Limits Chapter 1 Morning Test Review #3

1. $$\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$$

2. For $x \neq 4$, the function h(x) is equal to $\frac{x^2+x-20}{x-4}$. What value should be assigned to h(4) to make h(x) continuous at $x = 4$?

3. Given $f(x)=\begin{cases} x^2-2, & \text{if } x \leq 0 \\ 3x+k, & \text{if } x>0 \end{cases}$

Find the value for the constant k that will make the function continuous at x = 0.

4. Determine the values of a and b so that $f(x)$ is everywhere continuous. Justify your answer.

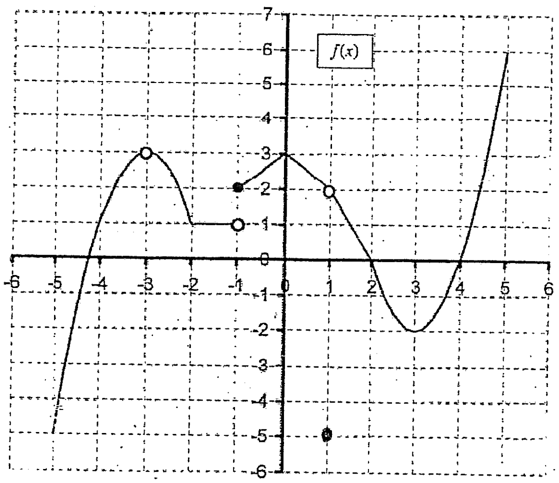
$$f(x)=\begin{cases} 5bx-6a & , \text{ if } x<-2 \\ -3b-4ax & , \text{ if } x=-2 \\ 5x-1 & , \text{ if } x>-2 \end{cases}$$

5. Let f be a continuous function. Selected values of f are given in the table below.

x	1	2	3	5	8
$f(x)$	-2	3	1	-5	7

What is the least number of solutions does the equation $f(x)=\frac{1}{2}$ have on the closed interval [1, 8]?

6.



$\lim\limits_{x \to -3^-} f(x) =$ b) $\lim\limits_{x \to -3^+} f(x) =$ c) $\lim\limits_{x \to -3} f(x) =$

$\lim\limits_{x \to -1^-} f(x) =$ e) $\lim\limits_{x \to -1^+} f(x) =$ f) $\lim\limits_{x \to -1} f(x) =$

Use continuity conditions to justify if graph is continuous at x = 1. Determine the type of discontinuity.

7. Find the value of **K** that makes f(x) continuous for all real numbers if:

$$g(x) = \begin{cases} \dfrac{x^2 + x - 6}{x + 3}, & x \neq -3 \\ k, & x = -3 \end{cases}$$

8.

Find $A = \lim\limits_{x \to 3^+} \dfrac{x(x+1)}{3-x}$ and $B = \lim\limits_{x \to \infty} \dfrac{(x^2+2)(3x^2-5)}{x^4+6}$.

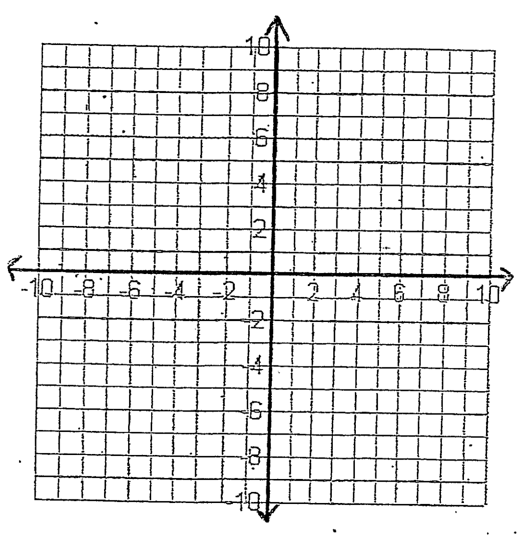
a) $A = -\infty$ and $B = -\dfrac{5}{3}$

b) $A = 0$ and $B = 0$

c) $A = \infty$ and $B = \infty$

d) $A = -\infty$ and $B = 3$

9. Evaluate $\lim\limits_{x \to 0} \dfrac{3 - \sqrt{x+9}}{x}$



9) Sketch graph satisfying the given values

a) $\lim\limits_{x \to -\infty} h(x) = -\infty$

b) $\lim\limits_{x \to -4^-} h(x) = -3$

c) $\lim\limits_{x \to -4^+} h(x) = 1$

d) $h(6) =$ undefined

e) $\lim\limits_{x \to 0^-} h(x) = -\infty$

f) $\lim\limits_{x \to 0} h(x) =$ DNE

g) $\lim\limits_{x \to 3^+} h(x) = -4$

h) $\lim\limits_{x \to 4} h(x) = -\infty$

i) $\lim\limits_{x \to 6} h(x) = 3$

j) $\lim\limits_{x \to \infty} h(x) = -2$

Key

Limits Chapter 1 Morning Test Review #3

1.

$$\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$$

$$\lim_{x \to 0} \frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} = \frac{1}{\sqrt{2+0}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$$

2.

