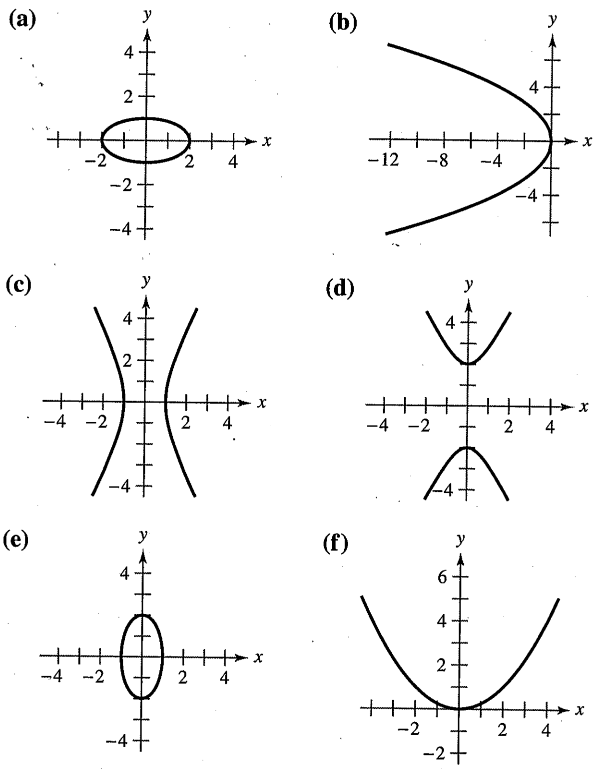
Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Matching **In Exercises 1–6, match the equation with the correct graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]**



1. $4x^2 + y^2 = 4$
2. $4x^2 - y^2 = 4$
3. $y^2 = -4x$
4. $y^2 - 4x^2 = 4$
5. $x^2 + 4y^2 = 4$
6. $x^2 = 4y$

Identifying a Conic **In Exercises 7–14, identify the conic, analyze the equation (center, radius, vertices, foci, eccentricity, directrix, and asymptotes, if possible), and sketch its graph. Use a graphing utility to confirm your results.**

7. $16x^2 + 16y^2 - 16x + 24y - 3 = 0$
8. $y^2 - 12y - 8x + 20 = 0$
9. $3x^2 - 2y^2 + 24x + 12y + 24 = 0$
10. $5x^2 + y^2 - 20x + 19 = 0$
11. $3x^2 + 2y^2 - 12x + 12y + 29 = 0$
12. $12x^2 - 12y^2 - 12x + 24y - 45 = 0$
13. $x^2 - 6x - 8y + 1 = 0$
14. $9x^2 + 25y^2 + 18x - 100y - 116 = 0$

Finding an Equation of a Parabola **In Exercises 15 and 16, find an equation of the parabola.**

15. Vertex: $(0, 2)$
    Directrix: $x = -3$
16. Vertex: $(2, 6)$
    Focus: $(2, 4)$

Finding an Equation of an Ellipse **In Exercises 17–20, find an equation of the ellipse.**

17. Center: $(0, 0)$
    Focus: $(5, 0)$
    Vertex: $(7, 0)$
18. Center: $(0, 0)$
    Major axis: vertical
    Points on the ellipse: $(1, 2), (2, 0)$
19. Vertices: $(3, 1), (3, 7)$
    Eccentricity: $\frac{2}{3}$
20. Foci: $(0, \pm 7)$
    Major axis length: 20

Finding an Equation of a Hyperbola **In Exercises 21–24, find an equation of the hyperbola.**

21. Vertices: $(0, \pm 8)$
    Asymptotes: $y = \pm 2x$
22. Vertices: $(\pm 2, 0)$
    Asymptotes: $y = \pm 32x$
23. Vertices: $(\pm 7, -1)$
    Foci: $(\pm 9, -1)$
24. Center: $(0, 0)$
    Vertex: $(0, 3)$
    Focus: $(0, 6)$

25. **Satellite Antenna** A cross section of a large parabolic antenna is modeled by the graph of

$$y = \frac{x^2}{200}, \quad -100 \le x \le 100.$$

The receiving and transmitting equipment is positioned at the focus.

