# Calculus Section 8.1 Basic Integration Rules

-Review procedures for fitting an integrand to one of the basic integration rules

In the sections following this one, we will expand on the basic integration techniques in order to learn how to solve higher level integrals. A listing of all the basic integration rules can be found on the inside front cover of this book. The key to solving an integration problem is identifying which integration technique to use. We will review the basic techniques in this section before moving to higher level integrals.

**Example) A Comparison of Three Similar Integrals**

Find each integral.

a) $\int \frac{4}{x^2+9}\,dx$ b) $\int \frac{4x}{x^2+9}\,dx$ c) $\int \frac{4x^2}{x^2+9}\,dx$

Here is a list of procedures that you use to make an integral fit one of the basic rules.

1) Expand a function $(1+e^x)^2 = 1+2e^x+e^{2x}$
2) Separate the numerator $\frac{1+x}{x^2+1} = \frac{1}{x^2+1} + \frac{x}{x^2+1}$
3) Complete the square $\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$
4) Long division $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$
5) Add and subtract terms in numerator $\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{x^2+2x+1}$
6) Use trigonometric identities $\cot^2 x = \csc^2 x - 1$
7) Multiply and divide by Pythagorean conjugate

$$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}$$

**Examples)**

Find $\int \frac{x^2}{\sqrt{16-x^6}}\,dx$

$\int \frac{1}{1+e^x}\,dx$

$\int (\cot x)[\ln(\sin x)]dx$

$\int \tan^2(2x)dx$

Key

## AP Calculus BC Ch. 8.1: Basic Integration Rules

-Review procedures for fitting an integrand to one of the basic integration rules

In the sections following this one, we will expand on the basic integration techniques in order to learn how to solve higher level integrals. A listing of all the basic integration rules can be found on the inside front cover of this book. The key to solving an integration problem is identifying which integration technique to use. We will review the basic techniques in this section before moving to higher level integrals.

**Example) A Comparison of Three Similar Integrals**

Find each integral.

a) $\int \frac{4}{x^2+9}dx$

*arctan rule

$\int \frac{du'}{a^2+u^2}du = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

$4\int \frac{dx}{(x)^2+(3)^2}$ $u=x$ $a=3$ $\frac{du}{dx}=1$ $dx=du$

$4\int \frac{du}{a^2+u^2} = 4\cdot\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$

$\boxed{\frac{4}{3}\arctan\left(\frac{x}{3}\right)+C}$

b) $\int \frac{4x}{x^2+9}dx$

$u=x^2+9$

$\frac{du}{dx}=2x$ $dx=\frac{du}{2x}$

$\int \frac{4\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}$

$2\int \frac{1}{u}du$

$\boxed{2\ln|x^2+9|+C}$

c) $\int \frac{4x^2}{x^2+9}dx$

$$\begin{array}{r} 4 - \frac{36}{x^2+9} \\ x^2+9\overline{)\,4x^2\qquad} \\ -(4x^2+36) \\ -36 \end{array}$$

$\int 4\,dx - \int \frac{36}{x^2+9}dx$

$\int 4\,dx - \int \frac{36}{(x)^2+(3)^2}dx$

$4x - 36\cdot\frac{1}{3}\arctan\left(\frac{x}{3}\right)+C$

$\boxed{4x-12\arctan\left(\frac{x}{3}\right)+C}$

Here is a list of procedures that you use to make an integral fit one of the basic rules.

1) Expand a function $(1+e^x)^2 = 1+2e^x+e^{2x}$
2) Separate the numerator $\frac{1+x}{x^2+1} = \frac{1}{x^2+1}+\frac{x}{x^2+1}$
3) Complete the square $\frac{1}{\sqrt{2x-x^2}} = \frac{1}{\sqrt{1-(x-1)^2}}$
4) Long division $\frac{x^2}{x^2+1} = 1-\frac{1}{x^2+1}$
5) Add and subtract terms in numerator $\frac{2x}{x^2+2x+1} = \frac{2x+2-2}{x^2+2x+1} = \frac{2x+2}{x^2+2x+1} - \frac{2}{x^2+2x+1}$
6) Use trigonometric identities $\cot^2 x = \csc^2 x - 1$
7) Multiply and divide by Pythagorean conjugate

$$\frac{1}{1+\sin x} = \left(\frac{1}{1+\sin x}\right)\left(\frac{1-\sin x}{1-\sin x}\right) = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x} = \sec^2 x - \frac{\sin x}{\cos^2 x}$$

**Class Examples**

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$$

1) Find $\int \frac{x^2}{\sqrt{16-x^6}}dx$

$$\int \frac{x^2 dx}{\sqrt{(4)^2-(x^3)^2}}$$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$dx = \frac{du}{3x^2}$

$$\int \frac{\cancel{x^2}\cancel{dx}}{\sqrt{4^2-(x^3)^2}} \cdot \frac{du}{3\cancel{x^2}}$$

$$\boxed{\frac{1}{3}\arcsin\left(\frac{x^3}{4}\right)+C}$$

2) $\int \frac{1}{1+e^x}dx$

$$\int \frac{1+e^x-e^x}{1+e^x}dx$$

$$\int \frac{1+e^x}{1+e^x} - \frac{e^x}{1+e^x}$$

$$\int 1 - \frac{e^x}{1+e^x}dx$$

$u = 1+e^x$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$$x - \int \frac{\cancel{e^x}}{u} \cdot \frac{du}{\cancel{e^x}}$$

$$\boxed{x - \ln|1+e^x| + C}$$

$$\int u\,d\cancel{v} = \cancel{uv} - \int v\,du$$

3) $\int (\cot x)[\ln(\sin x)]dx$

$u = \ln(\sin x)$

$\frac{du}{dx} = \frac{\cos x}{\sin x}$

$\frac{du}{dx} = \cot x$

$dx = \frac{du}{\cot x}$

$d\cancel{v = \cot x}$

$$\int \cancel{\cot x} \cdot u \cdot \frac{du}{\cancel{\cot x}}$$

$$\frac{u^2}{2}+C$$

$$\boxed{\frac{1}{2}[\ln(\sin x)]^2 + C}$$

4) $\int \tan^2(2x)dx$

$1+\tan^2 u = \sec^2 u$

$$\int \sec^2(2x) - 1\,dx$$

$$\int \sec^2(2x)dx - \int 1\,dx$$

$u = 2x$ $\quad dx = \frac{du}{2}$

$\frac{du}{dx} = 2$

$$\int \sec^2 u \cdot \frac{du}{2}$$

$$\boxed{\frac{1}{2}\tan(2x) - x + C}$$