For $x \neq 4$, the function h(x) is equal to $\frac{x^2+x-20}{x-4}$. What value should be assigned to h(4) to make h(x) continuous at x = 4?

$$\lim_{x \to 4} \frac{x^2+x-20}{x-4} = \lim_{x \to 4} \frac{(x+5)\cancel{(x-4)}}{\cancel{(x-4)}} = 9$$

y-value of the hole in graph

Define $h(4)=9$ to make graph continuous.

3.

Given $f(x)=\begin{cases} x^2-2, & \text{if } x \le 0 \\ 3x+k, & \text{if } x > 0 \end{cases}$

Find the value for the constant k that will make the function continuous at x = 0.

*set functions equal to each other at $x=0$

$x^2-2 = 3x+k$ at $x=0$

$0-2 = 0+k$

$\boxed{-2=k}$

$f(x) = \begin{cases} x^2-2, & x \le 0 \\ 3x-2, & x > 0 \end{cases}$

4. (OMIT)

Determine the values of a and b so that $f(x)$ is everywhere continuous. Justify your answer.

$$f(x)=\begin{cases} 5bx-6a & , \text{ if } x<-2 \\ -3b-4ax & , \text{ if } x=-2 \\ 5x-1 & , \text{ if } x>-2 \end{cases}$$

$5bx-6a=-3b-4ax$ at $x=2$

$5(2)b-6a=-3b-8a$

$10b-6a=-3b-8a$

$13b+2a=0$

$-4(13b+2a=0)$ $\quad -52b-8a=0$

$-3b+8a=-11$ $\quad -3b+8a=-11$

$-55b=-11$

$b=\frac{-11}{-55}$

$\boxed{b=\frac{1}{5}}$

$-3b-4ax=5x-1$ at $x=-2$

$-3b+8a=-10-1$

$-3b+8a=-11$

$-3(\frac{1}{5})+8a=-11$

$8a=\frac{+3}{5}-\frac{55}{5}$

$8a=\frac{-52}{5}$

$a=\frac{-52}{5}\cdot\frac{1}{8}=\frac{-52}{40}$

$\boxed{a=\frac{-13}{10}}$

5.

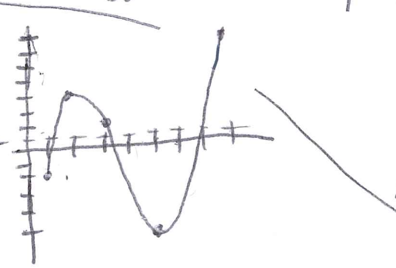
Let f be a continuous function. Selected values of f are given in the table below.

x	1	2	3	5	8
$f(x)$	-2	3	1	-5	7

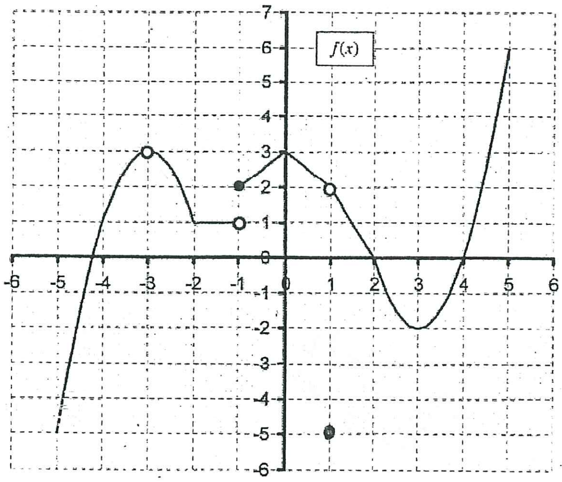
What is the least number of solutions does the equation $f(x)=\frac{1}{2}$ have on the closed interval [1, 8]?

$\boxed{\text{3 times}}$

Due to IVT



6.