(a) Find the coordinates of the focus.

(b) Find the surface area of the antenna.

26. **Using an Ellipse** Consider the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1.$

(a) Find the area of the region bounded by the ellipse.

(b) Find the volume of the solid generated by revolving the region about its major axis.

Using Parametric Equations **In Exercises 27–34, sketch the curve represented by the parametric equations (indicate the orientation of the curve), and write the corresponding rectangular equation by eliminating the parameter.**

27. $x = 1 + 8t, \ y = 3 - 4t$
28. $x = t - 6, \ y = t^2$
29. $x = e^t - 1, \ y = e^{3t}$
30. $x = e^{4t}, \ y = t + 4$
31. $x = 6 \cos \theta, \ y = 6 \sin \theta$
32. $x = 2 + 5 \cos t, \ y = 3 + 2 \sin t$
33. $x = 2 + \sec \theta, \ y = 3 + \tan \theta$
34. $x = 5 \sin^3 \theta, \ y = 5 \cos^3 \theta$

Finding Parametric Equations **In Exercises 35 and 36, find two different sets of parametric equations for the rectangular equation.**

35. $y = 4x + 3$
36. $y = x^2 - 2$

37. Rotary Engine The rotary engine was developed by Felix Wankel in the 1950s. It features a rotor that is a modified equilateral triangle. The rotor moves in a chamber that, in two dimensions, is an epitrochoid. Use a graphing utility to graph the chamber modeled by the parametric equations

$x = \cos 3\theta + 5\cos\theta$

and

$y = \sin 3\theta + 5\sin\theta.$

38. Serpentine Curve Consider the parametric equations $x = 2\cot\theta$ and $y = 4\sin\theta\cos\theta,\ 0 < \theta < \pi.$

(a) Use a graphing utility to graph the curve.

(b) Eliminate the parameter to show that the rectangular equation of the serpentine curve is $(4 + x^2)y = 8x.$

Finding Slope and Concavity **In Exercises 39–46, find dy/dx and d^2y/dx^2, and find the slope and concavity (if possible) at the given value of the parameter.**

Parametric Equations	Parameter
39. $x = 2 + 5t,\quad y = 1 - 4t$	$t = 3$
40. $x = t - 6,\quad y = t^2$	$t = 5$
41. $x = \dfrac{1}{t},\quad y = 2t + 3$	$t = -1$
42. $x = \dfrac{1}{t},\quad y = t^2$	$t = -2$
43. $x = 5 + \cos\theta,\quad y = 3 + 4\sin\theta$	$\theta = \dfrac{\pi}{6}$
44. $x = 10\cos\theta,\quad y = 10\sin\theta$	$\theta = \dfrac{\pi}{4}$
45. $x = \cos^3\theta,\quad y = 4\sin^3\theta$	$\theta = \dfrac{\pi}{3}$
46. $x = e^t,\quad y = e^{-t}$	$t = 1$

Finding an Equation of a Tangent Line **In Exercises 47 and 48, (a) use a graphing utility to graph the curve represented by the parametric equations, (b) use a graphing utility to find $dx/d\theta$, $dy/d\theta$, and dy/dx at the given value of the parameter, (c) find an equation of the tangent line to the curve at the given value of the parameter, and (d) use a graphing utility to graph the curve and the tangent line from part (c).**

Parametric Equations	Parameter
47. $x = \cot\theta,\quad y = \sin 2\theta$	$\theta = \dfrac{\pi}{6}$
48. $x = \dfrac{1}{4}\tan\theta,\quad y = 6\sin\theta$	$\theta = \dfrac{\pi}{3}$

Horizontal and Vertical Tangency **In Exercises 49–52, find all points (if any) of horizontal and vertical tangency to the curve. Use a graphing utility to confirm your results.**

