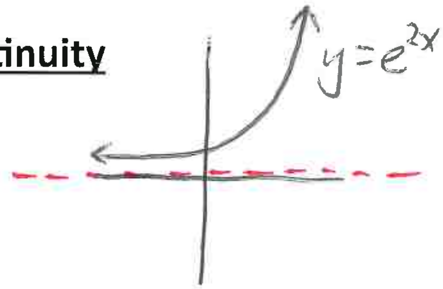
Ch. 1 AP Practice Problems (p. 159) – Limits & Continuity



1. Which line is an asymptote to the graph of $f(x)=e^{2x}$?

(A) $x=0$ $\quad$ $\boxed{\text{(B)}}$ $y=0$ $\quad$ (C) $y=2$ $\quad$ (D) $y=x$

2. $\lim\limits_{x\to\infty} \dfrac{3x^3+4x}{5-2x^4} =$

(A) $-\frac{3}{2}$ $\quad$ $\boxed{\text{(B)}}$ 0 $\quad$ (C) $\frac{3}{5}$ $\quad$ (D) $\frac{3}{2}$

3. If $f(x)=5x^3-1$, then $\lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x^3} =$

(A) 0 $\quad$ (B) 1 $\quad$ $\boxed{\text{(C)}}$ 5 $\quad$ (D) The limit does not exist.

$$\lim_{x\to 0} \frac{5x^3-1-(-1)}{x^3} \to \frac{5x^3-0}{x^3} \to \boxed{5}$$

4. $\lim\limits_{\theta\to 0} \dfrac{\theta^2}{1-\cos\theta} =$

(A) 0 $\quad$ (B) 1 $\quad$ $\boxed{\text{(C)}}$ 2 $\quad$ (D) 4

$$\lim_{\theta\to 0} \frac{\theta^2}{1-\cos\theta}\cdot\frac{(1+\cos\theta)}{(1+\cos\theta)}$$

$$\lim_{\theta\to 0} \frac{\theta^2(1+\cos\theta)}{1-\cos^2\theta} \to \lim_{\theta\to 0} \frac{\theta^2(1+\cos\theta)}{\sin^2\theta}$$

$$\lim_{\theta\to 0} \frac{\theta}{\sin\theta}\cdot\frac{\theta}{\sin\theta}\cdot\frac{1+\cos\theta}{1} \to$$

$$(1)(1)(2)=\boxed{2}$$

5. The table gives values of three functions:

x	-0.15	-0.1	-0.05	0	0.05	0.1	0.15
$f(x)$	0.075	0.05	0.025	-4	0.025	0.05	0.075
$g(x)$	-8.3	-8.2	-8.1	undefined	-7.9	-7.8	-7.7
$h(x)$	1.997	1.99	1.9975	1	1.005	1.02	1.045

For which of these functions does the table suggest that the limit as x approaches 0 exists?

(A) f only $\quad$ (B) h only

$\boxed{\text{(C)}}$ f and g only $\quad$ (D) f and h only

6. If a function f is continuous on the closed interval $[1, 4]$ and if $f(1)=6$ and $f(4)=-1$, then which of the following must be true?

(A) $f(c)=0$ for some number c in the open interval $(-1, 6)$.

$\boxed{\text{(B)}}$ $f(c)=1$ for some number c in the open interval $(1, 4)$.

(C) $f(c)=1$ for some number c in the open interval $(-1, 6)$.

(D) $f(c)\neq -2$ for any number c in the open interval $(1, 4)$.

By IVT,

since $f(4)=-1<1<6=f(1)$,

then $f(c)=1$ on interval $(1,4)$

7. Which are the equations of the asymptotes of the graph of the function $f(x) = \dfrac{x}{x(x^2-9)}$?

(A) $x=-3, x=0, x=3, y=0$
(B) $x=-3, x=0, x=3, y=1$
$\boxed{\text{(C)}}$ $x=-3, x=3, y=0$
(D) $x=-3, x=3, y=1$

$y = \dfrac{\cancel{x}(1)}{\cancel{(x)}(x+3)(x-3)}$

VA: $x=-3,\ x=3$
HA: $y=0$

8. Find the value of k that makes the function

$$f(x) = \begin{cases} x^2+2 & \text{if } x \le -1 \\ kx+4 & \text{if } x > -1 \end{cases}$$

continuous for all real numbers.

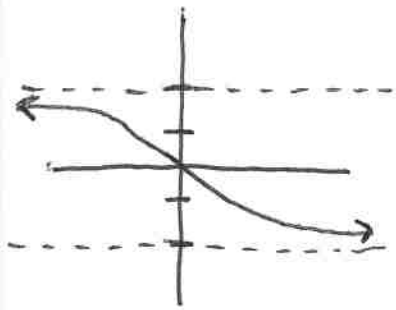
(A) -3 (B) -1 $\boxed{\text{(C)}}$ 1 (D) 3

*continuity conditions:
i) $f(-1) = (-1)^2 + 2 = 3$
ii) $\lim\limits_{x\to -1^-} x^2+2 = \boxed{3}$ $\lim\limits_{x\to -1^+} kx+4 \to \boxed{-k+4}$
$3 = -k+4 \to \boxed{k=1}$

9. An odd function f is continuous for all real numbers. If $\lim\limits_{x\to\infty} f(x) = -2$, then which of the statements must be true?

I. f has no vertical asymptotes.
II. $\lim\limits_{x\to 0} f(x) = 0$
III. The horizontal asymptotes of the graph of f are $y=-2$ and $y=2$.

(A) I only (B) III only
(C) I and III only $\boxed{\text{(D)}}$ I, II, and III



10. $\lim\limits_{x\to 0} \dfrac{\sin(2x)}{\tan(3x)} =$

(A) 0 $\boxed{\text{(B)}}$ $\dfrac{2}{3}$ (C) 1 (D) $\dfrac{3}{2}$

$\dfrac{\sin(2x)}{\frac{\sin(3x)}{\cos(3x)}} \to \dfrac{\sin(2x)}{1} \cdot \dfrac{\cos(3x)}{\sin(3x)}$

$\to \dfrac{2}{3} \cdot \dfrac{\sin 2x}{2x} \cdot \dfrac{3x}{\sin(3x)} \cdot \cos(3x)$

$\dfrac{2}{3} \cdot (1) \cdot (1) \cdot \cos(?x) \to \boxed{\dfrac{2}{3}}$

11. Suppose the function f is continuous for all real numbers. If $f(x) = \dfrac{x^3+8}{x+2}$ when $x \neq -2$, then $f(-2) =$

(A) 0 (B) 4 (C) 8 $\boxed{\text{(D)}}$ 12

$* a^3+b^3 = (a+b)(a^2-ab+b^2)$
$x^3+2^3 = (x+2)(x^2-2x+4)$

* step through continuity conditions
i) $f(-2) = k$
ii) $\lim\limits_{x\to -2} \dfrac{x^3+8}{x+2} \to \dfrac{0}{0} \to \lim\limits_{x\to -2} \dfrac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}} \to (-2)^2 - 2(-2) + 4 = \boxed{12}$

$\boxed{k=12}$