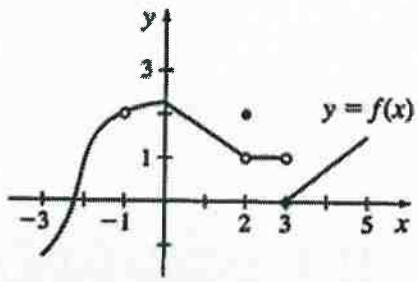
Key

1.3 AP Practice Problems (pg. 116-117)

1. The graph of a function f is shown below. Where on the open interval $(-3, 5)$ is f discontinuous?

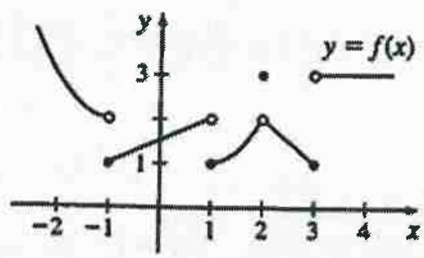


(A) 3 only (B) −1 and 3 only

(C) 1 only $\boxed{\text{(D)}}$ −1, 2, and 3

* removable discontinuity at $x=-1$, $x=2$
* nonremovable discontinuity at $x=3$

2. The graph of a function f is shown below.



If $\lim_{x \to c} f(x)$ exists and f is not continuous at c, then c equals

(A) −1 (B) 1 $\boxed{\text{(C)}}$ 2 (D) 3

* The only removable discontinuity exists at $x=2$.

3. How should the function $f(x) = \dfrac{x^2 - 25}{x+5}$ be defined at −5 to make it continuous at −5?

$\boxed{\text{(A)}}$ −10 (B) −5 (C) 0 (D) 10

*step thru continuity conditions

i) $f(-5) = \_\_\,?$

ii) $\lim_{x \to -5} \frac{x^2-25}{x+5} \to \lim_{x \to -5} \frac{\cancel{(x+5)}(x-5)}{\cancel{(x+5)}} = -5-5 = -10$

iii) $f(-5) = \lim_{x \to -5} f(x) = -10$

let $f(-5) = -10$ to make $f(x)$ continuous at $x = -5$

*continuity conditions [provides framework for working thru these problems]

4. If $f(x)=\begin{cases}\dfrac{\sqrt{2x+5}-\sqrt{x+15}}{x-10} & \text{if } x\neq 10\\ k & \text{if } x=10\end{cases}$

and if f is continuous at $x=10$, then $k=$

(A) 0 $\quad$ $\boxed{\text{(B)}}$ $\frac{1}{10}$ $\quad$ (C) 1 $\quad$ (D) 10

i) $f(10)=k$

ii) $\lim\limits_{x\to 10}\dfrac{\sqrt{2x+5}-\sqrt{x+15}}{x-10}\to\dfrac{0}{0}$

$\lim\limits_{x\to 10}\dfrac{\sqrt{2x+5}-\sqrt{x+15}}{x-10}\cdot\dfrac{(\sqrt{2x+5}+\sqrt{x+15})}{(\sqrt{2x+5}+\sqrt{x+15})}$

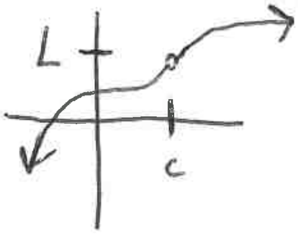
$\lim\limits_{x\to 10}\dfrac{\cancel{2x+5-(x+15)}\ (1)}{\cancel{(x-10)}(\sqrt{2x+5}+\sqrt{x+15})}\to\to\dfrac{1}{\sqrt{25}+\sqrt{25}}=\boxed{\dfrac{1}{10}}$

iii) $f(10)=\lim\limits_{x\to 10}f(x)=\boxed{\dfrac{1}{10}}$

5. If $\lim\limits_{x\to c}f(x)=L$, where L is a real number, which of the following must be true?

(A) f is defined at $x=c$. $\quad$ (B) f is continuous at $x=c$.

(C) $f(c)=L$. $\quad$ $\boxed{\text{(D)}}$ None of the above.



