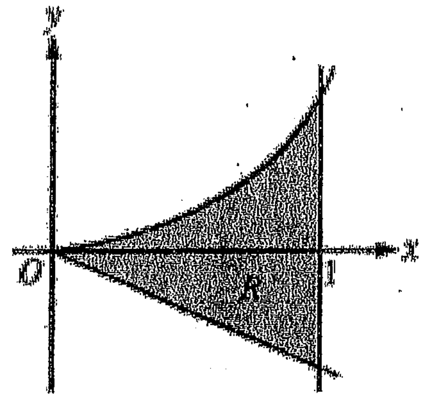
8

4)

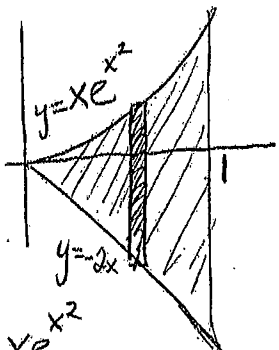
Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.



a)



$y = \underline{xe^{x^2}}$

$y = \underline{-2x}$

Top/bottom

u-substitation

$$\text{Area} = \int_0^1 xe^{x^2} - (-2x)\,dx = \int_0^1 xe^{x^2} + 2x\,dx$$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \cancel{x} e^u \frac{du}{2\cancel{x}}$

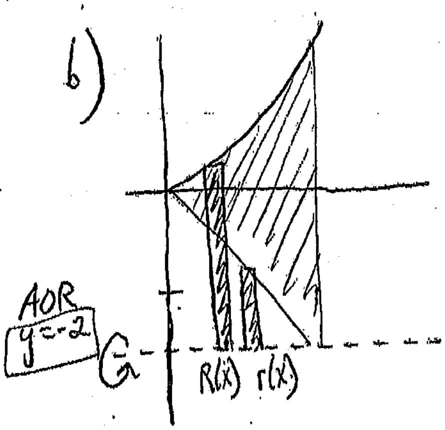
$\frac{1}{2}\int e^u du = \frac{1}{2}e^{x^2}$

$\left[\frac{1}{2}e^{x^2} + \frac{\cancel{2}x^2}{\cancel{2}}\right]_0^1$

$\frac{1}{2}e^1 + 1 - \left(\frac{1}{2}e^0 + 0^2\right)$

$\frac{1}{2}e + 1 - \frac{1}{2} = \boxed{\frac{1}{2}e + \frac{1}{2}}$

b)



washer method

Top/bottom

$y = \underline{xe^{x^2}}$

$y = \underline{-2x}$

$R(x) = xe^{x^2} - (-2)$

$r(x) = -2x - (-2)$

$= -2x + 2$

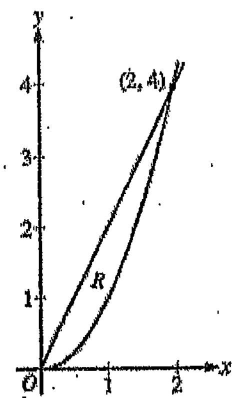
$$V = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2\,dx$$

$$\boxed{V = \pi \int_0^1 \left[xe^{x^2} + 2\right]^2 - \left[-2x + 2\right]^2 dx}$$

Key (9)

AB Calculus Chapter 7.1-7.2 Review Worksheet #4

1. (Non-Calculator)



Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

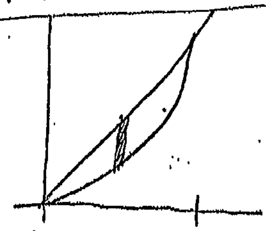
(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

d) Find Volume of solid by rotating R about line $x = -1$ (write expression)

a)



Top-bottom

$y = \underline{2x}$

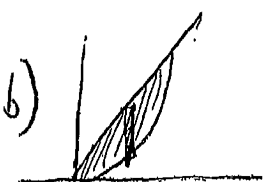
$y = \underline{x^2}$

$$\text{Area} = \int_0^2 2x - x^2\,dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\Big]_0^2 = 2^2 - \frac{2^3}{3} - (0 - 0)$$

$$= 4 - \frac{8}{3} = \frac{12}{3} - \frac{8}{3} = \boxed{\frac{4}{3}}$$

b)



