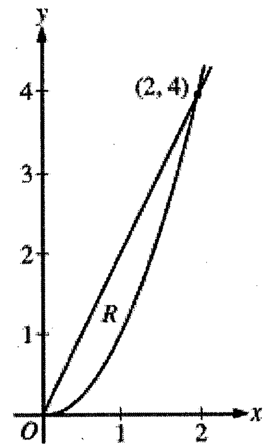
AB Calculus   Chapter 7.1-7.2 Review Worksheet #4

1. **(Non-Calculator)**

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.



(a) Find the area of R.

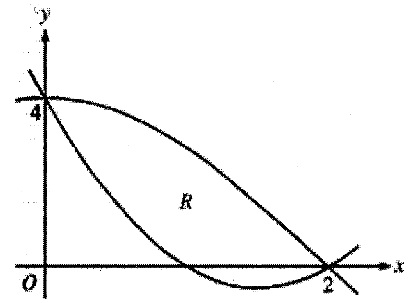
(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

(d) Write an Integral Expression that gives the Volume of the Solid generated when R is rotated about $x = -1$.

2) **(Non-Calculator)**

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.



(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

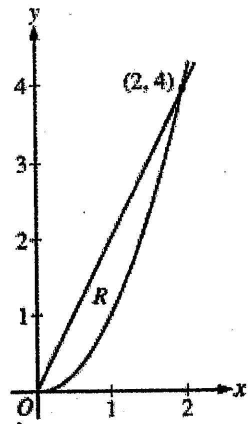
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

AB Calculus **Chapter 7.1-7.2 Review Worksheet #4**

Key

1. **(Non-Calculator)**

Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.



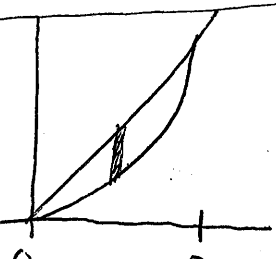
(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

d) Find Volume of solid by rotating R about line $x=-1$ (write expression)

a)



Top-bottom

$y = \underline{2x}$

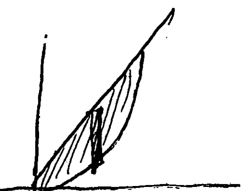
$y = \underline{x^2}$

$$\text{Area} = \int_0^2 2x - x^2\,dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\Big]_0^2 = 2^2 - \frac{2^3}{3} - (0-0)$$

$$= 4 - \frac{8}{3} = \frac{12}{3} - \frac{8}{3} = \boxed{\frac{4}{3}}$$

b)



$A(x) = \sin\left(\frac{\pi}{2}x\right)$

cross-section

$$V = \int [\text{Area}]\,dx$$

$$V = \int_0^2 \sin\left(\tfrac{\pi}{2}x\right)dx$$

$u = \frac{\pi}{2}x$

$\frac{du}{dx} = \frac{\pi}{2}$

$\pi\,dx = 2\,du$

$dx = \frac{2}{\pi}du$

$\int \sin u \cdot \frac{2}{\pi}du$

$\frac{2}{\pi}\int \sin u\,du$

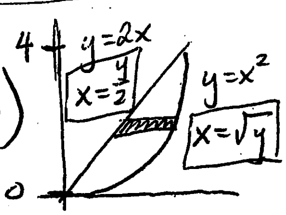
$= \frac{2}{\pi} \cdot -\cos u$

$-\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big]_0^2$

$-\frac{2}{\pi}\cos(\pi) - \left(-\frac{2}{\pi}\cos 0\right)$

$-\frac{2}{\pi}(-1) + \frac{2}{\pi} = \boxed{\frac{4}{\pi}}$

c)



Right/Left

$x = \underline{\sqrt{y}}$

$x = \underline{y/2}$

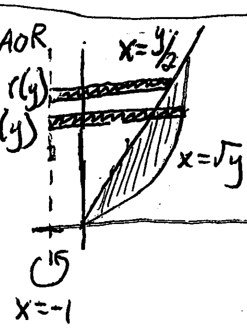
$\text{base} = \sqrt{y} - \frac{y}{2}$

$\underset{\text{square}}{\text{Area}} = [\text{base}]^2$

$\text{Area} = \left[\sqrt{y} - \frac{y}{2}\right]^2$

$$\boxed{V = \int_0^4 \left[\sqrt{y} - \frac{y}{2}\right]^2 dy}$$

d)



