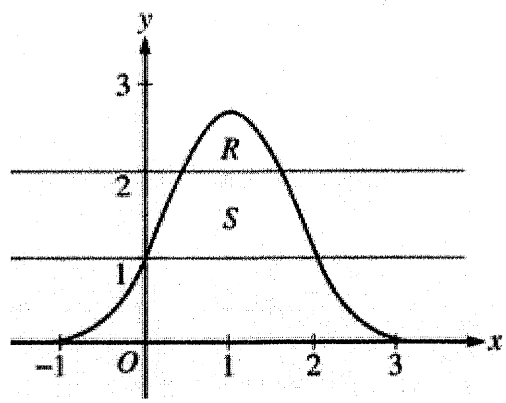
Ch. 7 Area/Volume AB FRQ Problems Worksheet #3

1)

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.



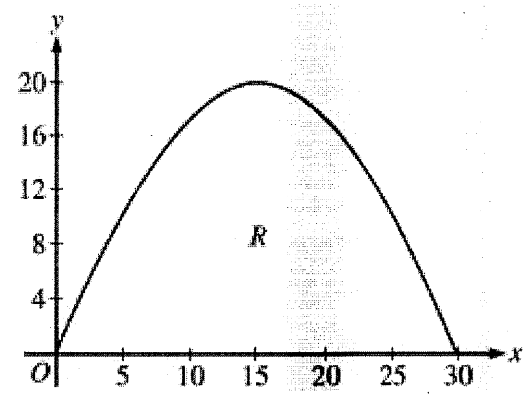
(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

2)

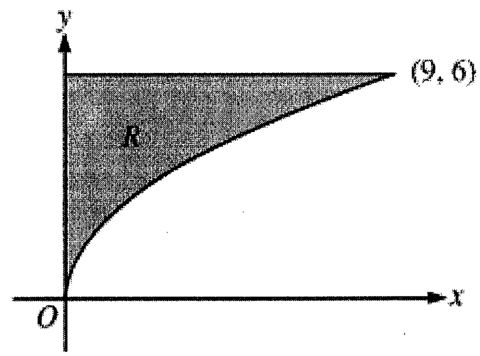
A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



(a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

(b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

3) (Non-Calculator)



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \leq y \leq 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

4)

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

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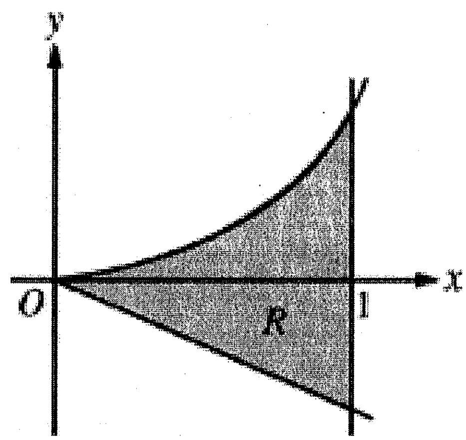

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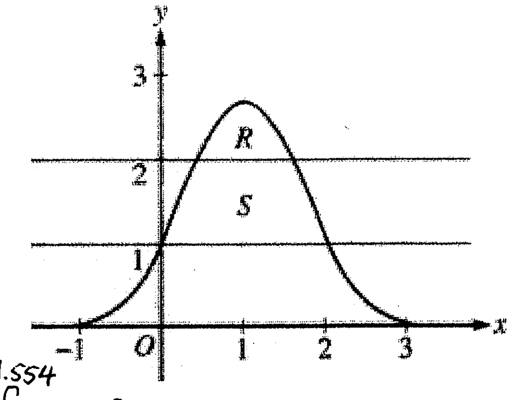
Key

1)



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(a) Find the area of R.

(b) Find the area of S.

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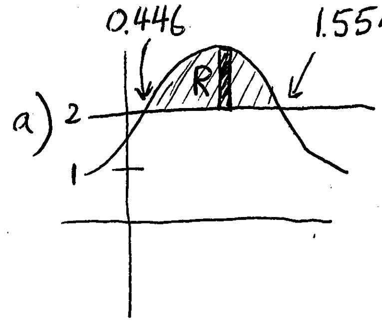
a)



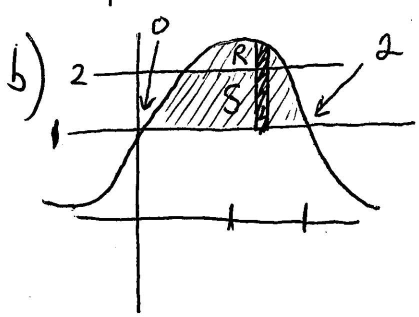
Top/bottom

$y = e^{2x-x^2}$

$y = 2$

$$\text{Area} = \int_{0.446}^{1.554} e^{2x-x^2} - 2\,dx = \boxed{0.514}$$

b)