$A(x) = \sin\left(\frac{\pi}{2}x\right)$

cross-section

$$V = \int [\text{Area}]\,dx$$

$$V = \int_0^2 \sin\left(\frac{\pi}{2}x\right)dx$$

$u = \frac{\pi}{2}x$

$\frac{du}{dx} = \frac{\pi}{2}$

$\pi\,dx = 2\,du$

$dx = \frac{2}{\pi}du$

$\int \sin u \cdot \frac{2}{\pi}du$

$\frac{2}{\pi}\int \sin u\,du$

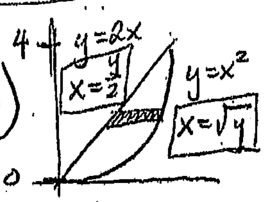
$= \frac{2}{\pi} \cdot -\cos u$

$-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big]_0^2$

$-\frac{2}{\pi}\cos(\pi) - \left(-\frac{2}{\pi}\cos 0\right)$

$-\frac{2}{\pi}(-1) + \frac{2}{\pi} = \boxed{\frac{4}{\pi}}$

c)



Right/Left

$x = \underline{\sqrt{y}}$

$x = \underline{y/2}$

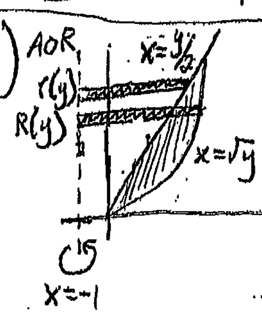
$\text{base} = \sqrt{y} - \frac{y}{2}$

$\text{Area}_{\text{square}} = [\text{base}]^2$

$\text{Area} = \left[\sqrt{y} - \frac{y}{2}\right]^2$

$$\boxed{V = \int_0^4 \left[\sqrt{y} - \frac{y}{2}\right]^2 dy}$$

d)



*washer method

*Right/Left

$x = \underline{\sqrt{y}}$

$x = \underline{y/2}$

$R(y) = \sqrt{y} - (-1)$

$r(y) = y/2 - (-1)$

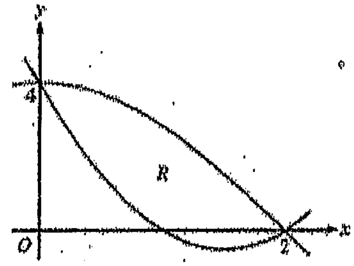
$$V = \pi\int_{y_1}^{y_2} [R(y)]^2 - [r(y)]^2\,dy$$

$$V = \pi\int_0^4 \left[\sqrt{y} + 1\right]^2 - \left[\frac{y}{2} + 1\right]^2 dy$$

(10)

2) (Non-Calculator)

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

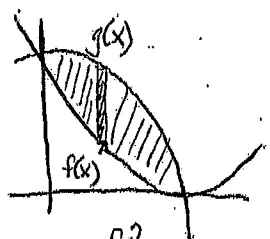


(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

a)



$$\text{Area} = \int_0^2 g(x) - f(x)\,dx$$

$$\text{Area} = \int_0^2 4\cos\left(\tfrac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right) dx$$

$$\text{Area} = \int_0^2 4\cos\left(\tfrac{\pi}{4}x\right) - 2x^2 + 6x - 4\,dx$$

$u = \frac{\pi}{4}x$ | $dx = \frac{4}{\pi}du$

$\frac{du}{dx} = \frac{\pi}{4}$ | $4\int \cos u \cdot \frac{4}{\pi}du$

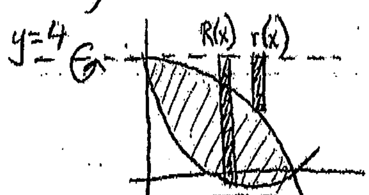
$\pi dx = 4du$

$$\left. 4 \cdot \tfrac{4}{\pi}\sin\left(\tfrac{\pi}{4}x\right) - \tfrac{2x^3}{3} + \tfrac{6x^2}{2} - 4x \right]_0^2$$

