Ch. 1.3a Exercise Problems: Continuity and types of Discontinuity

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In Problems 13–18, use the graph of $y = f(x)$ (top right).

(a) *Determine if f is continuous at c.*

(b) *If f is discontinuous at c, state which condition(s) of the definition of continuity is (are) not satisfied.*

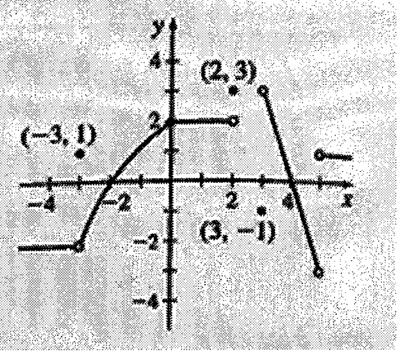
(c) *If f is discontinuous at c, determine if the discontinuity is removable.*

(d) *If the discontinuity is removable, define (or redefine) f at c to make f continuous at c.*

13. $c = -3$ **14.** $c = 0$

15. $c = 2$ **16.** $c = 3$

17. $c = 4$ **18.** $c = 5$



$\boxed{c=-3}$

13) a) i) $f(-3) = 1$

ii) $\lim_{x\to -3} f(x) = -2$

iii) $f(-3) \neq \lim_{x\to 3} f(x)$ therefore $f(x)$ not continuous at $c = -3$

b) $f(x)$ not continuous since $f(-3) \neq \lim_{x\to 3} f(x)$

c) Removable discontinuity since $\lim_{x\to 3} f(x)$ exists.

d) Redefine $f(-3) = -2$ so that $f(-3) = \lim_{x\to 3} f(x)$ and therefore continuous at $x = 3$.

15) $\boxed{c=2}$

a) i) $f(2) = 3$

ii) $\lim_{x\to 2} f(x)$ does not exist $\left(\text{since } \lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)\right)$

Therefore $f(x)$ not continuous at $x = 2$.

b) Not continuous since $\lim_{x\to 2} f(x)$ does not exist.

c) Nonremovable discontinuity at $x = 2$

d) Not possible

17) $\boxed{c=4}$

a) i) $f(4) = 0$ ✓ iii) $f(4) = \lim_{x\to 4} f(x) = 0$ ✓ | Therefore $f(x)$ continuous at $x = 4$.

ii) $\lim_{x\to 4} f(x) = 0$ ✓

Determine whether $f(x)$ is continuous at c.

25) $f(x) = \begin{cases} 3x-1 & , x<1 \\ 4 & , x=1 \\ 2x & , x>1 \end{cases}$ $\boxed{c=1}$

*continuity conditions:

i) $f(1) = 4$ ✓

ii) $\lim_{x\to 1^-} 3x-1 = 2$ $\lim_{x\to 1^+} 2x = 2$, $\lim_{x\to 1} f(x) = 2$ ✓

iii) Since $f(1) \neq \lim_{x\to 1} f(x)$, $f(x)$ not continuous at $x=1$.

29) $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ 2x & \text{if } x>0 \end{cases}$ $\boxed{c=0}$

i) $f(0) = (0)^2 = 0$ ✓

ii) $\lim_{x\to 0^-} x^2 = 0$ $\lim_{x\to 0^+} 2x = 0$ $\rightarrow$ $\lim_{x\to 0} f(x) = 0$ ✓

iii) Since $f(0) = \lim_{x\to 0} f(x)$, then $f(x)$ continuous at $c=0$.

35) Redefine $f(c)$ so that f is continuous at c.

$f(x) = \begin{cases} 1+x & , x<1 \\ 4 & , x=1 \\ 2x & , x>1 \end{cases}$ $\boxed{c=1}$

*step through continuity conditions:

i) $f(1) = 4$

ii) $\lim_{x\to 1^-} 1+x = 2$ $\lim_{x\to 1^+} 2x = 2$ $\rightarrow$ $\lim_{x\to 1} f(x) = 2$

iii) $f(1) \neq \lim_{x\to 1} f(x)$, ∴ $f(x)$ not continuous at $x=1$

Re-assign $f(1) = 2$ to make $f(x)$ continuous at $x=1$

Ch. 1.3a Exercise Problems → Continuity and types of discontinuity

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37) $f(x) = \frac{x^2-9}{x-3}$ on interval $[-3, 3)$

$f(x) = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$ hole at $x = 3$, so $f(x)$ not continuous at $x = 3$

$\boxed{f(x) \text{ continuous on interval } [-3, 3)}$

45) Determine continuity of $f(x)$ and state domain

$f(x) = \frac{x-9}{\sqrt{x}-3}$

$\boxed{\text{Domain: } \mathbb{R} \text{ except } x \neq 9 \text{ and } x \geq 0 \text{ or } [0, 9) \cup (9, \infty)}$

$\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} \to \frac{0}{0}$ $\quad \lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{(\sqrt{x}+3)}{(\sqrt{x}+3)}$

$\lim_{x \to 9} \frac{\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{(x-9)}} \to \sqrt{9}+3 = 6$

hole at point $(9, 6)$.