AP CALCULUS AB **Ch. 7 Test review WS #1** **Area/Volume Practice Questions**

Additional Notes: Volume problems involving x = k

Steps:

1. Set up the definite integral equation with **k** as a bound.
2. Use appropriate integral method for problem and find the bound k.

Example 1: Let R be the region bounded by x-axis , the graph of $y = \sqrt{x}$, and the line x = k.

a) Find the line x = k if the area of the region is 8 units^2

b) Find the line x = k if the volume of region revolved about the x-axis is 18π

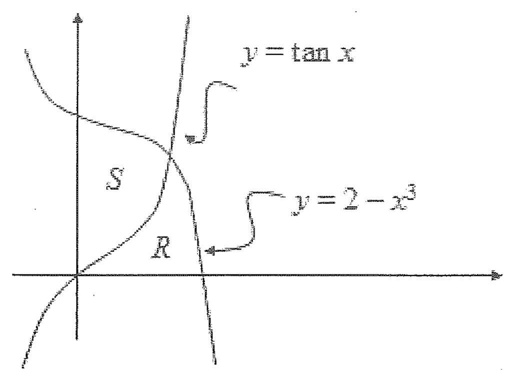
2.

1998 AP Calculus AB Scoring Guidelines

1. Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   (a) Find the area of the region R.
   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.
   (c) Find the volume of the solid generated when R is revolved about the x–axis.
   (d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.

3. 2001 FRQ #1

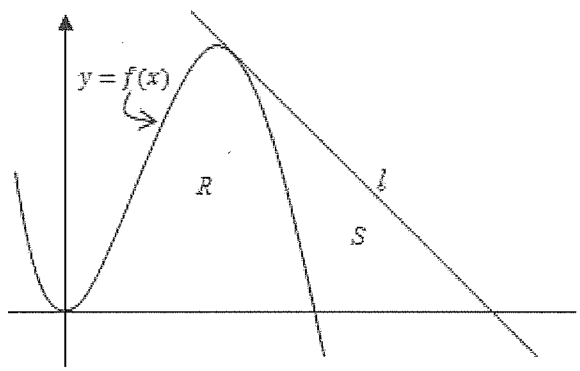
Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



a) Find the area of R
b) Find the area of S
c) Find the volume of the solid generated when S is revolved about the x-axis

4. 2003 Form B FRQ #1

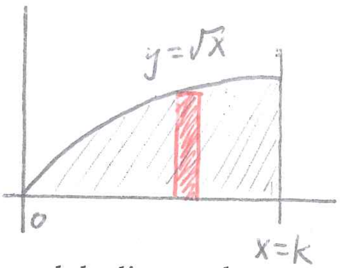
Let R be the region bounded by the graph of $f(x) = 4x^2 - x^3$ and the x-axis. Let l be the line tangent to f at $x = 3$ and let S be the region bounded by the graph of f, the line l, and the x-axis as shown in the figure above.



a) Find the equation of line l written in slope-intercept form.
b) Find the area of R.
c) Find the area of S.
d) Find the volume of the solid generated when R is revolved about the x-axis

AP CALCULUS AB   Ch. 7 Test review WS #1   Area/Volume Practice Questions

Additional Notes: Volume problems involving x = k



Steps:
1. Set up the definite integral equation with **k** as a bound.
2. Use appropriate integral method for problem and find the bound k.

$\left[\sqrt[3]{27}\right]^2$

Example 1: Let R be the region bounded by x-axis , the graph of $y = \sqrt{x}$, and the line x = k.

a) Find the line x = k if the area of the region is 18 units^2

$A=\int_0^k \underbrace{\sqrt{x}}_{\text{top}}-\underbrace{0}_{\text{bottom}}\,dx$ $\left.\frac{x^{3/2}}{3/2}\right]_0^k$

$\int_0^k x^{1/2}dx=18$ $\left.\frac{2}{3}x^{3/2}\right]_0^k=\frac{2}{3}(k)^{3/2}-0=18$