$$\tfrac{16}{\pi}\sin\left(\tfrac{\pi}{4}\cdot 2\right) - \tfrac{2(2)^3}{3} + \tfrac{6(2)^2}{2} - 4(2) - \left[\tfrac{16}{\pi}\sin(0) - 0 + 0 - 0\right]$$

$$\boxed{\tfrac{16}{\pi}(1) - \tfrac{16}{3} + \tfrac{24}{2} - 8} \quad \text{or} \quad \boxed{\tfrac{16}{\pi} - \tfrac{4}{3}}$$

b) AOR: $y = 4$



* washer method

* Top/bottom

$y = \underline{4\cos\left(\frac{\pi}{4}x\right)}$

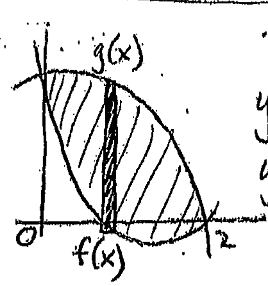
$y = \underline{2x^2 - 6x + 4}$

$R(x) = 4 - \left(2x^2 - 6x + 4\right) = 4 - 2x^2 + 6x - 4 = \boxed{-2x^2 + 6x}$

$r(x) = 4 - 4\cos\left(\frac{\pi}{4}x\right)$

$$\boxed{V = \pi\int_0^2 \left[-2x^2 + 6x\right]^2 - \left[4 - 4\cos\left(\tfrac{\pi}{4}x\right)\right]^2 dx}$$

c)



Top/bottom

$y = \underline{4\cos\left(\frac{\pi}{4}x\right)}$

$y = \underline{2x^2 - 6x + 4}$

base $= 4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)$

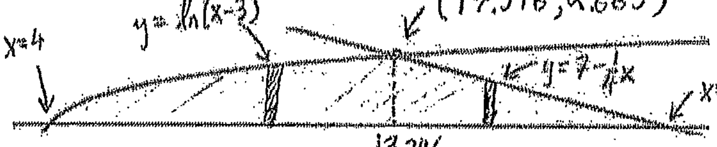
Area square $= \left[\text{base}\right]^2$

$$V = \int_0^2 \left[4\cos\left(\tfrac{\pi}{4}x\right) - 2x^2 + 6x - 4\right]^2 d$$

Key 11

A.P. Calculus AB Chapter 7.1-7.2 Area & Volume Unit Review WS #3

1) Given the region below enclosed by $f(x) = \ln(x-3)$, the line $y = 7 - \frac{1}{4}x$, and the x − axis.



Method 1: (Top-Bottom) *2 separate integrals*

$$Area = \int_4^{17.346} \ln(x-3) - 0\,dx + \int_{17.346}^{28} 7 - \tfrac{1}{4}x - 0\,dx$$

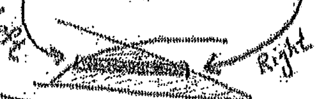
$$Area = 24.864 + 14.188 = \boxed{39.052 \text{ units}^2}$$

Method 2: (Right-Left)

$e^y = e^{\ln(x-3)}$; $y = 7 - \frac{1}{4}x$

$e^y = x - 3$; $(y - 7 = -\frac{1}{4}x) \cdot -4$

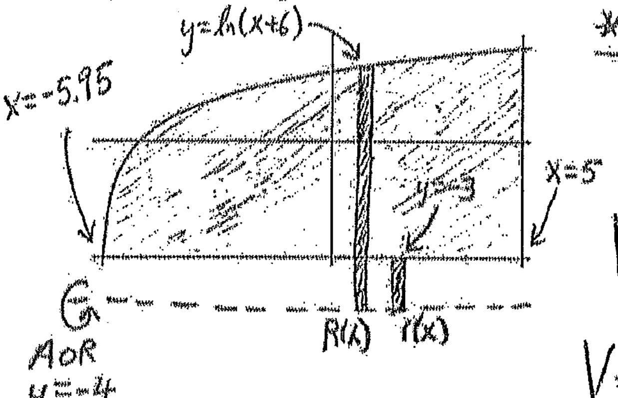
$3 + e^y = x$; $28 - 4y = x$



$$Area = \int_0^{2.663} 28 - 4y - (3 + e^y)\,dy = \boxed{39.052 \text{ units}^2}$$

