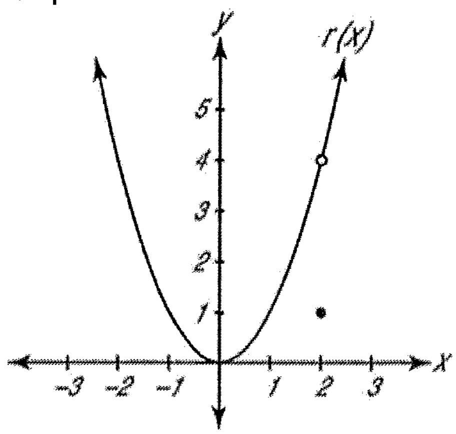
Calculus AB Ch. 1.2 Notes on Limits

Definition: **The Limit** is the y-value that a function or graph approaches as the x-value moves closer to a given constant

Function Value is finding the location of the y-value of the graph at a specific x-value.

Example 1:



* The y-value of the graph when x =2 is 1"

Notation:

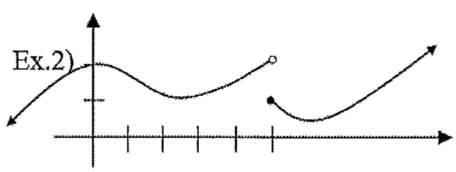
* "The Limit of r(x) as x approaches 2 is 4 "

Notation:

*In order for a limit to exist, the graph MUST approach the same **Real Number** y-value from both sides of the target x-value constant.

Examples where the Limit does not exist:

Ex.2)

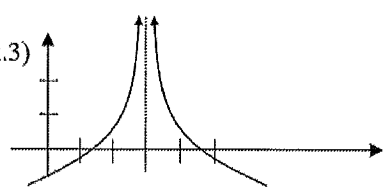


*Jump discontinuity

Example 2:

$$\lim_{x \to 5} f(x) =$$

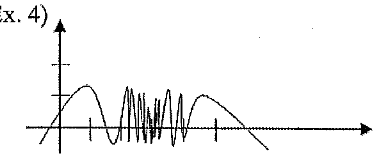
Ex.3)



*Vertical Asymptote

Example 3: $\lim_{x \to 3} f(x) =$

Ex. 4)



*Graphs with oscillating behavior

Example 4: $\lim_{x \to 3} \sin\left(\frac{1}{x}\right) =$

Example 5: Find the limit using a table of values given that $f(x) = \frac{x^3-1}{x-1}$

x	0.9	.99	.999	1	1.0001	1.001	1.01	1.1
f(x)	2.71	2.97	2.997	Undefined	3.0003	3.003	3.03	3.31

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} =$$

(2)

