Ch. 6 Unit Review AP Practice Problems (p.459-460)

1. The closed interval $[a, b]$ is partitioned into n subintervals each of width $\Delta x = \dfrac{b-a}{n}$. If u_i is any number in the ith subinterval, what is $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \sqrt[3]{u_i}\,\Delta x$?

(A) $\dfrac{3}{2}[b^{2/3} - a^{2/3}]$ (B) $\dfrac{3}{4}[b^{4/3} - a^{4/3}]$

(C) $\dfrac{4}{3}[b^{4/3} - a^{4/3}]$ (D) $\dfrac{1}{3}\left[\dfrac{1}{b^{2/3}} - \dfrac{1}{a^{2/3}}\right]$

2. If $\int_1^8 f(x)\,dx = 3$, find $\int_1^8 f(9-x)\,dx$.

(A) -3 (B) 6 (C) 3 (D) 9

3. The area under the graph of the function $f(x) = \dfrac{2}{x}$, from $x = k$ to $x = 4k$, $k > 0$ is

(A) $\ln 8$ (B) $2\dfrac{\ln(4k)}{\ln k}$ (C) $2\ln 4$ (D) $2\ln(4k)$

4. If $\int_{-3}^{4} f(x)\,dx = 8$ and $\int_{6}^{4} f(x)\,dx = -6$, what is $\int_{-3}^{6} f(x)\,dx$?

(A) -14 (B) 2 (C) 14 (D) 16

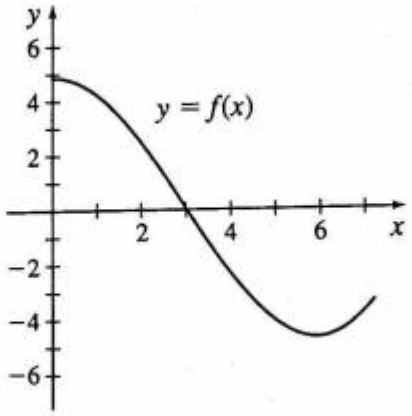
5. Find $\int_{-2}^{6} f(x)\,dx$ when $f(x) = \begin{cases} -x & \text{if } -2 \le x < 2 \\ x+3 & \text{if } 2 \le x \le 6 \end{cases}$.

(A) 24 (B) 28 (C) 32 (D) 44

6. An object is moving along the x-axis. If its velocity v at time t (in minutes) is $v(t) = t^2 - 2t$ (in feet per minute), what is the total distance the object travels between $t = 0$ and $t = 4$ minutes?

(A) 0 ft (B) $\frac{16}{3}$ ft (C) 8 ft (D) $\frac{40}{3}$ ft

7. The graph of a function f that is differentiable is shown below. If $F(x) = \int_0^x f(t)\,dt$, which of the following is true?



(A) $F(3) < F'(3) < F''(3)$ (B) $F'(3) < F''(3) < F(3)$

(C) $F''(3) < F(3) < F'(3)$ (D) $F''(3) < F'(3) < F(3)$

8. If $\int_0^6 f(3x+1)\,dx = 9$, then

(A) $\int_0^6 f(u)\,du = 3$ (B) $\int_1^{19} f(u)\,du = 9$

(C) $\int_0^{18} f(u)\,du = 27$ (D) $\int_1^{19} f(u)\,du = 27$

9. $\int x^3 e^{-x^4}\,dx =$

(A) $-\frac{1}{4}e^{-x^4} + C$ (B) $-\frac{1}{4}e^{x} + C$

(C) $\frac{x^4}{4}e^{-4x^3} + C$ (D) $\frac{x^4}{4}e^{-x^4} + C$

10. If at every point (x, y) on the graph of a function f, the slope of the tangent line is given by $y = 3 - 4x$ and if the point $(2, 3)$ is on the graph of f, then

(A) $f(x) = -5x + 7$ (B) $f(x) = -2x^2 + 3x - 11$

(C) $f(x) = -2x^2 + 3x$ (D) $f(x) = -2x^2 + 3x + 5$

11. $\int \frac{x^2 - 3x + 2\sqrt{x} - 1}{x} dx =$

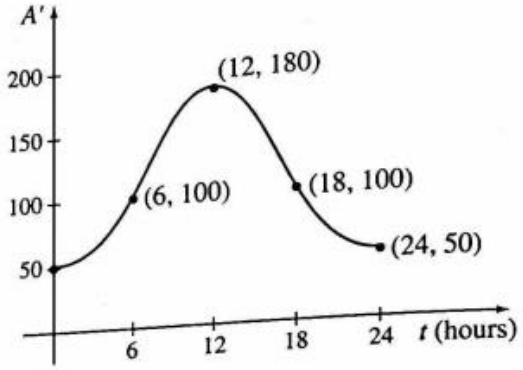
(A) $\frac{1}{2}x^2 - 3x + 4x^{1/2} - \ln|x| + C$

(B) $\frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{4}{3}x^{3/2} - \ln|x| + C$

(C) $\frac{1}{2}x^2 - 3x + \frac{6}{5}x^{5/2} - \ln|x| + C$

(D) $\frac{1}{2}x^2 - 3x + x^{1/2} - \ln|x| + C$

12. On a typical day, a dam releases water at a rate of $\frac{dA}{dt}$ (hundred thousand gallons per hour) as shown in the graph. Use a Left Riemann sum with four equal subintervals to approximate the total amount A (in hundreds of thousands of gallons) of water released in a day.



(A) 1720 (B) 2580 (C) 2760 (D) 3660

Free Response Questions

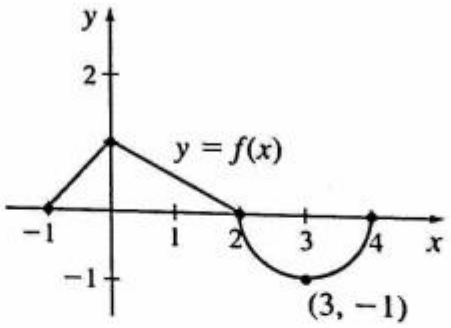
13. For the function $f(x) = \dfrac{\sin x}{x^2 + 1}$

(a) Find the derivative of f when $x = 1$.

(b) Find the area under the graph of f from 0 to π.

(c) What is the average value of f over the closed interval $[0, \pi]$?

14. Use the graph of the function f shown below to answer the questions.



(a) Use a Right Riemann sum with five subintervals of equal width to approximate $\int_{-1}^{4} f(x)\,dx$.

(b) Write $\int_{-1}^{4} f(x)\,dx$ as a sum of three integrals using properties of definite integrals.

(c) Find $\int_{-1}^{4} f(x)\,dx$ using geometry.

(d) Find $\int_{-1}^{4} f(x)\,dx$ using technology.

(e) Explain why $\int_{-1}^{2} f(x)\,dx > \int_{-1}^{4} f(x)\,dx$.