2) Given the region below enclosed by $f(x) = \ln(x+6)$, the line $y = -3$, and $x = 5$.

a) Find the Volume of solid generated when the enclosed region is revolved about the line $y = -4$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



*Washer Method (Top-Bottom)

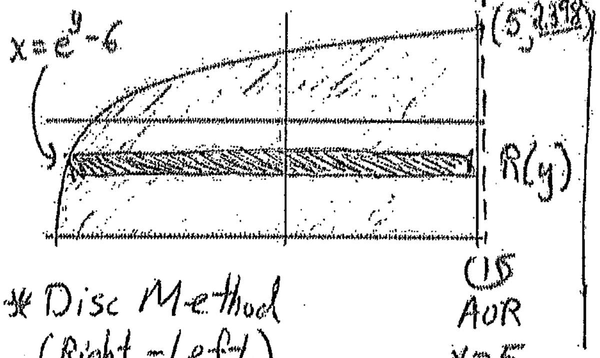
$R(x) = \ln(x+6) - (-4) = \ln(x+6) + 4$

$r(x) = -3 - (-4) = 1$

$$V = \pi \int_{x_1}^{x_2} R(x)^2 - r(x)^2\,dx$$

$$V = \pi \int_{-5.95}^{5} [\ln(x+6) + 4]^2 - [1]^2\,dx = \boxed{320.510\pi \text{ units}^3}$$

b) Find the Volume of solid generated when the enclosed region is revolved about the line x = 5 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



*Disc Method (Right-Left)

$y = \ln(x+6)$

$e^y = e^{\ln(x+6)}$

$e^y = x + 6$

$e^y - 6 = x$

$R(y) = 5 - (e^y - 6)$

$R(y) = 11 - e^y$

*find upper bound:

$y(x) = \ln(x+6)$

$y(5) = \ln(5+6) = \ln 11 = 2.398$

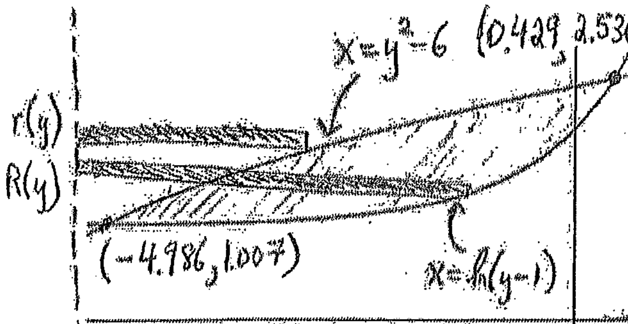
$$V = \pi \int_{y_1}^{y_2} [R(y)]^2\,dy$$

$$V = \pi \int_{-3}^{2.398} [11 - e^y]^2\,dy = \boxed{472.739\pi \text{ units}^3}$$

12

3) Given the region below enclosed by $f(x) = \sqrt{x+6}$, the $g(x) = e^x + 1$

a) Find the Volume of solid generated when the enclosed region is revolved about the line x = −6 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



AOR $x=-6$

*Washer Method (Right − Left)