Calculus Ch. 1.2 Classwork Problems — Evaluating Limits Graphically

1) $\lim_{x \to -5} f(x) =$

2) $\lim_{x \to -4} f(x) =$

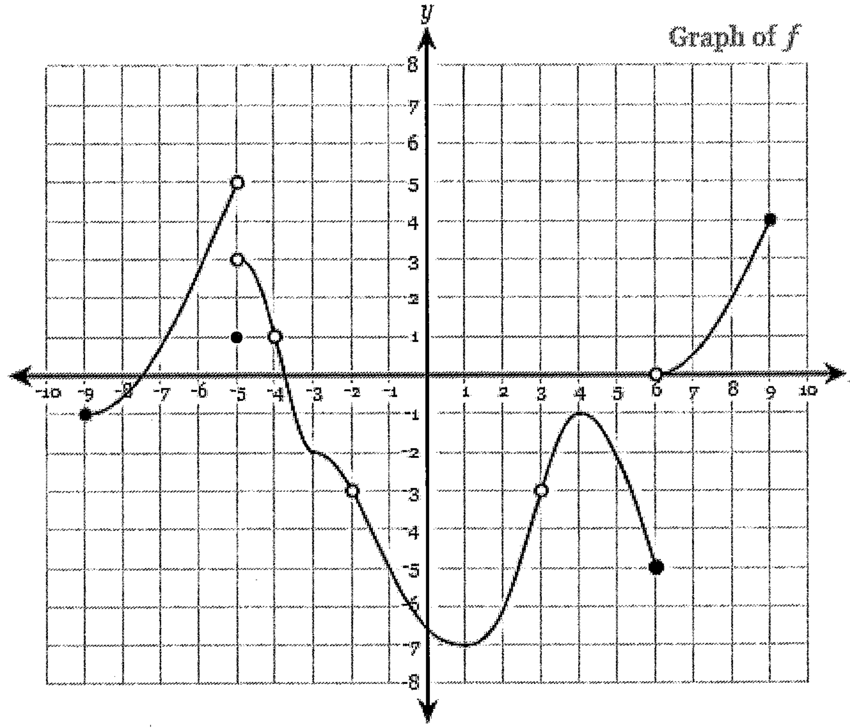
3) $f(-3) =$

4) $\lim_{x \to -3} f(x) =$

5) $f(3) =$

6) $\lim_{x \to 3} f(x) =$

7) $\lim_{x \to 6} f(x) =$



8) $\lim_{x \to -8} f(x) =$

9) $\lim_{x \to -7} f(x) =$

10) $f(-3) =$

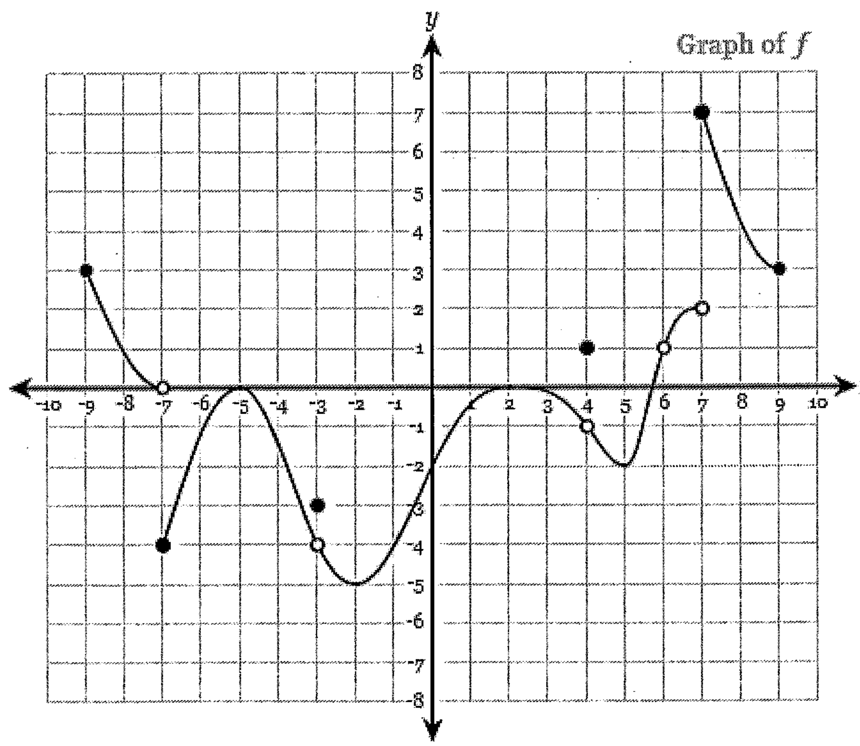
11) $\lim_{x \to 4} f(x) =$

12) $f(4) =$

13) $f(6) =$

14) $\lim_{x \to 6} f(x) =$

15) $\lim_{x \to 7} f(x) =$



Ch. 1.2 WS #1 Continued

16) $\lim_{x \to -9} f(x) =$

