Ch. 1.2 Exercise Problems

p. 99-102 #31-49 odd, 53, 59, 73, 75, 79, 85, 89

31) $\lim_{x\to 4} \frac{x^2+4}{\sqrt{x}} \to \frac{4^2+4}{4} = \frac{20}{4} = \boxed{5}$

direct substitution

33) $\lim_{x\to -2} \frac{2x^3+5x}{3x-2} \to \frac{-16-10}{-6-2} \to \frac{-26}{-8} \to \boxed{\frac{13}{4}}$

35) $\lim_{x\to 2} \frac{x^2-4}{x-2} \to \frac{0}{0} \to \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} \to 2+2 = \boxed{4}$

indeterminate form (hole in graph)

37) $\lim_{x\to -1} \frac{x^3-x}{x+1} \to \frac{0}{0} \to \lim_{x\to -1} \frac{x(x^2-1)}{x+1} \to \lim_{x\to -1} \frac{x(x-1)\cancel{(x+1)}}{\cancel{(x+1)}} \to (-1)(-2) = \boxed{2}$

39) $\lim_{x\to -8} \left(\frac{2x}{x+8} + \frac{16}{x+8}\right) \to \lim_{x\to -8} \frac{2x+16}{x+8} \to \frac{0}{0} \to \lim_{x\to -8} \frac{2\cancel{(x+8)}}{\cancel{x+8}} \to \boxed{2}$

41) $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2} \to \frac{0}{0} \to$ *Apply conjugate method $\to \lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2} \cdot \frac{(\sqrt{x}+\sqrt{2})}{(\sqrt{x}+\sqrt{2})}$

$\lim_{x\to 2} \frac{\cancel{(x-2)}(1)}{\cancel{(x-2)}(\sqrt{x}+\sqrt{2})} \to \frac{1}{\sqrt{2}+\sqrt{2}} \to \boxed{\frac{1}{2\sqrt{2}}}$

43) $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{(x-4)(x+1)} \to \frac{0}{0} \to \lim_{x\to 4} \frac{\sqrt{x+5}-3}{(x-4)(x+1)} \cdot \frac{(\sqrt{x+5}+3)}{(\sqrt{x+5}+3)}$

$\lim_{x\to 4} \frac{x+5-9}{(x-4)(x+1)(\sqrt{x+5}+3)} \to \lim_{x\to 4} \frac{\cancel{(x-4)}(1)}{\cancel{(x-4)}(x+1)(\sqrt{x+5}+3)} \to \frac{1}{(4+1)(\sqrt{9}+3)} \to \boxed{\frac{1}{30}}$

45) $\lim_{x\to 3^-} (x^2-4) \to (3)^2-4 = \boxed{5}$

47) $\lim_{x\to 3^-} \frac{x^2-9}{x-3} \to \frac{0}{0} \to \lim_{x\to 3^-} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} \to 3+3 = \boxed{6}$

49) $\lim_{x\to 3^-} \left(\sqrt{9-x^2}+x\right)^2 \to \left(\sqrt{9-3^2}+3\right)^2 = \boxed{9}$

53) $\lim_{x\to c} [g(x)]^3 \to \left[\lim_{x\to c} g(x)\right]^3 \to 2^3 = \boxed{8}$

(given) * $\lim_{x\to c} g(x) = 2$

59) (see next page)

73) $f(x) = \begin{cases} 2x-3, & x \le 1 \\ 3-x, & x > 1 \end{cases}$ at $c=1$

$\lim_{x\to 1^-} 2x-3 = 2-3 = -1$

$\lim_{x\to 1^+} 3-x = 3-1 = 2$

Since $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

$\boxed{\lim_{x\to 1} f(x) \text{ does not exist}}$

75) $f(x) = \begin{cases} 3x-1, & x<1 \\ 4, & x=1 \\ 2x, & x>1 \end{cases}$ at $c=1$

$\lim_{x\to 1^-} 3x-1 = 2$

$\lim_{x\to 1^+} 2x = 2$

Since $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\boxed{\text{then } \lim_{x\to 1} f(x) = 2}$