$y=\sqrt{x+6}$

$(y)^2=(\sqrt{x+6})^2$

$y^2=x+6$

$y^2-6=x$

$y=e^x+1$

$y-1=e^x$

$\ln(y-1)=\ln e^x$

$\ln(y-1)=x\cancel{\ln e}$

$\ln(y-1)=x$

$V=\pi\int_{y_1}^{y_2}[R(y)^2-r(y)^2]\,dy$

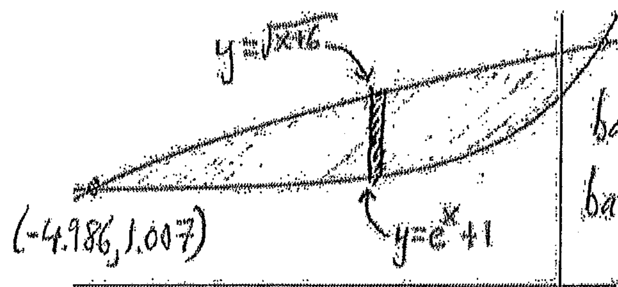
$R(y)=\ln(y-1)-(-6)=\ln(y-1)+6$

$r(y)=y^2-6-(-6)=y^2$

$V=\pi\int_{1.007}^{2.536}[\ln(y-1)+6]^2-[y^2]^2\,dy$

$\boxed{V=26.032\pi \text{ units}^3}$

b) The enclosed region is the base of a solid. The cross section of the solid taken parallel to the y-axis is a isosceles right triangle with hypotenuse on base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



*Vertical Base (Top − Bottom)

base $=\sqrt{x+6}-(e^x+1)$

base $=\sqrt{x+6}-e^x-1$

Area $=\frac{1}{4}(\text{base})^2$

Area $=\frac{1}{4}(\sqrt{x+6}-e^x-1)^2$

$V=\int_{x_1}^{x_2}[\text{Area}]\,dx$

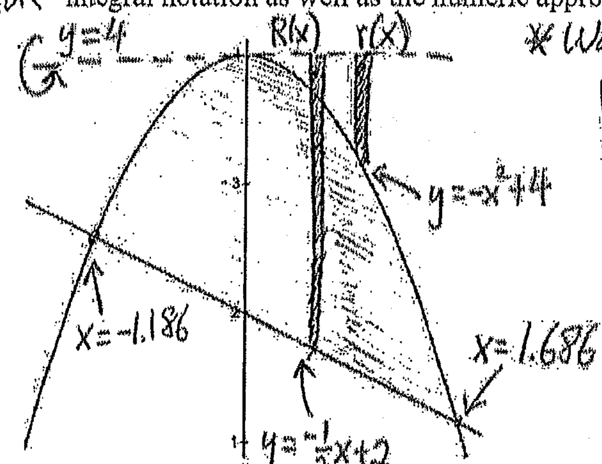
$V=\int_{-4.986}^{0.429}\frac{1}{4}[\sqrt{x+6}-e^x-1]^2\,dx$

$\boxed{V=0.581 \text{ units}^3}$

4) Given the region below enclosed by $f(x) = -x^2 + 4$ and $g(x) = -\frac{1}{2}x + 2$

AOR $y=4$

Find the Volume of solid generated when the enclosed region is revolved about the line y = 4 (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



*Washer Method (Top − Bottom)

$R(x)=4-(-\frac{1}{2}x+2)=4+\frac{1}{2}x-2=2+\frac{1}{2}x$

$r(x)=4-(-x^2+4)=4+x^2-4=x^2$

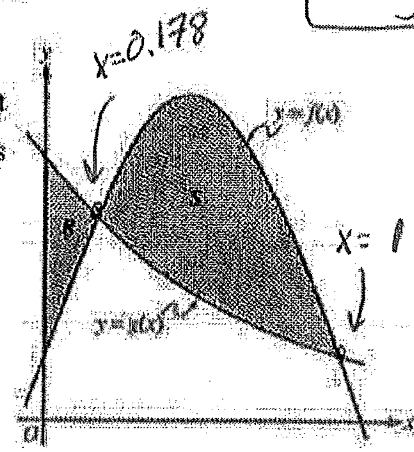
$V=\pi\int_{x_1}^{x_2}R(x)^2-r(x)^2\,dx$

$V=\pi\int_{-1.186}^{1.686}[2+\frac{1}{2}x]^2-[x^2]^2\,dx=\boxed{10.268\pi \text{ units}^3}$