17) $\lim_{x \to -6} f(x) =$

18) $\lim_{x \to -4} f(x) =$

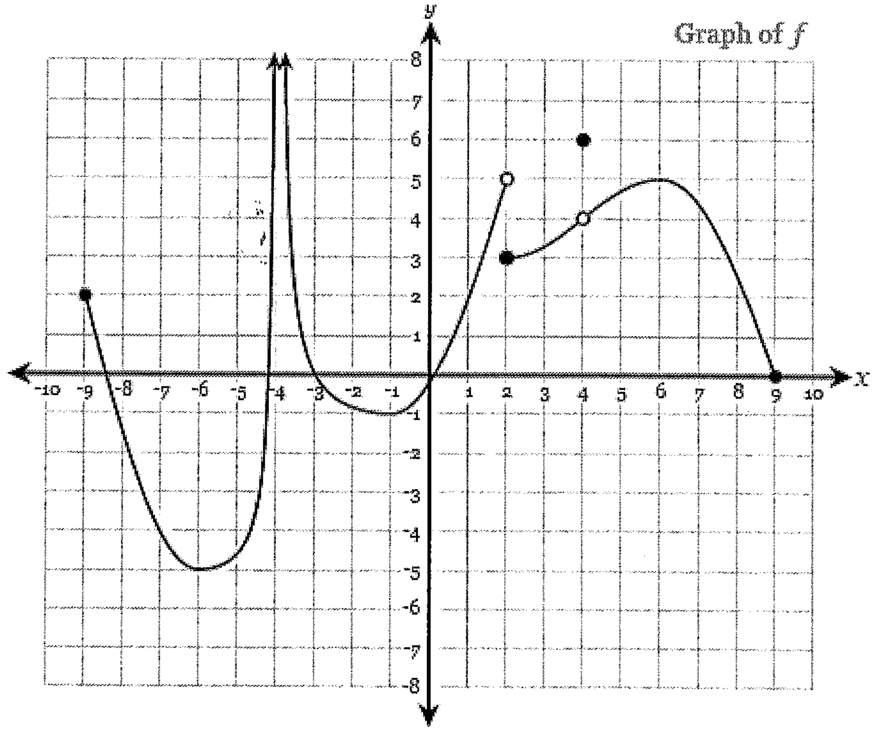
19) $f(-4) =$

20) $\lim_{x \to 2} f(x) =$

21) $f(2) =$

22) $\lim_{x \to 4} f(x) =$

23) $f(4) =$



24) $\lim_{x \to -6} f(x) =$

25) $\lim_{x \to -4} f(x) =$

26) $f(-4) =$

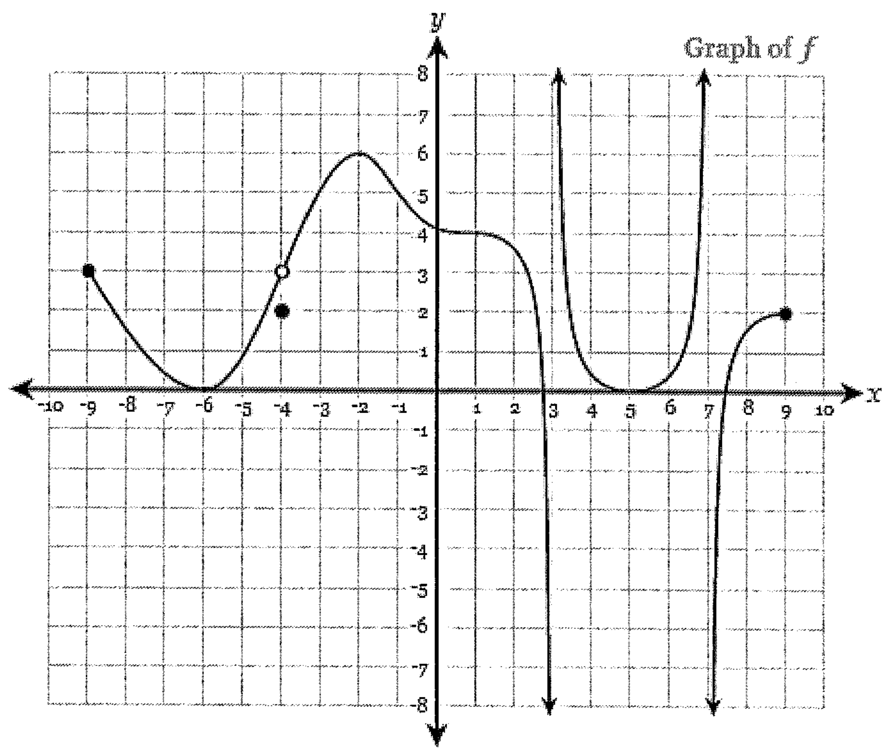
27) $f(3) =$

28) $\lim_{x \to 3} f(x) =$

29) $\lim_{x \to 5} f(x) =$

30) $\lim_{x \to 7} f(x) =$

31) $\lim_{x \to 9} f(x) =$



Calculus Ch. 1.2 Classwork Problems Worksheet #2

Sketch graph of a function satisfying the given descriptions:

