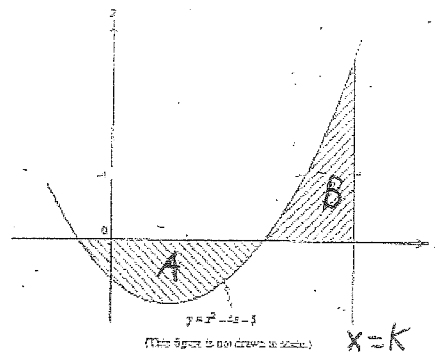
AP Calculus AB Morning Test Review (Ch. 7 and 8.2)

1. a) The sketch above shows the graphs of $f(x) = x^2 - 4x - 5$ and the line $x = k$. The regions labeled A and B have equal areas if k = ______

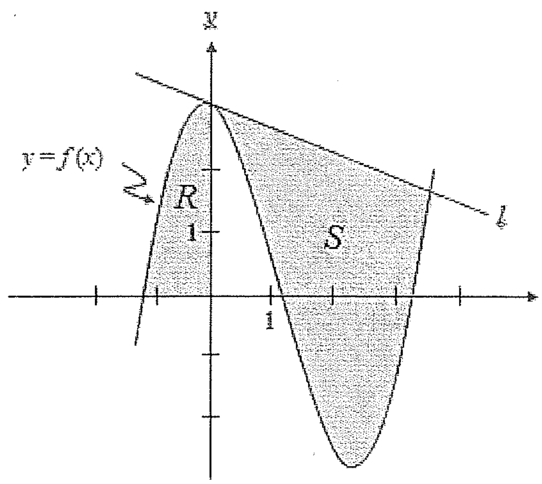


b) Find the volume of shaded region A revolved about y = 2

c) The region A is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle of height that is half the length with the length lying across the base. Find the volume of this solid.

2. 2006 Form B #1

Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line l, the line tangent to f at $x = 0$, as shown above.



a. Find the equation of the tangent line

b. Find the area of R.

c. Find the area of S

d. Find the volume of the solid generated when R is rotated about the line y = -2

3. Find the volume of the solid formed by revolving the region bounded by $y = \ln x$, the x-axis, and $x = 6$ about the line $x = -2$.

4. Evaluate $\int \ln x^3 \, dx$

5. Evaluate $\int \frac{\ln y}{y^2} dy$

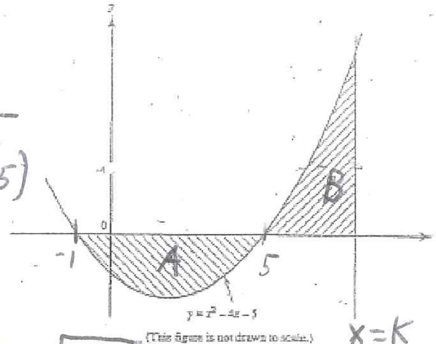
6. Evaluate $\int x^3 e^{-2x} dy$

7. Determine the area of the region bounded by $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$, and $x = 5$.

AP Calculus AB Morning Test Review (Ch. 7 and 8.2)

$x^2-4x-5=0 \quad (x-5)(x+1) \quad x=-1,5$

1. a) The sketch above shows the graphs of $f(x) = x^2 - 4x - 5$ and the line $x = k$. The regions labeled A and B have equal areas if $k =$ ______



Area of A $= \int_{-1}^{5} 0-(x^2-4x-5)\,dx = 36$

Area of B $\rightarrow \int_{5}^{k} x^2-4x-5-0\,dx = 36$

$\left.\frac{x^3}{3}-\frac{4x^2}{2}-5x\right]_5^k = \frac{k^3}{3}-2k^2-5k-\left(\frac{125}{3}-2(25)-25\right)$

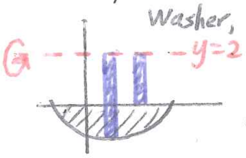
$\frac{k^3}{3}-2k^2-5k+\frac{100}{3}=36$

*Find x-int:

$\frac{k^3}{3}-2k^2-5k-\frac{8}{3}=0 \quad \boxed{k=8}$

b) Find the volume of shaded region A revolved about y = 2

Washer, Top/bottom



$R(x)=2-(x^2-4x-5)=-x^2+4x+7$

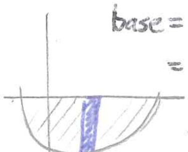
$r(x)=2-0=2$

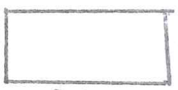
$V=\pi\int_{-1}^{5} R(x)^2-r(x)^2\,dx$

$V=\pi\int_{-1}^{5}[-x^2+4x+7]^2-2^2\,dx = \boxed{\frac{2016}{5}\pi}$ units^3

c) The region *A* is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is a rectangle of height that is half the length with the length lying across the base. Find the volume of this solid.

base $=0-(x^2-4x-5)$
$=-x^2+4x+5$





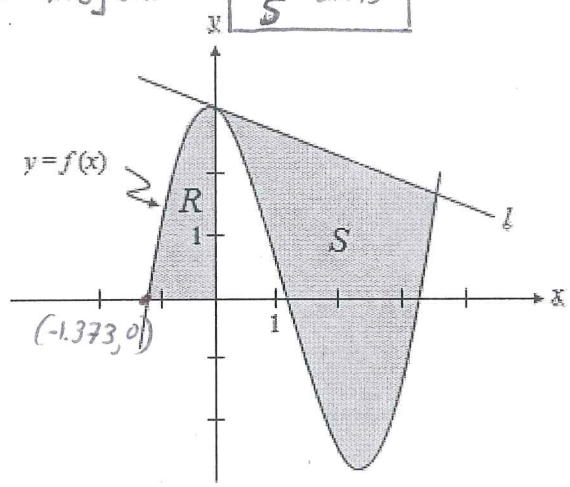
Area $=(-x^2+4x+5)\cdot\frac{1}{2}(-x^2+4x+5)$