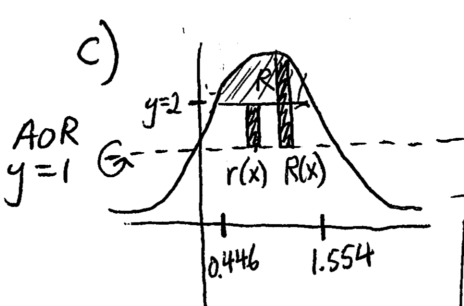
Area of S = Area of R+S − Area of R

$$\text{Area}(R+S) = \int_0^2 e^{2x-x^2} - 1\,dx = 2.06016$$

(Top/bottom)

$$\text{Area of } S = 2.06016 - 0.514 = \boxed{1.546}$$

c)



$R(x) = e^{2x-x^2} - 1$

$r(x) = 2 - 1 = 1$

$$V = \pi \int_{x_1}^{x_2} R(x)^2 - r(x)^2\,dx$$

$$\boxed{V = \pi \int_{0.446}^{1.554} \left[e^{2x-x^2} - 1\right]^2 - [1]^2\,dx}$$

Washer Method

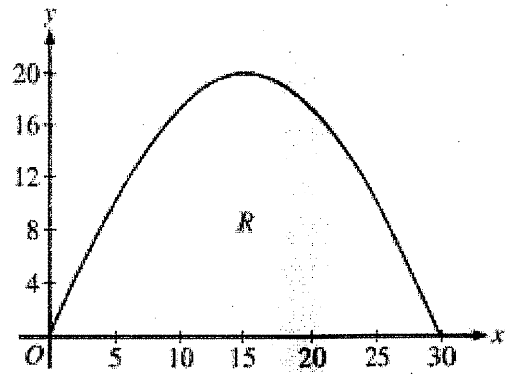
Top/bottom

$y = e^{2x-x^2}$

$y = 2$

2)

A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.



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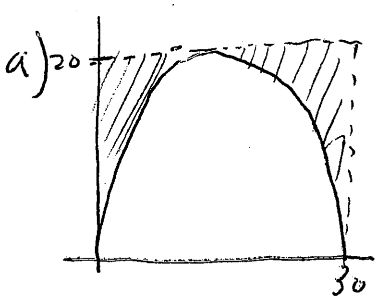
a)

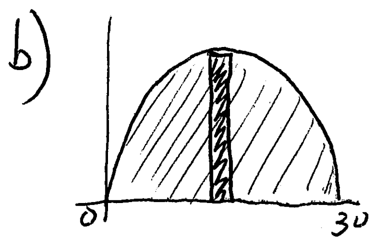


Area = Area of Box − Area under parabola (Top-bottom) $y = 20\sin\left(\frac{\pi x}{30}\right)$, $y = 0$

$$= 30(20) - \int_0^{30} 20\sin\left(\frac{\pi x}{30}\right) - 0\,dx$$

$$= 600 - 381.972 = \boxed{218.028\text{ cm}^2}$$

b)



Top/bottom

$y = 20\sin\left(\frac{\pi x}{30}\right)$

$y = 0$

$$\text{base} = 20\sin\left(\frac{\pi x}{30}\right) - 0$$

$$\text{base} = 20\sin\left(\frac{\pi x}{30}\right)$$

$$\underset{\text{(semicircle)}}{\text{Area}} = \frac{\pi}{8}[\text{base}]^2$$

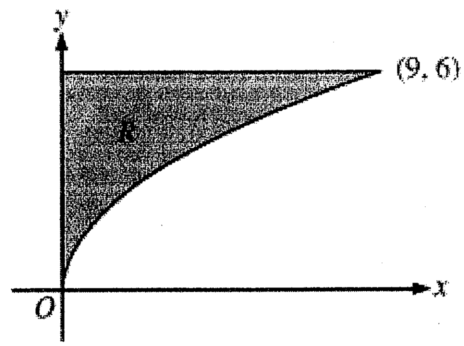
$$= \frac{\pi}{8}\left[20\sin\left(\frac{\pi x}{30}\right)\right]^2$$

$$\text{Volume} = \int_0^{30} \frac{\pi}{8}\left[20\sin\left(\frac{\pi x}{30}\right)\right]^2 dx$$

$$V = 2356.194\text{ cm}^3$$

$$\frac{0.05\text{ grams}}{1\,\cancel{\text{cm}^3}} \cdot 2356.194\,\cancel{\text{cm}^3} = \boxed{117.809\text{ grams (of chocolate)}}$$

3) (Non-calculator)



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(a) Find the area of R.