1) $\lim_{x \to -5} f(x) = 3$

2) $f(-5) = -2$

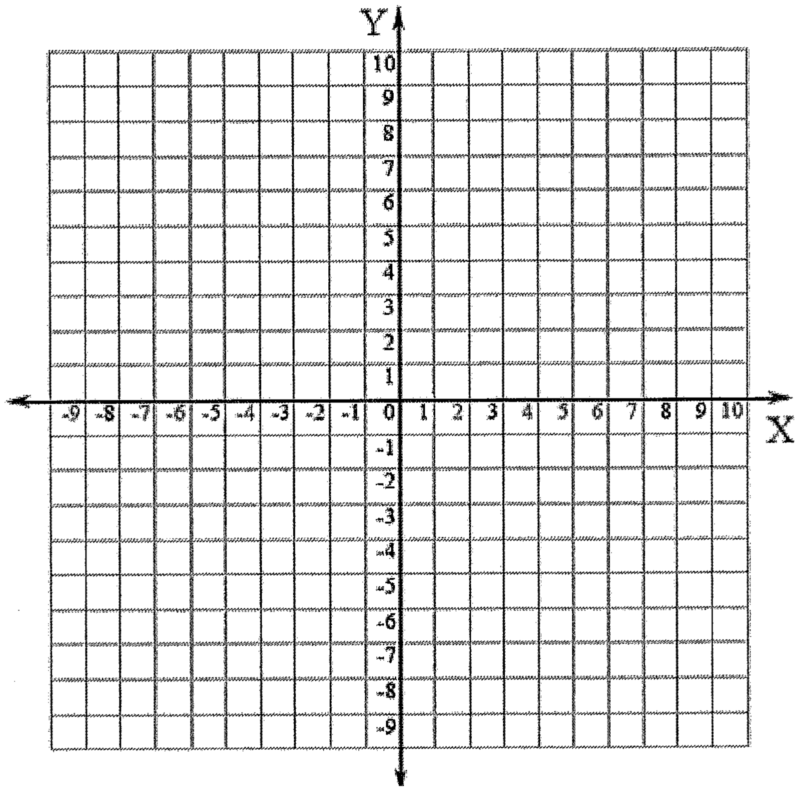
3) $f(-1) = 6$

4) $\lim_{x \to -3} f(x) = -\infty$

5) $f(3) = undefined$

6) $\lim_{x \to 3} f(x)$ *does not exist*

7) $\lim_{x \to 6} f(x) = 4$



8) $\lim_{x \to -8} f(x) = DNE$

9) $\lim_{x \to -7} f(x) = -5$

10) $f(-3) = 5$

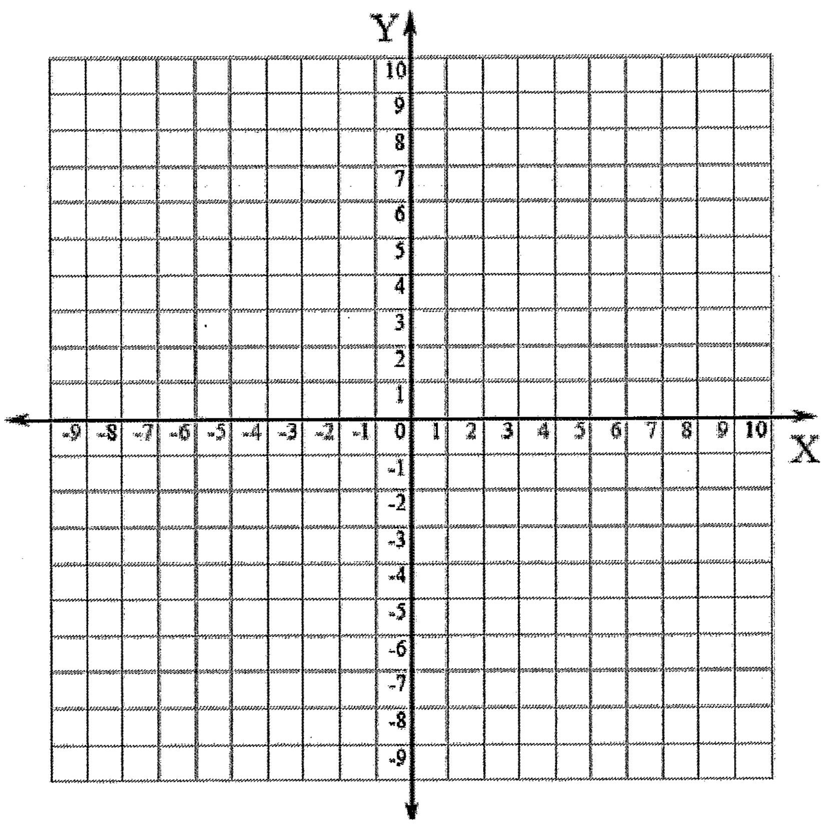
11) $\lim_{x \to 4} f(x) = 2$

12) $f(4) = undefined$

13) $f(6) = 4$

14) $\lim_{x \to 6} f(x) = \infty$

15) $\lim_{x \to 7} f(x) = -3$



Ch. 1.3a Evaluating Limits Algebraically

Rules:

1) $\lim_{x\to c} b = b$ 2) Suppose $\lim_{x\to c} f(x) = L$ then $\lim_{x\to c} \boldsymbol{b}f(x) = \boldsymbol{b}L$

I. **Direct Substitution Method:** To find limits for a function, first try to evaluate the argument in the expression (plug in the value). If the resulting value is a Real Number, then the value is the limit (answer).

Example 1:

a) $\lim_{x\to 2} x^2 + 3x =$	c) $\lim_{x\to -1} 3x^5 - 2x^2 + 7x + 4 =$
b) $\lim_{x\to 2} 5 =$	d) $\lim_{x\to \pi} x\cos x =$

II. **Simplify/Reduction Method** (Factor/Simplify/Substitute) Steps:

1) Evaluate argument First! (plug in value into expression)
2) If direct substitution produces $\frac{0}{0}$ (indeterminate form), we need to evaluate further

*Note: $\frac{0}{0}$ does not mean the Limit is Undefined. $\frac{0}{0}$ just means our problem is incomplete and unfinished. (It's true that the function value is undefined because there's a hole in the graph, but the limit most times does still exist)

3) Factor/Reduce/Simplify: Try finding common factors in order to reduce expression
4) Using the Reduced expression, re-evaluate the limit
5) Confirm resulting value is now a Real Number, therefore the limit (answer)

Example 2:

a) $\lim_{x\to -2} \frac{x^2 + 5x + 6}{x + 2} =$	b) $\lim_{x\to 1} \frac{x^2 + 5x + 6}{x - 1} =$

