

* Limits at Infinity and Infinite Limits

Ch. 1.5 Problems 9-23 odds (pg. 140-143)

#27-49 odds, 59, 60, 73

In Problems 9-16, use the accompanying graph of $y = f(x)$.

9. Find $\lim_{x \rightarrow \infty} f(x)$.

10. Find $\lim_{x \rightarrow -\infty} f(x)$.

11. Find $\lim_{x \rightarrow -1^-} f(x)$.

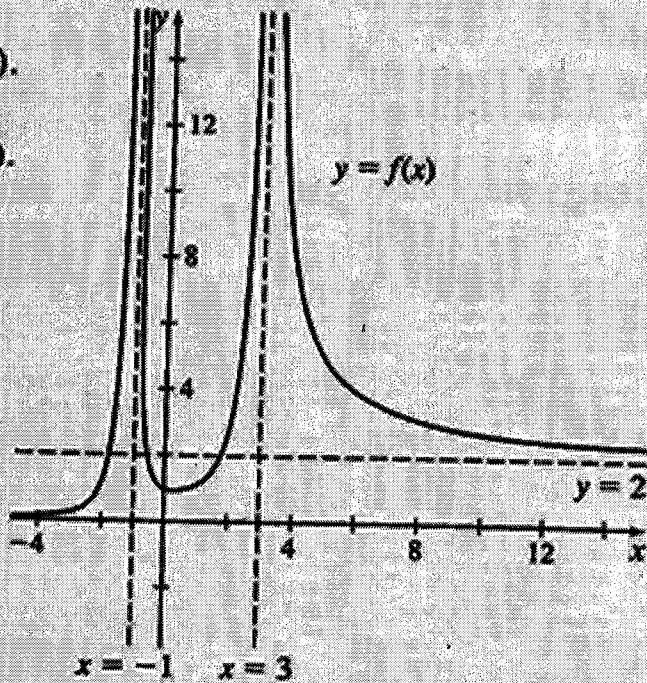
12. Find $\lim_{x \rightarrow -1^+} f(x)$.

13. Find $\lim_{x \rightarrow 3^-} f(x)$.

14. Find $\lim_{x \rightarrow 3^+} f(x)$.

15. Identify all vertical asymptotes.

16. Identify all horizontal asymptotes.



9) $\lim_{x \rightarrow \infty} f(x) = \boxed{2}$

11) $\lim_{x \rightarrow -1^-} f(x) = \boxed{\infty}$

13) $\lim_{x \rightarrow 3^-} f(x) = \boxed{-\infty}$

15) Vertical asymptotes
at $x = -1, x = 3$.

In Problems 17-26, use the graph (top, right) of $y = f(x)$.

17. Find $\lim_{x \rightarrow \infty} f(x)$. $\boxed{-3}$

18. Find $\lim_{x \rightarrow -\infty} f(x)$.

19. Find $\lim_{x \rightarrow -3^-} f(x)$. $\boxed{+\infty}$

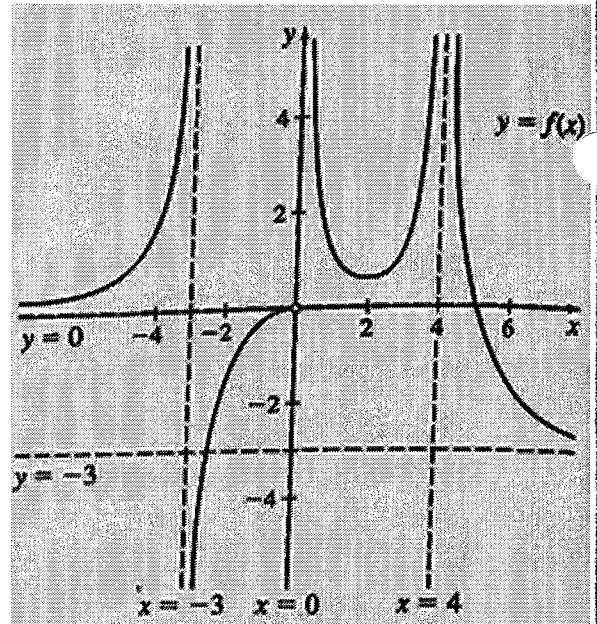
20. Find $\lim_{x \rightarrow -3^+} f(x)$.