49. $x = 5 - t,\quad y = 2t^2$

50. $x = t + 2,\quad y = t^3 - 2t$

51. $x = 2 + 2\sin\theta,\quad y = 1 + \cos\theta$

52. $x = 2 - 2\cos\theta,\quad y = 2\sin 2\theta$

Arc Length **In Exercises 53 and 54, find the arc length of the curve on the given interval.**

Parametric Equations	Interval
53. $x = t^2 + 1,\quad y = 4t^3 + 3$	$0 \le t \le 2$
54. $x = 6\cos\theta,\quad y = 6\sin\theta$	$0 \le \theta \le \pi$

Surface Area **In Exercises 55 and 56, find the area of the surface generated by revolving the curve about (a) the x-axis and (b) the y-axis.**

55. $x = t,\quad y = 3t,\quad 0 \le t \le 2$

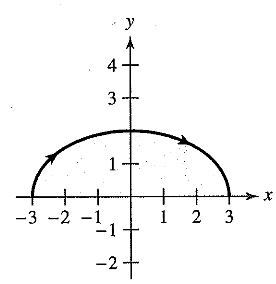
56. $x = 2\cos\theta,\quad y = 2\sin\theta,\quad 0 \le \theta \le \dfrac{\pi}{2}$

Area **In Exercises 57 and 58, find the area of the region.**

57. $x = 3\sin\theta$

$y = 2\cos\theta$

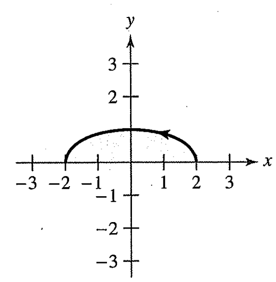
$-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$



58. $x = 2\cos\theta$

$y = \sin\theta$

$0 \le \theta \le \pi$



Polar-to-Rectangular Conversion **In Exercises 59–62, plot the point in polar coordinates and find the corresponding rectangular coordinates of the point.**

59. $\left(5, \dfrac{3\pi}{2}\right)$

60. $\left(-6, \dfrac{7\pi}{6}\right)$

61. $\left(\sqrt{3}, 1.56\right)$

62. $(-2, -2.45)$

Rectangular-to-Polar Conversion **In Exercises 63–66, the rectangular coordinates of a point are given. Plot the point and find *two* sets of polar coordinates of the point for $0 \le \theta < 2\pi$.**

63. $(4, -4)$

64. $(0, -7)$

65. $(-1, 3)$

66. $\left(-\sqrt{3}, -\sqrt{3}\right)$

Rectangular-to-Polar Conversion **In Exercises 67–72, convert the rectangular equation to polar form and sketch its graph.**

67. $x^2 + y^2 = 25$

68. $x^2 - y^2 = 4$

69. $y = 9$

70. $x = 6$

71. $x^2 = 4y$

72. $x^2 + y^2 - 4x = 0$

Polar-to-Rectangular Conversion **In Exercises 73–78, convert the polar equation to rectangular form and sketch its graph.**

73. $r = 3\cos\theta$

74. $r = 10$

75. $r = 6\sin\theta$

76. $r = 3\csc\theta$

77. $r = -2\sec\theta\tan\theta$

78. $\theta = \dfrac{3\pi}{4}$

Graphing a Polar Equation **In Exercises 79–82, use a graphing utility to graph the polar equation.**

79. $r = \dfrac{3}{\cos(\theta - \pi/4)}$

80. $r = 2\sin\theta\cos^2\theta$

81. $r = 4\cos 2\theta\sec\theta$

82. $r = 4(\sec\theta - \cos\theta)$

Horizontal and Vertical Tangency **In Exercises 83 and 84, find the points of horizontal and vertical tangency (if any) to the polar curve.**

83. $r = 1 - \cos\theta$

84. $r = 3\tan\theta$

Tangent Lines at the Pole **In Exercises 85 and 86, sketch a graph of the polar equation and find the tangents at the pole.**