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59. (a) $\lim_{x\to 4}[f(x)+g(x)]$

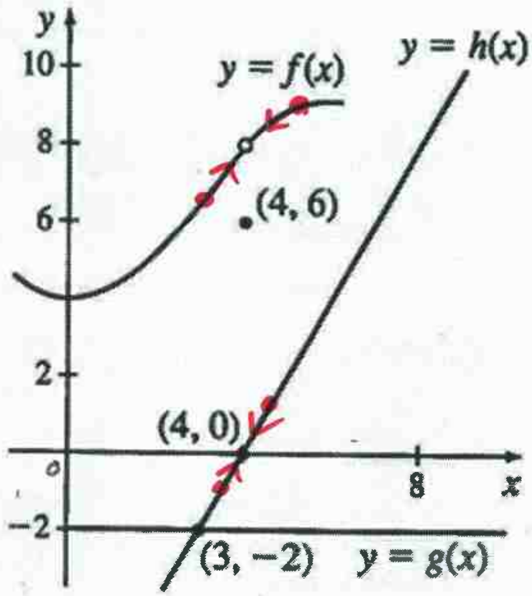
(b) $\lim_{x\to 4}\{f(x)[g(x)-h(x)]\}$

(c) $\lim_{x\to 4}[f(x)\cdot g(x)]$

(d) $\lim_{x\to 4}[2h(x)]$

(e) $\lim_{x\to 4}\dfrac{g(x)}{f(x)}$

(f) $\lim_{x\to 4}\dfrac{h(x)}{f(x)}$



a) $\lim_{x\to 4} f(x) + \lim_{x\to 4} g(x) = 8 + (-2) = \boxed{6}$

b) $8(-2-0) = \boxed{-16}$

c) $8(-2) = \boxed{-16}$

d) $2(0) = \boxed{0}$

e) $\dfrac{-2}{8} = \boxed{-\dfrac{1}{4}}$

f) $\dfrac{0}{8} = \boxed{0}$

* $\lim_{x\to 4} f(x) = 8$
* $\lim_{x\to 4} g(x) = -2$
* $\lim_{x\to 4} h(x) = 0$

79) $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$ at $c=3$ or... $\begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x < 3 \\ 6 & \text{if } x = 3 \\ \dfrac{x^2-9}{x-3} & \text{if } x > 3 \end{cases}$

$$\lim_{x\to 3^-} \frac{x^2-9}{x-3} \to \frac{3^2-9}{3-3} \to \frac{0}{0} \to \lim_{x\to 3^-} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} \to 3+3 = \boxed{6}$$

$$\lim_{x\to 3^+} \frac{x^2-9}{x-3} = \boxed{6}$$

Since $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$, then $\boxed{\lim_{x\to 3} f(x) = 6}$

85) $\lim_{h\to 0} \dfrac{\boxed{\frac{1}{x+h} - \frac{1}{x}}}{h} \to \dfrac{0}{0}$ → find common denominator, then combine the 2 fractions.

$$\lim_{h\to 0} \frac{\boxed{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}}{h} \to \lim_{h\to 0} \frac{\frac{x-x-h}{x(x+h)}}{h} \to \lim_{h\to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{x(x+h)} \to \frac{-1}{x(x+0)} \to \boxed{\frac{-1}{x^2}}$$

89) $\lim_{x\to 7} \dfrac{x-7}{\sqrt{x+2}-3} \to \dfrac{0}{0}$

$$\lim_{x\to 7} \frac{x-7}{\sqrt{x+2}-3} \cdot \frac{(\sqrt{x+2}+3)}{(\sqrt{x+2}+3)}$$

$$\lim_{x\to 7} \frac{(x-7)(\sqrt{x+2}+3)}{x+2-9}$$

$$\lim_{x\to 7} \frac{\cancel{(x-7)}(\sqrt{x+2}+3)}{\cancel{(x-7)}(1)}$$

$$\to \frac{\sqrt{7+2}+3}{1} \to \frac{\sqrt{9}+3}{1} = \boxed{6}$$