Key

A.P. Calculus AB Chapter 7.-7.2 Area & Volume Unit Review WS #2

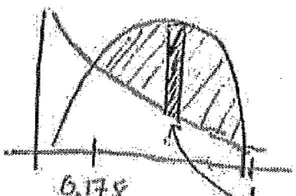
1)



Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

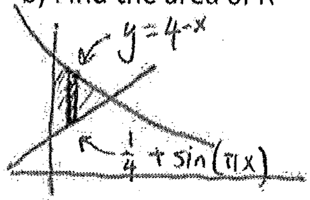
a) Find the area of S

$$A = \int_{0.178}^{1} \frac{1}{4} + \sin(\pi x) - (4^{-x})\,dx$$



$$\boxed{A = 0.410}\ \text{units}^2$$

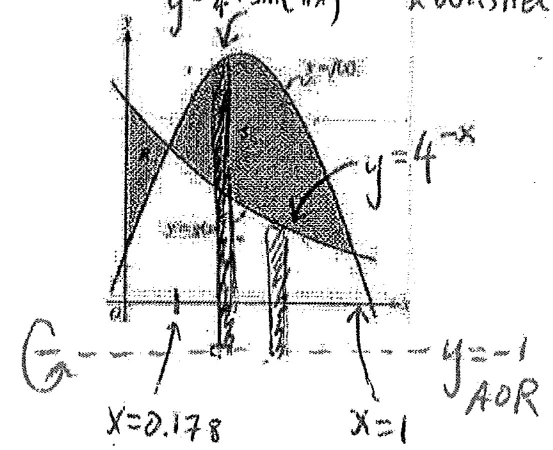
b) Find the area of R



$$\text{Area} = \int_0^{0.178} 4^{-x} - \left(\tfrac{1}{4} + \sin(\pi x)\right)dx = \boxed{0.0648\ \text{units}^2}$$

c) Find the volume of the solid generated when S is revolved about the horizontal line y = - 1.

*Washer Method



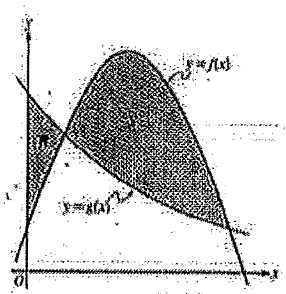
$$R(x) = \tfrac{1}{4} + \sin(\pi x) - (-1) = \tfrac{5}{4} + \sin(\pi x)$$

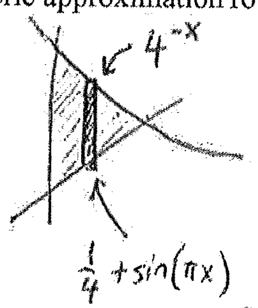
$$r(x) = 4^{-x} - (-1) = 4^{-x} + 1$$

$$V = \pi \int_{x_1}^{x_2} R(x)^2 - r(x)^2\,dx$$

$$V = \pi \int_{0.178}^{1} \left[\tfrac{5}{4} + \sin(\pi x)\right]^2 - \left[4^{-x} + 1\right]^2 dx = \boxed{1.451\pi\ \text{units}^3}$$

d) The enclosed region R is the base of a solid. The cross section of the solid taken <u>parallel to the y-axis</u> is a isosceles right triangle with leg on base. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)





$$\text{base} = 4^{-x} - \left(\tfrac{1}{4} + \sin(\pi x)\right)$$

$$\text{base} = 4^{-x} - \tfrac{1}{4} - \sin(\pi x)$$

$$\text{Area} = \tfrac{1}{2}(\text{base})^2 \rightarrow \tfrac{1}{2}\left[4^{-x} - \tfrac{1}{4} - \sin(\pi x)\right]^2$$

