Review Exercises for Chapter 11

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1 and 2, let $\mathbf{u} = \overrightarrow{PQ}$ and $\mathbf{v} = \overrightarrow{PR}$, and find (a) the component forms of u and v, (b) the magnitude of v, and (c) $2\mathbf{u} + \mathbf{v}$.

1. $P = (1, 2), Q = (4, 1), R = (5, 4)$
2. $P = (-2, -1), Q = (5, -1), R = (2, 4)$

In Exercises 3 and 4, find the component form of v given its magnitude and the angle it makes with the positive x-axis.

3. $\|\mathbf{v}\| = 8,\ \theta = 120°$
4. $\|\mathbf{v}\| = \frac{1}{2},\ \theta = 225°$

5. Find the coordinates of the point in the xy-plane four units to the right of the xz-plane and five units behind the yz-plane.
6. Find the coordinates of the point located on the y-axis and seven units to the left of the xz-plane.

In Exercises 7 and 8, determine the location of a point (x, y, z) that satisfies the condition.

7. $yz > 0$
8. $xy < 0$

In Exercises 9 and 10, find the standard equation of the sphere.

9. Center: $(3, -2, 6)$; Diameter: 15
10. Endpoints of a diameter: $(0, 0, 4), (4, 6, 0)$

In Exercises 11 and 12, complete the square to write the equation of the sphere in standard form. Find the center and radius.

11. $x^2 + y^2 + z^2 - 4x - 6y + 4 = 0$
12. $x^2 + y^2 + z^2 - 10x + 6y - 4z + 34 = 0$

In Exercises 13 and 14, the initial and terminal points of a vector are given. Sketch the directed line segment and find the component form of the vector.

13. Initial point: $(2, -1, 3)$
    Terminal point: $(4, 4, -7)$
14. Initial point: $(6, 2, 0)$
    Terminal point: $(3, -3, 8)$

In Exercises 15 and 16, use vectors to determine whether the points are collinear.

15. $(3, 4, -1), (-1, 6, 9), (5, 3, -6)$
16. $(5, -4, 7), (8, -5, 5), (11, 6, 3)$

17. Find a unit vector in the direction of $\mathbf{u} = \langle 2, 3, 5\rangle$.
18. Find the vector $\mathbf{v}$ of magnitude 8 in the direction $\langle 6, -3, 2\rangle$.

In Exercises 19 and 20, let $\mathbf{u} = \overrightarrow{PQ}$ and $\mathbf{v} = \overrightarrow{PR}$, and find (a) the component forms of u and v, (b) $\mathbf{u} \cdot \mathbf{v}$, and (c) $\mathbf{v} \cdot \mathbf{v}$.

19. $P = (5, 0, 0),\ Q = (4, 4, 0), R = (2, 0, 6)$
20. $P = (2, -1, 3),\ Q = (0, 5, 1),\ R = (5, 5, 0)$

In Exercises 21 and 22, determine whether u and v are orthogonal, parallel, or neither.

21. $\mathbf{u} = \langle 7, -2, 3\rangle$
    $\mathbf{v} = \langle -1, 4, 5\rangle$
22. $\mathbf{u} = \langle -4, 3, -6\rangle$
    $\mathbf{v} = \langle 16, -12, 24\rangle$

In Exercises 23–26, find the angle θ between the vectors.

23. $\mathbf{u} = 5[\cos(3\pi/4)\mathbf{i} + \sin(3\pi/4)\mathbf{j}]$
    $\mathbf{v} = 2[\cos(2\pi/3)\mathbf{i} + \sin(2\pi/3)\mathbf{j}]$
24. $\mathbf{u} = \langle 4, -1, 5\rangle,\quad \mathbf{v} = \langle 3, 2, -2\rangle$
25. $\mathbf{u} = \langle 10, -5, 15\rangle,\quad \mathbf{v} = \langle -2, 1, -3\rangle$
26. $\mathbf{u} = \langle 1, 0, -3\rangle,\quad \mathbf{v} = \langle 2, -2, 1\rangle$

27. Find two vectors in opposite directions that are orthogonal to the vector $\mathbf{u} = \langle 5, 6, -3\rangle$.
28. ***Work*** An object is pulled 8 feet across a floor using a force of 75 pounds. The direction of the force is 30° above the horizontal. Find the work done.

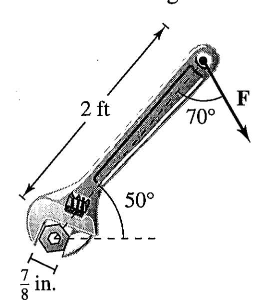
In Exercises 29–32, let $\mathbf{u} = \langle 3, -2, 1\rangle$, $\mathbf{v} = \langle 2, -4, -3\rangle$, and $\mathbf{w} = \langle -1, 2, 2\rangle$.

29. Show that $\mathbf{u} \cdot \mathbf{u} = \|\mathbf{u}\|^2$.
30. Find the angle between **u** and **v**.
31. Determine the projection of **w** onto **u**.
32. Find the work done in moving an object along the vector **u** if the applied force is **w**.

In Exercises 33–38, let $\mathbf{u} = \langle 3, -2, 1\rangle$, $\mathbf{v} = \langle 2, -4, -3\rangle$, and $\mathbf{w} = \langle -1, 2, 2\rangle$.

