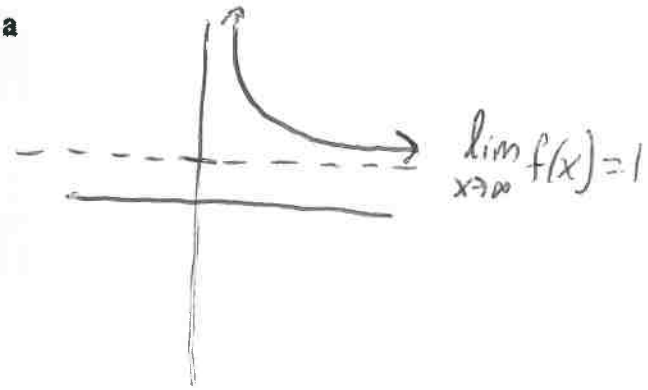
Key

1.5 AP Practice Problems (p. 143) – Infinite Limits (V.A.) & Limits at Infinity (H.A.)

1. For $x > 0$, the line $y = 1$ is an asymptote of the graph of a function f. Which of the following statements must be true?

(A) $f(x) \neq 1$ for $x > 0$ (B) $\lim_{x \to 1} f(x) = \infty$

(C) $\lim_{x \to \infty} f(x) = 1$ (D) $\lim_{x \to -\infty} f(x) = 1$



2. $\lim_{x \to \infty} \frac{3x^3 + 4x^2 - x + 10}{2x^4 - x^3 + 2x^2 - 2} =$

(A) -5 (B) 0 (C) $\frac{3}{2}$ (D) ∞

3. $\lim_{x \to \infty} \frac{5x^3 - x}{8 - x^3} =$ $\frac{5}{-1} = -5$

(A) -5 (B) $\frac{5}{8}$ (C) 5 (D) ∞

4. Find all the horizontal asymptotes of the graph of $y = \frac{2 + 3^x}{4 - 3^x}$.

(A) $y = -1$ only (B) $y = \frac{1}{2}$ only

(C) $y = -1$ and $y = 0$ (D) $y = -1$ and $y = \frac{1}{2}$

$\lim_{x \to -\infty} \frac{2 + 3^x}{4 - 3^x} \to \frac{2 + 3^{-\infty}}{4 - 3^{-\infty}}$

$\frac{2 + \frac{1}{3^\infty}}{4 - \frac{1}{\infty}} \to \frac{2+0}{4-0} \to \frac{1}{2}$

$\lim_{x \to \infty} \frac{2 + 3^x}{4 - 3^x} \to \frac{3^\infty}{-3^{-\infty}} = -1$

5. Find all the vertical asymptotes of the graph of

$$r(x) = \frac{x^2 + 5x + 6}{x^3 - 4x}$$

(A) $x = 0$ and $x = -2$ (B) $x = 0$ and $x = 2$

(C) $x = -2$ and $x = 2$ (D) $x = 0$, $x = -2$ and $x = 2$

$x(x^2 - 4)$

$\frac{(x+3)(x+2)}{x(x-2)(x+2)}$

VA at $x = 0$, $x = 2$

6. $\lim_{x\to-\infty} \frac{\sqrt{8x^2-4x}}{x+2} =$

(A) $-\infty$ (B) $-2\sqrt{2}$ (C) 4 (D) $2\sqrt{2}$

$\frac{-\sqrt{8}}{1} = -2\sqrt{2}$

7. The graph of which of the following functions has an asymptote of $y = 1$?

(A) $y = \cos x$ (B) $y = \frac{x-1}{x}$ ← degrees match b/t numerator and denominator

(C) $y = e^{-x}$ (D) $y = \ln x$

8. If the graph of $f(x) = \frac{ax-b}{x+c}$ has a vertical asymptote $x = -5$ and horizontal asymptote $y = -3$, then $a + c =$

(A) -8 (B) -2 (C) $\frac{3}{5}$ (D) 2

VA: $x = -5$

$\frac{ax-b}{x+5} \to \frac{-3x-b}{x+5}$

$a + c \to -3 + 5 = 2$

9. $\lim_{x\to1^-} \frac{x}{\ln x} =$

(A) $-\infty$ (B) -1 (C) 1 (D) ∞

test $x = 0.9$ $\frac{0.9}{\ln(0.9)} \to \frac{+}{-} \to -\infty$

10. The function $f(x) = \frac{2x}{|x|-1}$ has

(A) no vertical asymptote and one horizontal asymptote.
(B) one vertical asymptote and one horizontal asymptote.
(C) two vertical asymptotes and one horizontal asymptote.
(D) two vertical asymptotes and two horizontal asymptotes.

11. $\lim_{x\to2^-} \frac{5x+1}{2x-4} =$ $\frac{11}{0} \to$ VA at $x = 2$

(A) $-\infty$ (B) $-\frac{5}{2}$ (C) $\frac{5}{2}$ (D) ∞

$\lim_{x\to2^-} \frac{5x+1}{2x-4} \to \frac{5(1.9)+1}{2(1.9)-4} \to \frac{+}{-}$

test $x = 1.9$

$-\infty$