21. Find $\lim_{x \rightarrow 0^-} f(x)$. $\boxed{0}$

22. Find $\lim_{x \rightarrow 0^+} f(x)$.

23. Find $\lim_{x \rightarrow 4^-} f(x)$. $\boxed{+\infty}$

24. Find $\lim_{x \rightarrow 4^+} f(x)$.



27) $\lim_{x \rightarrow 2^-} \frac{3x}{x-2} \rightarrow \frac{6}{0}$ VA at $x=2$
 $\nearrow +\infty$
 or
 $\searrow -\infty$

* test using decimals

$x=1.9 \rightarrow \frac{3(1.9)}{1.9-2} \rightarrow \frac{+}{-} \rightarrow \boxed{-\infty}$

29) $\lim_{x \rightarrow 2^+} \frac{5}{x^2-4} \rightarrow \frac{5}{0} \rightarrow$ VA at $x=2 \rightarrow$ one-sided limit
 at a VA always either $+\infty$ or $-\infty$

* test using 2.1

$\frac{5}{(2.1)^2-4} \rightarrow \frac{+}{+} \rightarrow \boxed{+\infty}$

31) $\lim_{x \rightarrow -1^+} \frac{5x+3}{x(x+1)} \rightarrow \frac{-2}{0} \rightarrow$ VA at $x=-1$

* test $x=-0.9 \rightarrow \frac{5(-0.9)+3}{-0.9(-0.9+1)} \rightarrow \frac{-}{-} \rightarrow \boxed{+\infty}$

33) $\lim_{x \rightarrow -3^-} \frac{1}{x^2-9} \rightarrow \frac{1}{0} \rightarrow$ VA at $x=-3$

test $x=-3.1$

$\frac{1}{(-3.1)^2-9} \rightarrow \frac{+}{+} \rightarrow \boxed{+\infty}$

Ch. 1.5 Exercise Problems \rightarrow Limits at Infinity (H.A.) and Infinite Limits (V.A.)

p. 140-143 #9-23 odds, 27-49 odds, 59, 60, 73

$$35) \lim_{x \rightarrow 3} \frac{1-x}{(3-x)^2} \rightarrow \frac{-2}{0} \rightarrow \text{VA at } x=3$$

$$\lim_{x \rightarrow 3^-} \frac{1-x}{(3-x)^2} \rightarrow \frac{1-2.9}{(3-2.9)^2} \rightarrow \frac{-}{+} \rightarrow -\infty$$

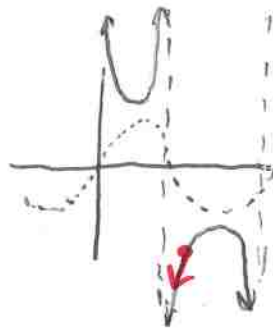
$$\lim_{x \rightarrow 3^+} \frac{1-x}{(3-x)^2} \rightarrow \frac{1-3.1}{(3-3.1)^2} \rightarrow \frac{-}{+} \rightarrow -\infty$$

$$\lim_{x \rightarrow 3} f(x) = \boxed{-\infty}$$

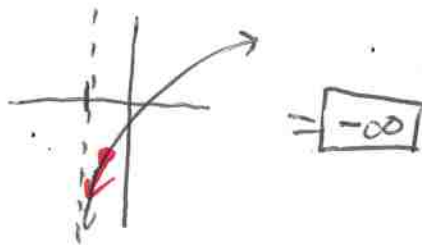
$$37) \lim_{x \rightarrow \pi^-} \cot x = \boxed{-\infty}$$



$$39) \lim_{x \rightarrow \frac{\pi}{2}^+} \csc(2x) \rightarrow \frac{1}{\sin(2x)} \rightarrow \boxed{-\infty}$$



$$41) \lim_{x \rightarrow -1^+} \ln(x+1) = \boxed{-\infty}$$



$$43) \lim_{x \rightarrow \infty} \frac{5}{x^2+4} \rightarrow \boxed{0}$$

$$45) \lim_{x \rightarrow \infty} \frac{2x+4}{5x}$$

* degrees match between numerator and denominator

$$\rightarrow \boxed{\frac{2}{5}}$$

$$47) \lim_{x \rightarrow \infty} \frac{x^3+x^2+2x-1}{x^3+x+1} \rightarrow \frac{1}{1} = \boxed{1}$$

$$49) \lim_{x \rightarrow -\infty} \frac{x^2+1}{x^3-1} = \boxed{0}$$

$$59) \lim_{x \rightarrow -\infty} \frac{5x^3}{x^2+1} \rightarrow \begin{matrix} +\infty \\ -\infty \end{matrix} \text{ or } \rightarrow \text{test } x = -100$$

$$\frac{5(-100)^3}{(-100)^2+1} \rightarrow \frac{-}{+} \rightarrow \boxed{-\infty}$$

$$60) \lim_{x \rightarrow -\infty} \frac{x^4}{x-2} \rightarrow \begin{matrix} +\infty \\ -\infty \end{matrix} \rightarrow \text{test } x = -100$$

$$\frac{(-100)^4}{-100-2} \rightarrow \frac{+}{-} = \boxed{-\infty}$$

73) Sketch a graph with given properties

i) $f(3) = 0$

ii) $\lim_{x \rightarrow \infty} f(x) = 1$ (H.A.)

iii) $\lim_{x \rightarrow -\infty} f(x) = 1$ (H.A.)

iv) $\lim_{x \rightarrow 1^-} f(x) = \infty$ (V.A.)

v) $\lim_{x \rightarrow 1^+} f(x) = -\infty$ (V.A.)