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(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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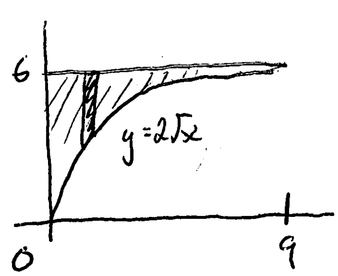


Top-bottom

$y = \underline{2\sqrt{x}}$

$y = \underline{6}$

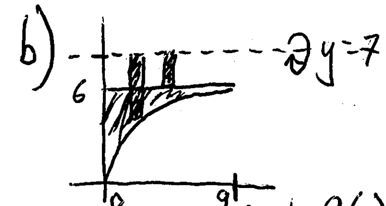
$$\text{Area} = \int_0^9 6 - 2\sqrt{x}\,dx$$

$$= \int_0^9 6 - 2x^{1/2}\,dx = 6x - \frac{2x^{3/2}}{3/2}\Bigg]_0^9$$

$$6x - \frac{4}{3}x^{3/2}\Bigg]_0^9 = 6(9) - \frac{4}{3}(9)^{3/2} - (0-0)$$

$$= 54 - \frac{4}{3}(3)^3 = 54 - 4(9) = \boxed{18}$$

b)



washer method

Top/bottom

$y = \underline{6}$

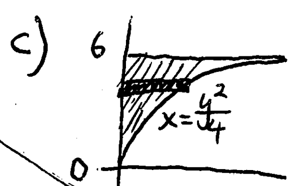
$y = \underline{2\sqrt{x}}$

$R(x) = 7 - 2\sqrt{x}$

$r(x) = 7 - 6 = 1$

$$\boxed{V = \pi\int_0^9 [7-2\sqrt{x}]^2 - [1]^2\,dx}$$

c)



$y = 2\sqrt{x} \rightarrow \frac{y}{2} = \sqrt{x}$

$\left(\frac{y}{2}\right)^2 = x$

Right/Left

$x = \underline{y^2/4}$

$x = \underline{0}$

base $= \frac{y^2}{4} - 0 = \frac{y^2}{4}$

height $= 3(\text{base}) = 3\left(\frac{y^2}{4}\right)$

Area $=$ base $\times$ height

$= \left(\frac{y^2}{4}\right) \times 3\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$

$$\boxed{V = \int_0^6 \frac{3}{16}y^4\,dy}$$

4)

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

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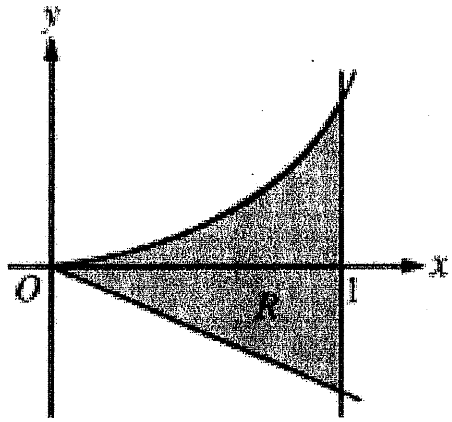
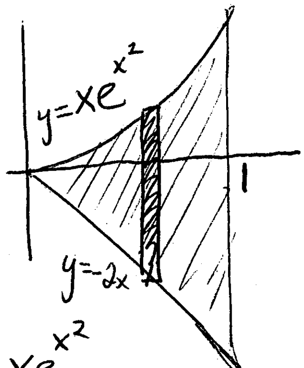


a)



$y = \underline{xe^{x^2}}$

$y = \underline{-2x}$

Top/bottom

u-substitution

$$\text{Area} = \int_0^1 xe^{x^2} - (-2x)\,dx = \int_0^1 xe^{x^2} + 2x\,dx$$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int xe^{u}\frac{du}{2\cancel{x}}$

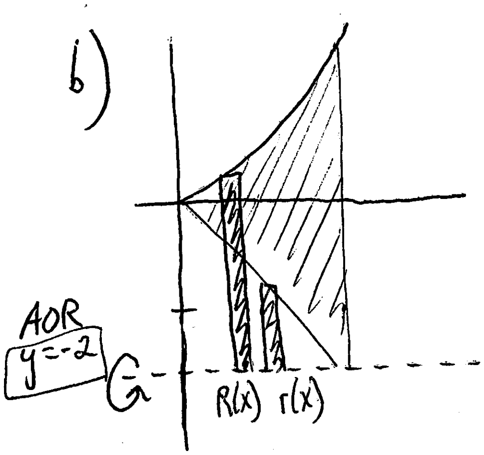
$\frac{1}{2}\int e^{u}du = \frac{1}{2}e^{x^2}$

$\left[\frac{1}{2}e^{x^2} + \frac{\cancel{2}x^2}{\cancel{2}}\right]_0^1$

$\frac{1}{2}e^{1} + 1 - \left(\frac{1}{2}e^{0} + 0^2\right)$

$\frac{1}{2}e + 1 - \frac{1}{2} = \boxed{\frac{1}{2}e + \frac{1}{2}}$

b)



washer method

Top/bottom

$y = \underline{xe^{x^2}}$

$y = \underline{-2x}$

$R(x) = xe^{x^2} - (-2)$

$r(x) = -2x - (-2)$

$= -2x + 2$

$$V = \pi\int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2\,dx$$

$$\boxed{V = \pi\int_0^1 \left[xe^{x^2}+2\right]^2 - \left[-2x+2\right]^2\,dx}$$