$\lim_{x\to-3^-} f(x)=3$ b) $\lim_{x\to-3^+} f(x)=3$ c) $\lim_{x\to-3} f(x)=3$

$\lim_{x\to-1^-} f(x)=1$ e) $\lim_{x\to-1^+} f(x)=2$ f) $\lim_{x\to-1} f(x)=$ DNE

Use continuity conditions to justify if graph is continuous at x = 1. Determine the type of discontinuity.

i) $f(1)=-5$

ii) $\lim_{x\to1} f(x)=2$

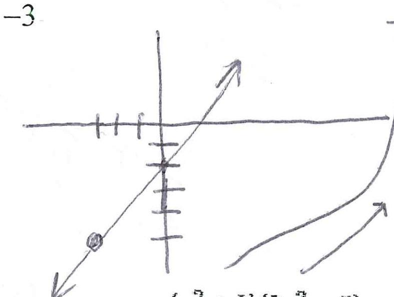
iii) $f(1)\neq\lim_{x\to1} f(x)$, removable discontinuity at $x=1$

7. Find the value of k that makes f(x) continuous for all real numbers if:

$$g(x)=\begin{cases}\dfrac{x^2+x-6}{x+3}, & x\neq-3\\ k, & x=-3\end{cases}$$

$\lim_{x\to-3}\dfrac{x^2+x-6}{x+3}=\lim_{x\to-3}\dfrac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}}=-5$ ← y-value of the location of hole on graph

$\boxed{k=-5}$



$\lim_{x\to\infty}\dfrac{3x^4-5x^2+6x^2-10}{x^4+6}=\dfrac{3}{1}=\boxed{3}$

8.

Find $A=\lim_{x\to3^+}\dfrac{x(x+1)}{3-x}$ and $B=\lim_{x\to\infty}\dfrac{(x^2+2)(3x^2-5)}{x^4+6}$.

a) $A=-\infty$ and $B=-\dfrac{5}{3}$

b) $A=0$ and $B=0$

c) $A=\infty$ and $B=\infty$

d) $A=-\infty$ and $B=3$

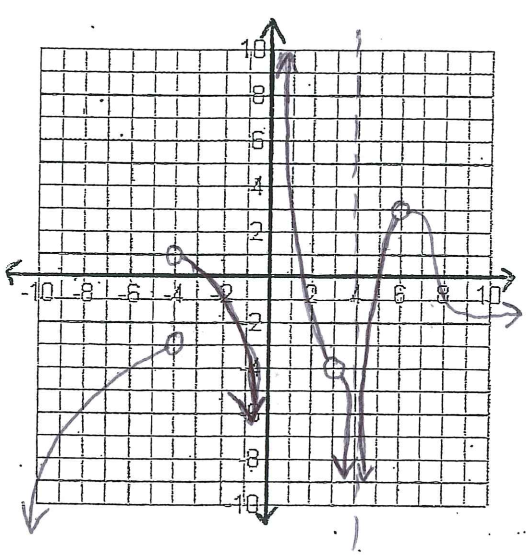
$\lim_{x\to3^+}\dfrac{x(x+1)}{3-x}=\dfrac{12}{0}$

$\lim_{x\to3^+}\dfrac{(3.1)(3.1+1)}{3-3.1}=\dfrac{+}{-}=\boxed{-\infty}$

9. Evaluate $\lim_{x\to0}\dfrac{3-\sqrt{x+9}}{x}=\dfrac{0}{0}$

$\lim_{x\to0}\dfrac{3-\sqrt{x+9}}{x}\cdot\dfrac{3+\sqrt{x+9}}{3+\sqrt{x+9}}=\dfrac{9-(x+9)}{x(3+\sqrt{x+9})}$

$\lim_{x\to0}\dfrac{-x}{x(3+\sqrt{x+9})}=\dfrac{-1}{3+\sqrt{0+9}}=\boxed{\dfrac{-1}{6}}$



9) Sketch graph satisfying the given values

✓a) $\lim_{x\to-\infty} h(x)=-\infty$

✓b) $\lim_{x\to-4^-} h(x)=-3$

✓c) $\lim_{x\to-4^+} h(x)=1$

✓d) $h(6)=$ undefined

✓e) $\lim_{x\to0^-} h(x)=-\infty$

✓f) $\lim_{x\to0} h(x)=$ DNE

✓g) $\lim_{x\to3^+} h(x)=-4$

✓h) $\lim_{x\to4} h(x)=-\infty$

i) $\lim_{x\to6} h(x)=3$

j) $\lim_{x\to\infty} h(x)=-2$