Key

1.2 AP Practice Problems (pg.102) – Analytical Techniques for Finding Limits

1. Consider the piecewise-defined function f given by

$$f(x)=\begin{cases} -x-2 & \text{if } x<-1 \\ x^2 & \text{if } -1\le x<2 \\ -4x+12 & \text{if } x\ge 2 \end{cases}$$

Investigate the limits below and decide which limit does NOT exist.

(A) $\lim_{x\to -1^+} f(x)$ (B) $\lim_{x\to 2^-} f(x)$

(C) $\lim_{x\to 2} f(x)$ $\boxed{\text{(D)}}$ $\lim_{x\to -1} f(x)$

$\lim_{x\to -1^-} -x-2=-1$

$\lim_{x\to -1^+} x^2 = 1$

Since $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$

then $\boxed{\lim_{x\to -1} f(x) \text{ dne}}$

2. $\lim_{t\to 5} \frac{(5-t)^2}{t-5} =$

(A) -5 $\boxed{\text{(B)}}$ 0 (C) 1 (D) 5

$\lim_{t\to 5} \frac{(5-t)^2}{t-5} = \frac{0}{0} \to \lim_{t\to 5} \frac{(\cancel{5-t})(5-t)}{(\cancel{t-5})(-1)} \to \frac{0}{-1} = \boxed{0}$

3. Find $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$ for the function $f(x)=3x^3-4$.

(A) 0 (B) 12 (C) 24 $\boxed{\text{(D)}}$ 36

$\lim_{x\to 2} \frac{3x^3-4-f(2)}{x-2} \to \frac{0}{0} \to \lim_{x\to 2} \frac{3x^3-4-20}{x-2} \to \lim_{x\to 2} \frac{3x^3-24}{x-2} \to \frac{3(x^3-8)}{x-2}$

$\lim_{x\to 2} \frac{3(\cancel{x-2})(x^2+2x+4)}{(\cancel{x-2})} \to 3(12) = \boxed{36}$

4. $\lim_{x\to s} \frac{x-s}{\sqrt{x}-\sqrt{s}} =$

(A) $2s$ $\boxed{\text{(B)}}$ $2\sqrt{s}$ (C) $\sqrt{2s}$ (D) s

$\lim_{x\to s} \frac{x-s}{\sqrt{x}-\sqrt{s}} \to \frac{0}{0} \to \lim_{x\to s} \frac{(x-s)}{\sqrt{x}-\sqrt{s}} \cdot \frac{\sqrt{x}+\sqrt{s}}{\sqrt{x}+\sqrt{s}}$

$\lim_{x\to \sqrt{s}} \frac{(\cancel{x-s})(\sqrt{x}+\sqrt{s})}{(\cancel{x-s})} \to \sqrt{s}+\sqrt{s} = \boxed{2\sqrt{s}}$

5. For $g(x) = \begin{cases} ax^2 - 5 \text{ if } x < 2 \\ ax + b \text{ if } x > 2 \end{cases}$

find values for a and b so that $\lim_{x\to 2} g(x) = 7$.

(A) $a=1, b=5$ (B) $a=2, b=3$

$\boxed{\text{(C)}}$ $a=3, b=1$ (D) $a=6, b=-5$

$\lim_{x\to 2^-} ax^2 - 5 = 7$

$a(2)^2 - 5 = 7$

$4a - 5 = 7$

$4a = 12$

$a = 3$

$\lim_{x\to 2^+} ax + b = 7$

$2a + b = 7$

$2(3) + b = 7$

$\boxed{b = 1}$

6. $\lim_{x\to 4^+} (5\sqrt{x^2 - 16} + 3x) =$

(A) -12 (B) 0 $\boxed{\text{(C)}}$ 12 (D) The limit does not exist.

$5\sqrt{4^2 - 16} + 3(4) = 0 + 12 = \boxed{12}$

7. If $\lim_{x\to 2} \sqrt{\frac{[f(x)]^2 - 8x + 3}{x+1}} = 9$ and $f(x) \geq 0$ for all x,

find $\lim_{x\to 2} f(x)$.

(A) $\sqrt{22}$ (B) $2\sqrt{10}$ $\boxed{\text{(C)}}$ 16 (D) 256

$\lim_{x\to 2} \sqrt{\frac{[f(2)]^2 - 8(2) + 3}{2+1}} = 9$

$\left[\sqrt{\frac{(f(2))^2 - 13}{3}}\right]^2 = [9]^2$

$\frac{(f(2))^2 - 13}{3} = 81$

$(f(2))^2 - 13 = 243$

$(f(2))^2 = 256$

$f(2) = \pm\sqrt{256}$

$f(2) = \pm 16$

Since $f(x) > 0$, $\boxed{f(2) = 16}$

8. $\lim_{x\to 3} [x^{-1/2}(5x - 7)^{1/3}] =$

(A) $3^{-1/2}$ $\boxed{\text{(B)}}$ $\frac{2}{3^{1/2}}$ (C) $\frac{8}{3^{1/2}}$ (D) $6^{-1/2}$

$\lim_{x\to 3} f(x) \to 3^{-1/2} \cdot (15 - 7)^{1/3}$

$\to \frac{8^{1/3}}{3^{1/2}} = \boxed{\frac{2}{3^{1/2}}}$