6

<u>Simplify/Reduction Method</u> (Factor/Simplify/Substitute) Steps:

1) Evaluate argument First! (plug in value into expression)
2) If direct substitution produces $\frac{0}{0}$ (indeterminate form), we need to evaluate further
3) Factor/Reduce/Simplify: Try finding common factors in order to reduce expression
4) Using the Reduced expression, re-evaluate the limit
5) Confirm resulting value is now a Real Number, therefore the limit (answer)

Example 2 (continued):

$c)\ \lim_{x\to 2}\frac{x^2+5x+6}{x+2} =$

$d)\ \lim_{x\to 2}\frac{x^2-4}{x+2} =$

Practice Problems:

1) $\lim_{x\to 1}\frac{2x^2-x-3}{x-1}$

2) $\lim_{x\to 3}\frac{4x^2-7x-2}{x-2}$

3) $\lim_{x\to -1}\frac{x^2-1}{x+1}$

4) $\lim_{x\to 1}\frac{x^2-4}{x^2-3x+2}$

5) $\lim_{x\to -3}\frac{x^2+x-6}{x^2-9}$

6) $\lim_{x\to 5}\frac{5-x}{x^2-25}$

Ch. 1.3b (More) Evaluating Limits Algebraically

Recap Steps: **Simplify/Reduction Method** (Factor/Simplify/Substitute) Steps:

1) Evaluate argument First! (plug in value into expression)
2) If direct substitution produces $\frac{0}{0}$ (indeterminate form), we need to evaluate further

*Note: $\frac{0}{0}$ does not mean the Limit is Undefined. $\frac{0}{0}$ just means our problem is incomplete and unfinished. (It's true that the function value is undefined because there's a hole in the graph, but the limit most times does still exist)

3) **Factor/Reduce/Simplify: Try finding common factors in order to reduce expression**
4) Using the Reduced expression, re-evaluate the limit
5) Confirm resulting value is now a Real Number, therefore the limit (answer)

I. Simplify using conjugate method

- If there is a sum or difference of 2 terms in the numerator, then multiply the numerator and denominator by the **conjugate** term.

