Limits Chapter 1 Morning Test Review #3

$\frac{\sqrt{2}-\sqrt{2}}{0} \neq \frac{0}{0} \rightarrow$ Limit does exist

Key

1.

$$\lim_{x\to 0^-} \frac{\sqrt{2+x}-\sqrt{2}}{x}$$

$$\lim_{x\to 0^-} \frac{(\sqrt{2+x}-\sqrt{2})}{(x)} \cdot \frac{(\sqrt{2+x}+\sqrt{2})}{(\sqrt{2+x}+\sqrt{2})}$$

$$\lim_{x\to 0^-} \frac{\cancel{2}+x\cancel{-2}\ 1}{\cancel{x}(\sqrt{2+x}+\sqrt{2})} = \frac{1}{\sqrt{2}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$$

2.

For $x \neq 4$, the function h(x) is equal to $\frac{x^2+x-20}{x-4}$. What value should be assigned to h(4) to make h(x) continuous at x = 4?

*continuity conditions

i) $h(4) = k$

ii) $\lim_{x\to 4} \frac{x^2+x-20}{x-4}$ $\frac{0}{0}$ $\lim_{x\to 4} \frac{(x+5)\cancel{(x-4)}}{\cancel{(x-4)}} \rightarrow 4+5 = 9$

iii) $h(4) = \lim_{x\to 4} h(x) \rightarrow \boxed{k=9}$

3.

Given $f(x)=\begin{cases} x^2-2, & \text{if } x\le 0 \\ 3x+k, & \text{if } x>0 \end{cases}$ ← c = 0

Find the value for the constant k that will make the function continuous at x = 0.

i) $f(0) = 0^2-2 = -2$

ii) $\lim_{x\to 0^-} x^2-2 = -2$

$\lim_{x\to 0^+} 3x+k = 3(0)+k = k$

$\boxed{k=-2}$

4. OMIT

Determine the values of a and b so that $f(x)$ is everywhere continuous. Justify your answer.

$$f(x)=\begin{cases} 5bx-6a & , \text{if } x<-2 \\ -3b-4ax & , \text{if } x=-2 \\ 5x-1 & , \text{if } x>-2 \end{cases}$$

Ex. 5b

x-values target y-value

$f(x) = x^2-5$

$[-1, 3]$ $f(c) = 1$

$f(x)$ continuous $[-1,3]$

$f(-1) = (-1)^2-5 = -4$ y-values

$f(3) = 3^2-5 = 4$

By IVT, since $f(-1) = -4 < 1 < 4 = f(3)$ on $[-1,3]$, $f(c) = 1$

5.

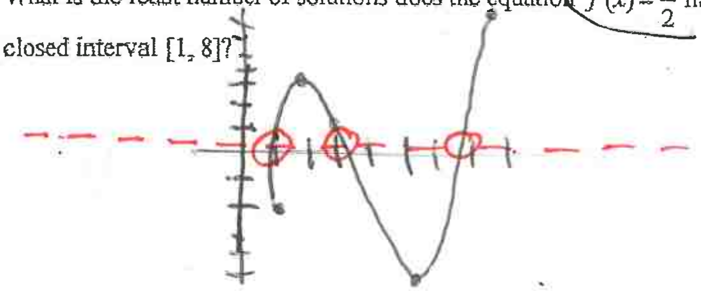
Let f be a continuous function. Selected values of f are given in the table below.

(IVT) how many times

x	1	2	3	5	8
$f(x)$	−2	3	1	−5	7

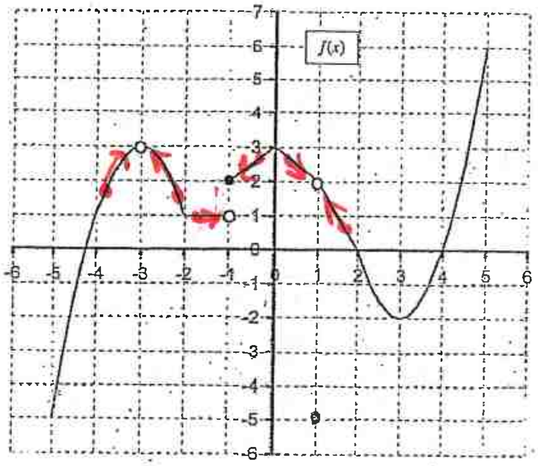
What is the least number of solutions does the equation $f(x) = \frac{1}{2}$ have on the closed interval [1, 8]?

$y = \frac{1}{2}$



3 times due to IVT

6.



$\lim_{x \to -3^-} f(x) = 3$ $\lim_{x \to -3^+} f(x) =$

a) $\lim_{x \to -3^-} f(x) =$ 3 b) $\lim_{x \to -3^+} f(x) =$ 3 c) $\lim_{x \to -3} f(x) =$ 3

d) $\lim_{x \to -1^-} f(x) =$ 1 e) $\lim_{x \to -1^+} f(x) =$ 2 f) $\lim_{x \to -1} f(x) =$ dne

Use continuity conditions to justify if graph is continuous at x = 1. Determine the type of discontinuity.

i) $f(1) = -5$

ii) $\lim_{x \to 1} f(x) = 1$

iii) $f(1) \neq \lim_{x \to 1} f(x)$

Removable discontinuity at $x = 1$

7. Find the value of K that makes f(x) continuous for all real-numbers if:

graph →
point →

$$g(x) = \begin{cases} \dfrac{x^2 + x - 6}{x+3}, & x \neq -3 \\ k, & x = -3 \end{cases}$$

i) $g(-3) = K$

ii) $\lim_{x \to -3} \dfrac{x^2 + x - 6}{x+3} \to \dfrac{0}{0}$ $\lim_{x \to -3} \dfrac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}} = -3 - 2 = -5$

iii) $g(-3) = \lim_{x \to -3} g(x)$

$\boxed{K = -5}$

8.

