Ch. 1.3b Exercise Problems → Intermediate Value Theorem (IVT)

1.4 → Squeeze Theorem

(1.3) p.112–117 #59, 63, 73

(1.4) p. 125 #3, 5-8 all

Use Intermediate Value Theorem to determine interval where a zero will exist. ($f(c)=0$)

59) $f(x)=x^3-3x \quad [-2,2]$

i) $f(x)$ continuous $[-2,2]$

ii) $f(-2)=-2$

$f(2)=2$

Since $f(-2)=-2<0<2=f(2)$ there must exist a c where $f(c)=0$ in interval $(-2,2)$

63) $f(x)=\dfrac{x^3-1}{x-1}$, $[0,2]$

$f(x)$ not continuous $[0,2]$ since $f(1)$ is undefined. Therefore, IVT does not apply due to discontinuity on $[0,2]$

73) $f(x)=\sqrt{x^2+4x}-2 \quad [0,1]$

i) $f(x)$ continuous $[0,1]$

ii) $f(0)=-2$

$f(1)=\sqrt{5}-2\approx 0.236$

Since $f(0)=-2<0<f(1)\approx 0.236$ there must exist a c where $f(c)=0$ on interval $(0,1)$

$\sqrt{x^2+4x}-2=0$

* use calculator to graph and estimate x-intercept

$x\approx 0.828$

$\boxed{c\approx 0.828}$

(1.4) p.125 #3, 5-8 all

3) $f(x) \le g(x) \le h(x)$ If $\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$, then

$$\boxed{\lim_{x\to c} g(x) = L}$$

5) $-x^2+1 \le g(x) \le x^2+1$ Find $\lim_{x\to 0} g(x)$

Since $\lim_{x\to 0} -x^2+1 = 1$ and $\lim_{x\to 0} x^2+1 = 1$, then by Squeeze theorem, then $\boxed{\lim_{x\to 0} g(x) = 1}$

6) $-(x-2)^2-3 \le g(x) \le (x-2)^2-3$ Find $\lim_{x\to 2} g(x)$

Since $\lim_{x\to 2} -(x-2)^2-3 = -3$ and $\lim_{x\to 2} (x-2)^2-3 = -3$, then $\boxed{\lim_{x\to 2} g(x) = -3}$

7) $\cos x \le g(x) \le 1$ Find $\lim_{x\to 0} g(x)$

$\lim_{x\to 0} \cos x = 1$ and $\lim_{x\to 0} 1 = 1$, then $\boxed{\lim_{x\to 0} g(x) = 1}$

8) $-x^2+1 \le g(x) \le \sec x$ Find $\lim_{x\to 0} g(x)$

$\lim_{x\to 0} -x^2+1 = 1$ $\lim_{x\to 0} \sec x = \frac{1}{\cos(0)} = 1$

$$\boxed{\lim_{x\to 0} g(x) = 1}$$