*washer method

*Right/Left

$x = \underline{\sqrt{y}}$

$x = \underline{y/2}$

$R(y) = \sqrt{y} - (-1)$

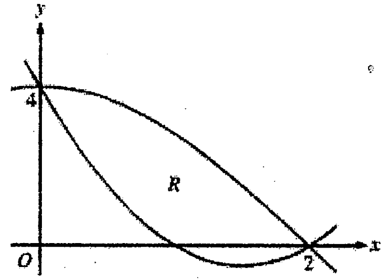
$r(y) = y/2 - (-1)$

$$V = \pi \int_{y_1}^{y_2} [R(y)]^2 - [r(y)]^2\,dy$$

$$\boxed{V = \pi \int_0^4 [\sqrt{y}+1]^2 - \left[\tfrac{y}{2}+1\right]^2 dy}$$

2) **(Non-Calculator)**

Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

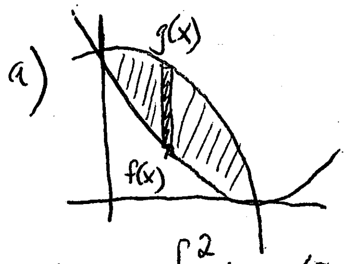


(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

a)



$$Area = \int_0^2 g(x) - f(x)\,dx$$

$$Area = \int_0^2 4\cos\left(\tfrac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right) dx$$

$$Area = \int_0^2 4\cos\left(\tfrac{\pi}{4}x\right) - 2x^2 + 6x - 4\,dx$$

$u = \frac{\pi}{4}x$ | $dx = \frac{4}{\pi}du$

$\frac{du}{dx} = \frac{\pi}{4}$ | $4\int \cos u \cdot \frac{4}{\pi}du$

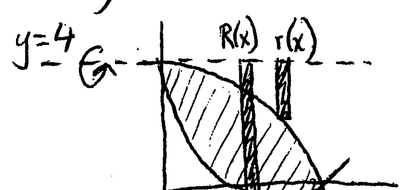
$\pi dx = 4du$

$$\left. 4 \cdot \tfrac{4}{\pi}\sin\left(\tfrac{\pi}{4}x\right) - \tfrac{2x^3}{3} + \tfrac{6x^2}{2} - 4x \right]_0^2$$

$$\tfrac{16}{\pi}\sin\left(\tfrac{\pi}{4}\cdot 2\right) - \tfrac{2(2)^3}{3} + \tfrac{6(2)^2}{2} - 4(2) - \left[\tfrac{16}{\pi}\sin(0) - 0 + 0 - 0\right]$$

$$\boxed{\tfrac{16}{\pi}(1) - \tfrac{16}{3} + \tfrac{24}{2} - 8} \quad \text{or} \quad \boxed{\tfrac{16}{\pi} - \tfrac{4}{3}}$$

b) AOR: $y = 4$



* washer method
* Top/bottom

$y = 4\cos\left(\frac{\pi}{4}x\right)$

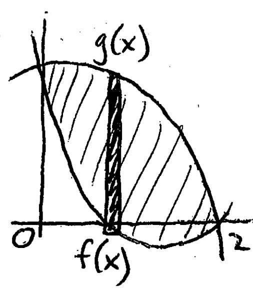
$y = 2x^2 - 6x + 4$

$R(x) = 4 - \left(2x^2 - 6x + 4\right) = 4 - 2x^2 + 6x - 4 = \boxed{-2x^2 + 6x}$

$r(x) = 4 - 4\cos\left(\frac{\pi}{4}x\right)$

$$\boxed{V = \pi\int_0^2 \left[-2x^2 + 6x\right]^2 - \left[4 - 4\cos\left(\tfrac{\pi}{4}x\right)\right]^2 dx}$$

c)



Top/bottom

$y = 4\cos\left(\frac{\pi}{4}x\right)$

$y = 2x^2 - 6x + 4$

base $= 4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)$

Area square $= \left[\text{base}\right]^2$

$$\boxed{V = \int_0^2 \left[4\cos\left(\tfrac{\pi}{4}x\right) - 2x^2 + 6x - 4\right]^2 d}$$