Find $A = \lim_{x \to 3^+} \dfrac{x(x+1)}{3-x}$ and $B = \lim_{x \to \infty} \dfrac{(x^2+2)(3x^2-5)}{x^4+6}$.

a) $A = -\infty$ and $B = -\frac{5}{3}$

b) $A = 0$ and $B = 0$

c) $A = \infty$ and $B = \infty$

d) $A = -\infty$ and $B = 3$

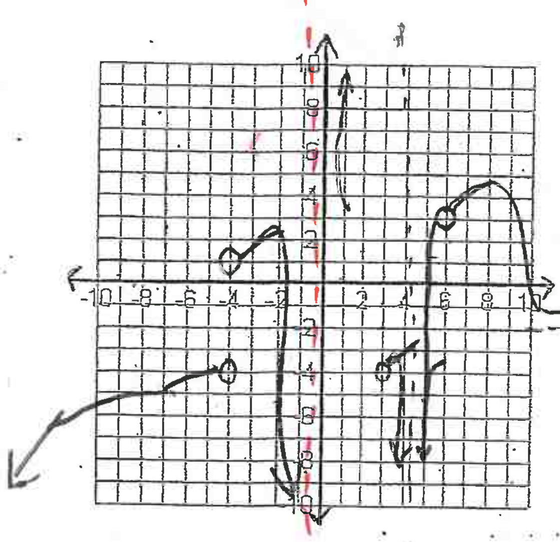
* Work shown on next page

9. Evaluate $\lim_{x \to 0} \dfrac{3 - \sqrt{x+9}}{x}$ $\to \dfrac{3 - \sqrt{9}}{0} = \dfrac{0}{0}$

$\lim_{x \to 0} \left(\dfrac{3 - \sqrt{x+9}}{x}\right)\left(\dfrac{3 + \sqrt{x+9}}{3 + \sqrt{x+9}}\right)$

$\cancel{9} - x - \cancel{9}$

$\lim_{x \to 0} \dfrac{9 - (x+9)}{x(3 + \sqrt{x+9})} \to \lim_{x \to 0} \dfrac{-\cancel{x}}{\cancel{x}(3 + \sqrt{x+9})} (-1) = \boxed{\dfrac{-1}{6}}$



9) Sketch graph satisfying the given values

a) $\lim_{x \to -\infty} h(x) = -\infty$

b) $\lim_{x \to -4^-} h(x) = -3$

c) $\lim_{x \to -4^+} h(x) = 1$

d) $h(6) =$ undefined

e) $\lim_{x \to 0^-} h(x) = -\infty$

f) $\lim_{x \to 0} h(x) =$ DNE

g) $\lim_{x \to 3^+} h(x) = -4$

h) $\lim_{x \to 4} h(x) = -\infty$ (VA)

i) $\lim_{x \to 6} h(x) = 3$

j) $\lim_{x \to \infty} h(x) = -2$ HA

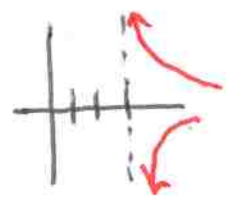
2) * step thru continuity conditions

i) $h(4) = K$

ii) $\displaystyle \lim_{x\to 4} \frac{x^2+x-20}{x-4} \xrightarrow{0/0} \lim_{x\to 4} \frac{(x+5)\cancel{(x-4)}}{\cancel{(x-4)}} = 4+5 = \boxed{9}$

iii) $\displaystyle h(4) = \lim_{x\to 4} f(x)$ $\boxed{K=9}$

8) $\displaystyle \lim_{x\to 3^{+}} \frac{x(x+1)}{3-x} \to \frac{3(3+1)}{3-3} = \frac{12}{0} \to VA \to$ limit dne



test $x=3.1$ | $\displaystyle \frac{3.1(3.1+1)}{3-3.1} = \frac{+}{-} = \boxed{-\infty}$

6) $\displaystyle \lim_{x\to\infty} \frac{(x^2+2)(3x^2-5)}{x^4+6} \to \lim_{x\to\infty} \frac{3x^4+1x^2-10}{1x^4+6} = \boxed{3}$

10) $\lim_{x\to\infty} \frac{\sqrt{x+1000}}{3^x}$

Comparative growth rates

L < R < P < E

$\frac{\text{Rad}}{\text{Exponent}} = \boxed{0}$

$\lim_{x\to\infty} \frac{3^x}{\sqrt{x+1000}}$ $\frac{\text{Exp}}{\text{Rad}}$ $\nearrow \boxed{+\infty}$

$\searrow -\infty$

test

$x=100$