$V=\int[\text{Area of cross section}]\,dx$

$V=\int_{-1}^{5}\frac{1}{2}[-x^2+4x+5]^2\,dx = \boxed{\frac{648}{5}\text{ units}^3}$

2. 2006 Form B #1

Let f be the function given by $f(x)=\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line l, the line tangent to f at $x = 0$, as shown above.



a. Find the equation of the tangent line $f(0)=3$

$f'(x)=\frac{3}{4}x^2-\frac{2}{3}x-\frac{1}{2}+3(-\sin x)$

$f'(0)=0-0-\frac{1}{2}-3\sin(0)=-\frac{1}{2} \quad m=-\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y-3=-\frac{1}{2}(x-0)\rightarrow y=-\frac{1}{2}x+3$

b. Find the area of *R*.

Area $=\int_{-1.373}^{0} f(x)\,dx = 2.903$

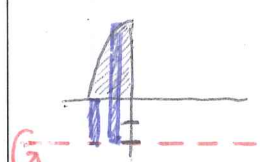
c. Find the area of S

*find bounds: set $f(x)=-\frac{1}{2}x+3$, $f(x)+\frac{1}{2}x-3=0$

$x=3.389$

Area $=\int_{0}^{3.389}\left[-\frac{1}{2}x+3\right]-f(x)\,dx = 6.982$ units^2

d. Find the volume of the solid generated when R is rotated about the line y = -2



$R(x)=f(x)-(-2)=f(x)+2$

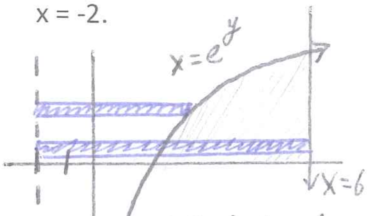
$r(x)=0-(-2)=2$

$V=\pi\int_{-1.373}^{0} R(x)^2-r(x)^2\,dx$

$V=\pi\int_{-1.373}^{0}[f(x)+2]^2-2^2\,dx$

$=\boxed{18.895\pi\text{ units}^3}$

3. Find the volume of the solid formed by revolving the region bounded by $y = \ln x$, the x-axis, and $x = 6$ about the line $x = -2$.



$R(y) = 6-(-2) = 8$

$r(y) = e^y - (-2) = e^y + 2$

$y = \log_e x$

$e^y = x$

$$V = \pi\int_0^{\ln 6} 8^2 - (e^y+2)^2\,dy = \boxed{70.006\pi \text{ units}^3}$$

*find bound

$e^y = 6$

$\ln e^y = \ln 6 \quad y = \ln 6$

4. Evaluate $\int \ln x^3\,dx$

$u = \ln x^3 \qquad dv = dx$

$du = \frac{3x^2}{x^3}dx \qquad v = x$

$du = \frac{3}{x}dx$

$x\ln x^3 - \int \frac{3}{x}(x)\,dx$

$-\int 3\,dx$

$\boxed{x\ln x^3 - 3x + C}$

5. Evaluate $\int \frac{\ln y}{y^2}dy$

$\int \ln y \cdot y^{-2}\,dy$

$u = \ln y \qquad dv = y^{-2}$

$du = \frac{1}{y}dy \qquad v = \frac{y^{-1}}{-1} = -\frac{1}{y}$

$\ln y \cdot \left(\frac{-1}{y}\right) - \int -\frac{1}{y}\cdot\frac{1}{y}\,dy$

$-\frac{\ln y}{y} + \int y^{-2}\,dy$

$-\frac{\ln y}{y} + \frac{y^{-1}}{-1}$

$= \boxed{\frac{-\ln y}{y} - \frac{1}{y} + C}$

6. Evaluate $\int x^3 e^{-2x}\,dy$

$u = x^3 \qquad dv = e^{-2x}$

	u	dv
+	x^3	e^{-2x}
−	$3x^2$	$-\frac{1}{2}e^{-2x}$
+	$6x$	$\frac{1}{4}e^{-2x}$
−	6	$-\frac{1}{8}e^{-2x}$
+	0	$\frac{1}{16}e^{-2x}$

$\int e^{-2x} = \int e^u \cdot \frac{du}{-2}$

$u = -2x$

$\frac{du}{dx} = -2$

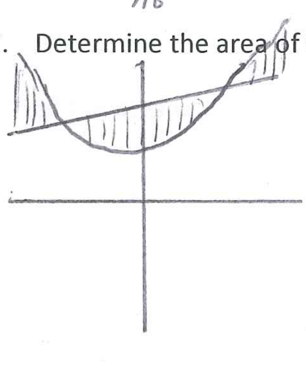
$dx = \frac{du}{-2}$

$= -\frac{1}{2}\int e^u\,du$

$= -\frac{1}{2}e^{-2x} + C$

$$\boxed{e^{-2x}\left(\frac{-1}{2}x^3 - \frac{3}{4}x^2 - \frac{3}{4}x - \frac{6}{16}\right) + C}$$

7. Determine the area of the region bounded by $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$, and $x = 5$.



$$\int_{-2}^{-1} 2x^2+10 - (4x+16)\,dx + \int_{-1}^{3} (4x+16) - (2x^2+10)\,dx + \int_{3}^{5} (2x^2+10) - (4x+16)\,dx$$

$$\frac{14}{3} + \frac{64}{3} + \frac{64}{3}$$

$$= \boxed{\frac{142}{3}}$$