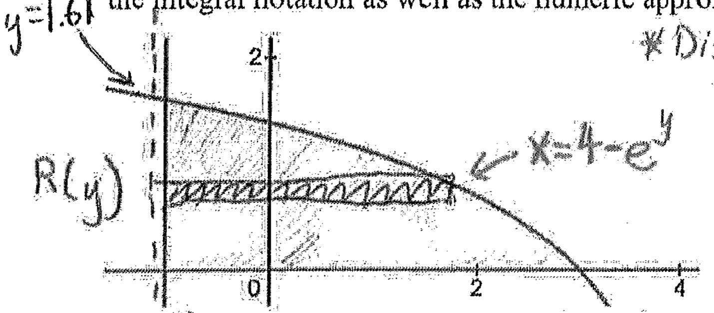
$$V = \int_{x_1}^{x_2} [\text{Area}]\,dx \rightarrow V = \int_0^{0.178} \tfrac{1}{2}\left[4^{-x} - \tfrac{1}{4} - \sin(\pi x)\right]^2 dx = \boxed{0.016\ \text{units}^3}$$

13b

2) Given the region below enclosed by $f(x) = \ln(4 - x)$, the line $x = -1$, and the x-axis.

AOR $x=-1$

a) Find the Volume of solid generated when the enclosed region is revolved about the line $x = -1$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



* Disc Method

$y = \ln(4-x)$

$e^y = e^{\ln(4-x)}$

$e^y = (4-x)$

$e^y = 4 - x$

$x = 4 - e^y$

$x=-1$

AoR

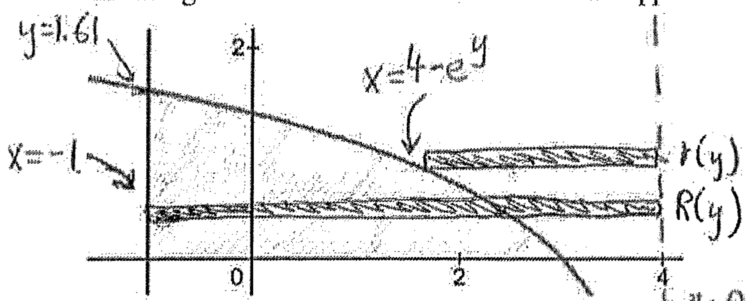
$R(y) = 4 - e^y - (-1)$

$R(y) = 5 - e^y$

$V = \pi \int_{y_1}^{y_2} R[y]^2 \, dy$

$V = \pi \int_0^{1.61} [5 - e^y]^2 \, dy = \boxed{12.236\pi \text{ units}^3}$

b) Find the Volume of solid generated when the enclosed region is revolved about the line $x = 4$ (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



*Washer Method

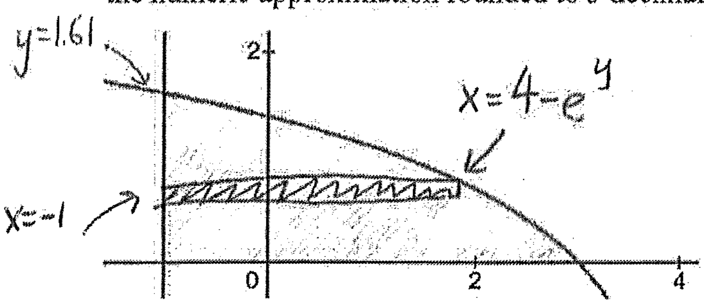
$R(y) = 4 - (-1) = 5$

$r(y) = 4 - (4 - e^y) = e^y$

$V = \pi \int_{y_1}^{y_2} R(y)^2 - r(y)^2 \, dy$

$V = \pi \int_0^{1.61} [5]^2 - [e^y]^2 \, dy = \boxed{28.236\pi \text{ units}^3}$

c) The enclosed region is the base of a solid. The cross section of the solid taken parallel to the x-axis is a rectangle whose height is 4. Find the volume of the given solid. (Write the integral notation as well as the numeric approximation rounded to 3 decimal places)



base $= 4 - e^y - (-1) = 5 - e^y$

Area $=$ (base)(height)

height $= 4$

$A = (5 - e^y)(4)$

$V = \int_{y_1}^{y_2} [\text{Area}] \, dy \rightarrow \int_0^{1.61} 4(5 - e^y) \, dy = \boxed{16.189 \text{ units}^3}$