85. $r = 4\sin 3\theta$

86. $r = 3\cos 4\theta$

Sketching a Polar Graph **In Exercises 87–96, sketch a graph of the polar equation.**

87. $r = 6$

88. $\theta = \dfrac{\pi}{10}$

89. $r = -\sec\theta$

90. $r = 5\csc\theta$

91. $r^2 = 4\sin^2 2\theta$

92. $r = 3 - 4\cos\theta$

93. $r = 4 - 3\cos\theta$

94. $r = 4\theta$

95. $r = -3\cos 2\theta$

96. $r = \cos 5\theta$

Finding the Area of a Polar Region **In Exercises 97–102, find the area of the region.**

97. One petal of $r = 3\cos 5\theta$

98. One petal of $r = 2\sin 6\theta$

99. Interior of $r = 2 + \cos\theta$

100. Interior of $r = 5(1 - \sin\theta)$

101. Interior of $r^2 = 4\sin 2\theta$

102. Common interior of $r = 4\cos\theta$ and $r = 2$

Finding the Area of a Polar Region **In Exercises 103–106, use a graphing utility to graph the polar equation. Find the area of the given region analytically.**

103. Inner loop of $r = 3 - 6\cos\theta$

104. Inner loop of $r = 2 + 4\sin\theta$

105. Between the loops of $r = 3 - 6\cos\theta$

106. Between the loops of $r = 2 + 4\sin\theta$

Finding Points of Intersection **In Exercises 107 and 108, find the points of intersection of the graphs of the equations.**

107. $r = 1 - \cos\theta$
$r = 1 + \sin\theta$

108. $r = 1 + \sin\theta$
$r = 3\sin\theta$

Finding the Arc Length of a Polar Curve **In Exercises 109 and 110, find the length of the curve over the given interval.**

	Polar Equation	Interval
109.	$r = 5\cos\theta$	$\dfrac{\pi}{2} \le \theta \le \pi$
110.	$r = 3(1 - \cos\theta)$	$0 \le \theta \le \pi$

Finding the Area of a Surface of Revolution **In Exercises 111 and 112, write an integral that represents the area of the surface formed by revolving the curve about the given line. Use the integration capabilities of a graphing utility to approximate the integral accurate to two decimal places.**

	Polar Equation	Interval	Axis of Revolution
111.	$r = 1 + 4\cos\theta$	$0 \le \theta \le \dfrac{\pi}{2}$	Polar axis
112.	$r = 2\sin\theta$	$0 \le \theta \le \dfrac{\pi}{2}$	$\theta = \dfrac{\pi}{2}$

Sketching and Identifying a Conic **In Exercises 113–118, find the eccentricity and the distance from the pole to the directrix of the conic. Then sketch and identify the graph. Use a graphing utility to confirm your results.**

113. $r = \dfrac{6}{1 - \sin\theta}$

114. $r = \dfrac{2}{1 + \cos\theta}$

115. $r = \dfrac{6}{3 + 2\cos\theta}$

116. $r = \dfrac{4}{5 - 3\sin\theta}$

117. $r = \dfrac{4}{2 - 3\sin\theta}$

118. $r = \dfrac{8}{2 - 5\cos\theta}$

Finding a Polar Equation **In Exercises 119–124, find a polar equation for the conic with its focus at the pole. (For convenience, the equation for the directrix is given in rectangular form.)**

	Conic	Eccentricity	Directrix
119.	Parabola	$e = 1$	$x = 4$
120.	Ellipse	$e = \dfrac{3}{4}$	$y = -2$
121.	Hyperbola	$e = 3$	$y = 3$

	Conic	Vertex or Vertices
122.	Parabola	$\left(2, \dfrac{\pi}{2}\right)$
123.	Ellipse	$(5, 0), (1, \pi)$
124.	Hyperbola	$(1, 0), (7, 0)$