$\frac{3}{2}\cdot\frac{2}{3}k^{3/2}=18\cdot\frac{3}{2}$

$k^{3/2}=18\cdot\frac{3}{2}$

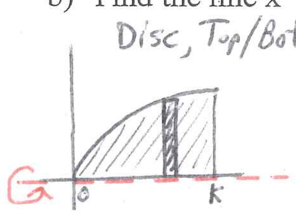
$k^{3/2}=27$

$\left(k^{3/2}\right)^{2/3}=(27)^{2/3}=3^2=9$

$\boxed{k=9}$

b) Find the line x = k if the volume of region revolved about the x-axis is 18π

Disc, Top/Bottom



$V=\pi\int_0^k R(x)^2dx=18\pi$

$R(x)=\sqrt{x}-0=\sqrt{x}$

$\pi\int_0^k(\sqrt{x})^2dx$

$=\pi\int_0^k x\,dx=18\pi$

$\left.\pi\frac{x^2}{2}\right]_0^k=\frac{k^2}{2}\pi-0=18\pi$

$\frac{k^2}{2}\cancel{\pi}=18\cancel{\pi}$

$\sqrt{k^2}=\sqrt{36}$ $\boxed{k=6}$

2.

1998 AP Calculus AB Scoring Guidelines

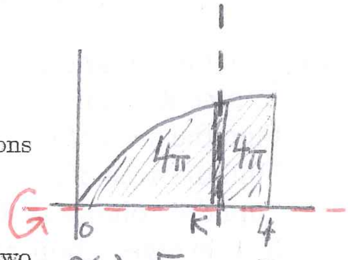
Let R be the region bounded by the x–axis, the graph of $y=\sqrt{x}$, and the line $x=4$.

(a) Find the area of the region R.

(b) Find the value of h such that the vertical line $x=h$ divides the region R into two regions of equal area.

(c) Find the volume of the solid generated when R is revolved about the x–axis.

(d) The vertical line $x=k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k.



$R(x)=\sqrt{x}-0=\sqrt{x}$

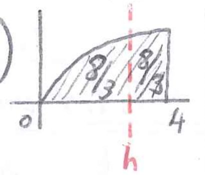
a) $A=\int_0^4\sqrt{x}-0\,dx$

$=\int x^{1/2}dx=\left.\frac{x^{3/2}}{3/2}\right]_0^4$

$\left.\frac{2}{3}x^{3/2}\right]_0^4=\frac{2}{3}(4)^{3/2}$

$=\frac{2}{3}(8)=\boxed{\frac{16}{3}\text{ units}^2}$

b)



$\int_0^h\sqrt{x}dx=\frac{8}{3}$

$\left.\frac{2}{3}x^{3/2}\right]_0^h=\frac{2}{3}(h)^{3/2}-0$

$\frac{3}{2}\cdot\frac{2}{3}(h)^{3/2}=\frac{8}{3}\cdot\frac{3}{2}$

$h^{3/2}=4$

$\left(h^{3/2}\right)^{2/3}=(4)^{2/3}$

$\boxed{h=\sqrt[3]{16}\approx2.520}$

c) $V=\pi\int_0^4(\sqrt{x})^2dx=\pi\int_0^4 x\,dx$

$\left.\pi\frac{x^2}{2}\right]_0^4=\frac{16}{2}-0=8\pi$

$\boxed{V=8\pi\text{ units}^3}$

d) $\pi\int_0^k(\sqrt{x})^2dx=4\pi$

$\left.\frac{x^2}{2}\right]_0^k=\pi\frac{k^2}{2}-0=4\pi$

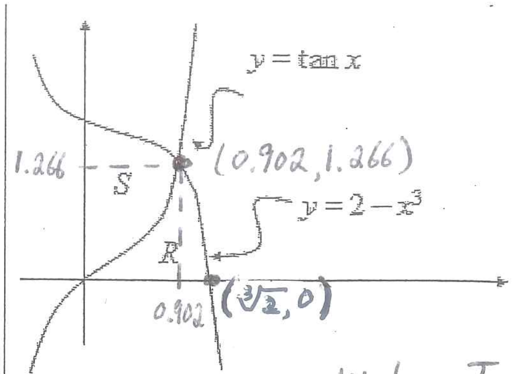
$\cancel{\pi}\frac{k^2}{2}=4\cancel{\pi}$