6. If $f(x)=x^3-2x+5$ and if $f(c)=0$ for only one real number c, then c is between

$\boxed{\text{(A)}}$ -4 and -2 $\quad$ (B) -2 and -1 $\quad$ (C) -1 and 1 $\quad$ (D) 1 and 3

*Intermediate Value Theorem

$f(x)$ continuous on $(-\infty,\infty)$

$\boxed{f(-4)=-51 \quad f(-2)=1}$

$f(-2)=1 \quad f(-1)=6 \quad f(1)=4$

$f(-1)=6 \quad f(1)=4 \quad f(3)=26$

$\boxed{f(-4)=-51<0<1=f(-2)}$

$f(c)=0$ when c is on interval $(-4,-2)$

7. The function f is continuous at all real numbers, and $f(-8)=3$ and $f(-1)=-4$. If f has only one real zero (root), then which number x could satisfy $f(x)=0$?

(A) -10 $\quad$ $\boxed{\text{(B)}}$ -5 $\quad$ (C) 0 $\quad$ (D) 2

* only $x=-5$ exist between $x=-1$ and $x=-8$ (among the answer choices)

8. Let f be the function defined by

$$f(x)=\begin{cases} x^2 & \text{if} \quad x<0 \\ \sqrt{x} & \text{if} \quad 0\le x<1 \\ 2-x & \text{if} \quad 1\le x<2 \\ x-3 & \text{if} \quad x\ge 2 \end{cases}$$

For what numbers x is f NOT continuous?

(A) 1 only
(B) 2 only
(C) 0 and 2 only
(D) 1 and 2 only

$\lim_{x\to 0^-} x^2 = 0$ $\lim_{x\to 1^-} \sqrt{x} = 1$ $\lim_{x\to 2^-} 2-x = 0$ $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

$\lim_{x\to 0^+} \sqrt{x} = 0$ $\lim_{x\to 1^+} 2-x = 1$ $\lim_{x\to 2^+} x-3 = -1$

9. The function f is continuous on the closed interval $[-2, 6]$. If $f(-2)=7$ and $f(6)=-1$, then the Intermediate Value Theorem guarantees that

(A) $f(0)=0$.
(B) $f(c)=2$ for at least one number c between -2 and 6.
(C) $f(c)=0$ for at least one number c between -1 and 7.
(D) $-1\le f(x)\le 7$ for all numbers in the closed interval $[-2, 6]$.

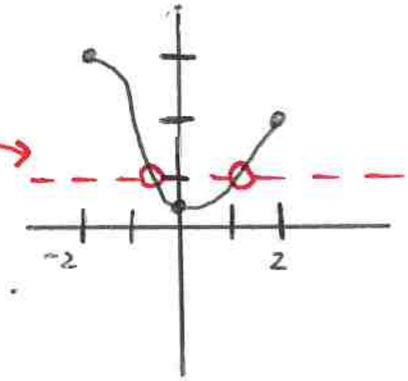
Since $f(6)=-1<2<7=f(-2)$
where $f(c)=2$ when c is on interval $(-2, 6)$

10. **The function f is continuous on the closed interval $[-2, 2]$. Several values of the function f are given in the table below.**

x	-2	0	2
$f(x)$	3	c	2

The equation $f(x) = 1$ must have at least two solutions in the interval $[-2, 2]$ if $c =$

(A) $\frac{1}{2}$ (B) 1 (C) 3 (D) 4



* sketch graph to help see answer

11. **The function f is defined by $f(x) = \begin{cases} x^2 - 2x + 3 & \text{if } x \le 1 \\ -2x + 5 & \text{if } x > 1 \end{cases}$**

(a) Is f continuous at $x = 1$?
(b) Use the definition of continuity to explain your answer.

a) No

b) $\lim_{x \to 1} f(x) = \text{dne}$

* step through continuity conditions

i) $f(1) = 1 - 2 + 3 = 2$

ii) $\lim_{x \to 1^-} x^2 - 2x + 3 = 2$ $\lim_{x \to 1^+} -2x + 5 = 3$

$\boxed{\lim_{x \to 1} f(x) = \text{does not exist since } \lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x)}$

$\boxed{\text{Nonremovable discontinuity at } x = 1}$