33. Determine a unit vector perpendicular to the plane containing **v** and **w**.
34. Show that $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$.
35. Find the volume of the solid whose edges are **u**, **v**, and **w**.
36. Show that $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$.
37. Find the area of the parallelogram with adjacent sides **u** and **v**.
38. Find the area of the triangle with adjacent sides **v** and **w**.

39. ***Torque*** The specifications for a tractor state that the torque on a bolt with head size $\frac{7}{8}$ inch cannot exceed 200 foot-pounds. Determine the maximum force $\|\mathbf{F}\|$ that can be applied to the wrench in the figure.



40. ***Volume*** Use the triple scalar product to find the volume of the parallelepiped having adjacent edges $\mathbf{u} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 2\mathbf{j} + \mathbf{k}$, and $\mathbf{w} = -\mathbf{j} + 2\mathbf{k}$.

In Exercises 41 and 42, find sets of (a) parametric equations and (b) symmetric equations of the line through the two points. (For each line, write the direction numbers as integers.)

41. $(3, 0, 2)$, $(9, 11, 6)$ **42.** $(-1, 4, 3)$, $(8, 10, 5)$

In Exercises 43–46, find (a) a set of parametric equations and (b) a set of symmetric equations for the line.

43. The line passes through the point $(1, 2, 3)$ and is perpendicular to the xz-plane.

44. The line passes through the point $(1, 2, 3)$ and is parallel to the line given by $x = y = z$.

45. The intersection of the planes $3x - 3y - 7z = -4$ and $x - y + 2z = 3$

46. The line passes through the point $(0, 1, 4)$ and is perpendicular to $\mathbf{u} = \langle 2, -5, 1\rangle$ and $\mathbf{v} = \langle -3, 1, 4\rangle$.

In Exercises 47–50, find an equation of the plane.

47. The plane passes through

$(-3, -4, 2)$, $(-3, 4, 1)$, and $(1, 1, -2)$.

48. The plane passes through the point $(-2, 3, 1)$ and is perpendicular to $\mathbf{n} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$.

49. The plane contains the lines given by

$$\frac{x-1}{-2} = y = z + 1$$

and

$$\frac{x+1}{-2} = y - 1 = z - 2.$$

50. The plane passes through the points $(5, 1, 3)$ and $(2, -2, 1)$ and is perpendicular to the plane $2x + y - z = 4$.

51. Find the distance between the point $(1, 0, 2)$ and the plane $2x - 3y + 6z = 6$.

52. Find the distance between the point $(3, -2, 4)$ and the plane $2x - 5y + z = 10$.

53. Find the distance between the planes $5x - 3y + z = 2$ and $5x - 3y + z = -3$.

54. Find the distance between the point $(-5, 1, 3)$ and the line given by $x = 1 + t$, $y = 3 - 2t$, and $z = 5 - t$.

In Exercises 55–64, describe and sketch the surface.

55. $x + 2y + 3z = 6$

56. $y = z^2$

57. $y = \frac{1}{2}z$

58. $y = \cos z$

59. $\dfrac{x^2}{16} + \dfrac{y^2}{9} + z^2 = 1$

60. $16x^2 + 16y^2 - 9z^2 = 0$

61. $\dfrac{x^2}{16} - \dfrac{y^2}{9} + z^2 = -1$

62. $\dfrac{x^2}{25} + \dfrac{y^2}{4} - \dfrac{z^2}{100} = 1$

63. $x^2 + z^2 = 4$

64. $y^2 + z^2 = 16$

65. Find an equation of a generating curve of the surface of revolution $y^2 + z^2 - 4x = 0$.

66. Find an equation for the surface of revolution generated by revolving the curve $z^2 = 2y$ in the yz-plane about the y-axis.

In Exercises 67 and 68, convert the point from rectangular coordinates to (a) cylindrical coordinates and (b) spherical coordinates.

67. $(-2\sqrt{2}, 2\sqrt{2}, 2)$ **68.** $\left(\dfrac{\sqrt{3}}{4}, \dfrac{3}{4}, \dfrac{3\sqrt{3}}{2}\right)$

In Exercises 69 and 70, convert the point from cylindrical coordinates to spherical coordinates.

69. $\left(100, -\dfrac{\pi}{6}, 50\right)$ **70.** $\left(81, -\dfrac{5\pi}{6}, 27\sqrt{3}\right)$

In Exercises 71 and 72, convert the point from spherical coordinates to cylindrical coordinates.

71. $\left(25, -\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$

72. $\left(12, -\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$

In Exercises 73 and 74, convert the rectangular equation to an equation in (a) cylindrical coordinates and (b) spherical coordinates.

73. $x^2 - y^2 = 2z$

74. $x^2 + y^2 + z^2 = 16$

In Exercises 75 and 76, find an equation in rectangular coordinates for the equation given in cylindrical coordinates, and sketch its graph.

75. $r = 4 \sin \theta$ **76.** $z = 4$

In Exercises 77 and 78, find an equation in rectangular coordinates for the equation given in spherical coordinates, and sketch its graph.

77. $\theta = \dfrac{\pi}{4}$ **78.** $\rho = 2 \cos \theta$