$\sqrt{k^2}=\sqrt{8}$

$\boxed{k=\sqrt{8}=2\sqrt{2}\approx2.828}$

3. 2001 FRQ #1

Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



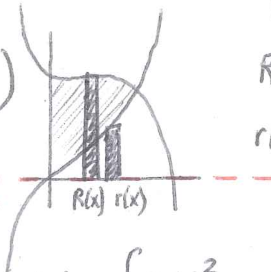
a) Find the area of R
b) Find the area of S
c) Find the volume of the solid generated when S is revolved about the x-axis

$$\text{a) } A_R = \int_0^{0.902} \tan x - 0\,dx + \int_{0.902}^{\sqrt[3]{2}} 2 - x^3 - 0\,dx$$

$$= 0.478 + 0.251 = \boxed{0.729 \text{ units}^2}$$

$$\text{b) } A_S = \int_0^{0.902} \underbrace{2 - x^3}_{\text{top}} - \underbrace{\tan x}_{\text{bottom}}\,dx = \boxed{1.161 \text{ units}^2}$$

c)



Washer, Top/bottom

$R(x) = 2 - x^3 - 0 = 2 - x^3$

$r(x) = \tan x - 0 = \tan x$

$$V = \pi \int R(x)^2 - r(x)^2\,dx$$

$$V = \pi \int_0^{0.902} [2 - x^3]^2 - [\tan x]^2\,dx$$

$$= \boxed{2.652\pi \text{ units}^3}$$

4. 2003 Form B FRQ #1

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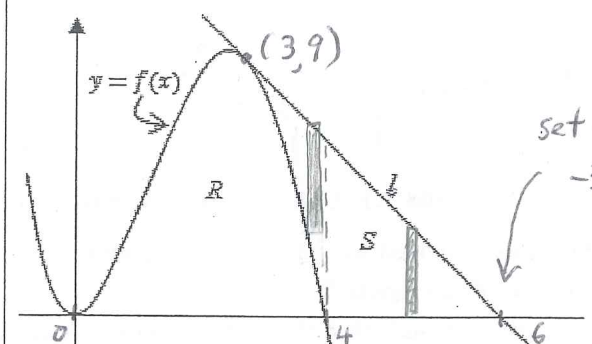


set $-3x + 18 = 0$

$-3x = -18$

$x = 6$

a) Find the equation of line l written in slope-intercept form.
b) Find the area of R.
c) Find the area of S.
d) Find the volume of the solid generated when R is revolved about the x-axis

a) $f(3) = 4(3)^2 - 3^3 = 9$ point $(3,9)$

$f'(x) = 8x - 3x^2$

$f'(3) = 8(3) - 3(3)^2 = 24 - 27 = -3$ $m = -3$

$y - y_1 = m(x - x_1)$

$y - 9 = -3(x - 3)$

$y - 9 = -3x + 9$

$\boxed{y = -3x + 18}$

b) set $4x^2 - x^3 = 0$ to find bounds

$x^2(4 - x) = 0$ $x = 0, 4$

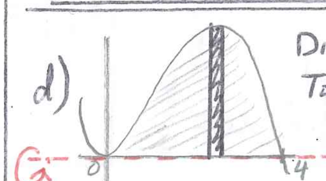
$$A = \int_0^4 \underbrace{4x^2 - x^3}_{\text{top}} - \underbrace{0}_{\text{bottom}}\,dx = \boxed{\frac{64}{3} \text{ units}^2}$$

$$\text{c) } A_S = \int_3^4 \underbrace{(-3x + 18)}_{\text{top}} - \underbrace{(4x^2 - x^3)}_{\text{bottom}}\,dx + \int_4^6 \underbrace{(-3x + 18)}_{\text{top}} - \underbrace{0}_{\text{bottom}}\,dx$$

$$= 1.917 + 6$$

$$\boxed{A_S = 7.917 \text{ units}^2}$$

d)



Disc, Top/bottom

$R(x) = 4x^2 - x^3 - 0 = 4x^2 - x^3$

$$V = \pi \int_0^4 R(x)^2\,dx$$

$$V = \pi \int_0^4 [4x^2 - x^3]^2\,dx$$

$$\boxed{V = 156.038\pi}$$