Example 1: $\lim\limits_{x \to 4} \frac{6-\sqrt{x+32}}{x-4}$

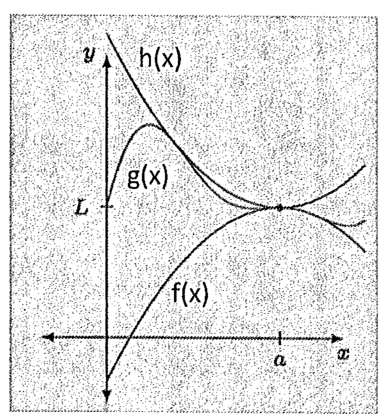
II. Simplify by finding Common Denominator

Example 2: $\lim\limits_{x \to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

III. Squeeze Theorem

In the graph below, the lower and upper functions have the same limit value at x=a. The middle function has the same limit value because it is trapped between the two outer functions.

The middle function is "squeezed" to Limit **L** as x approaches **a**



Definition: *Suppose* $f(x) \leq g(x) \leq h(x)$ *for all* x *in an open interval*

Suppose that $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} h(x) = L$ *Then,* $\lim\limits_{x \to a} g(x) = L$

Example 3: Let $h(x) = 1$, $f(x) = x^2 + 1$. *If* $f(x) \leq g(x) \leq h(x)$ *find* $\lim\limits_{x \to 0} g(x)$

8

1.3b Practice Problems:

Simplify/Reduction Method (Factor/Simplify/Substitute) Steps:

a) Evaluate argument First! (plug in value into expression)
b) If direct substitution produces $\frac{0}{0}$ (indeterminate form), we need to evaluate further
c) Factor/Reduce/Simplify: Try finding common factors in order to reduce expression
d) Using the Reduced expression, re-evaluate the limit
e) Confirm resulting value is now a Real Number, therefore the limit (answer)

1)
$$\lim_{x\to4}\frac{\sqrt{x}-2}{x-4}$$

2)
$$\lim_{x\to0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x}$$

3)
$$\lim_{x\to0}\frac{x^4-5x^2}{x^2}$$

4)
$$\lim_{x\to16}\frac{4-\sqrt{x}}{x-16}$$

5)
$$\lim_{x\to4}\frac{x^2+7x-44}{x^2-6x+8}$$

6)
$$\lim_{x\to0}\frac{\sqrt{1+2x}-1}{3x}$$

7)
$$\lim_{x\to0}\frac{\frac{1}{4}+\frac{1}{x-4}}{x}$$

8)
$$\lim_{x\to5}\frac{\sqrt{x-1}-2}{x^2-25}$$

Ch. 1.2-1.3 Limits Quiz Review Worksheet

1) Find the values

a. $\lim_{x \to -7} g(x) =$

b. $g(-7) =$

c. $\lim_{x \to -4} g(x) =$

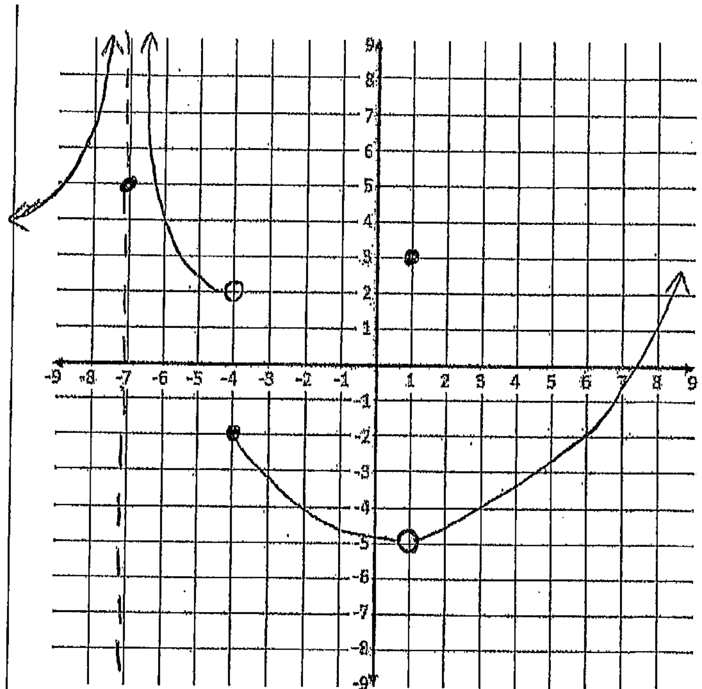
d. $g(-4) =$

e. $\lim_{x \to 1} g(x) =$

f. $g(1) =$

g. $g(6) =$

h. $\lim_{x \to 6} g(x) =$



2) Sketch a graph with the following characteristics:

a) $\lim_{x \to -5} f(x) = -3$

b) $g(-5) = undefined$

c) $g(-2) = -1$

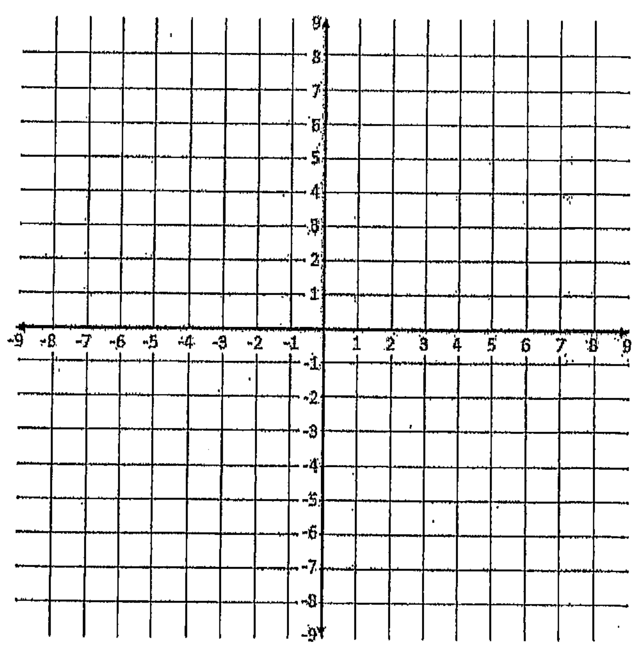
d) $\lim_{x \to -2} f(x) = -\infty$

e) $g(2) = 3$

f) $\lim_{x \to 2} f(x) = 3$

g) $g(6) = 7$

h) $\lim_{x \to 6} f(x)$ *does not exist*



(10)

Evaluate the Limit:

3)

$$\lim_{x \to 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$$

4)

$$\lim_{x \to 1} \frac{x^2 - x - 2}{x - 1}$$

5)

$$\lim_{x \to 2} \frac{4 - \sqrt{18 - x}}{x - 2}$$

6)

$$\lim_{x \to 1} \frac{3x^2 - x - 2}{x - 2}$$

7)

$$\lim_{x \to 5} \frac{4x^2 - 22x + 10}{x - 5}$$

8)

$$\lim_{x \to 0} \frac{\sqrt{3 + x} - \sqrt{3}}{x}$$

9)

$$\lim_{x \to 0} \frac{\frac{1}{4-x} - \frac{1}{4}}{x}$$

10)

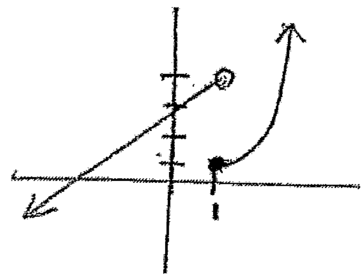
$$\lim_{x \to 1} \frac{\frac{3}{x} - 3}{x - 1}$$

Calculus **Ch. 1.4a Notes** **Continuity and One-Sided Limits**

Definition: **One-Sided Limits** – describes the function's behavior from the left or the right side of an x-value

Example 1:

$$f(x)=\begin{cases} x^2 & , \quad x \geq 1 \\ x+3 & , \quad x<1 \end{cases}$$



a) $\lim_{x\to 1} f(x) =$

b) left handed limit: $\lim_{x\to 1^-} f(x) =$

In other words: "The Limit (y-value that the graph approaches) **from the left side** of x = 1 is ____

c) right handed limit: $\lim_{x\to 1^+} f(x) =$

In other words: "The Limit (y-value that the graph approaches) **from the right side** of x = 1 is ____

- Recall that the limit of $f(x)$ as $x \to c$ exists only if $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$.

Continuity

can I walk along the graph without any interruptions? Can I draw the graph without ever lifting my pen/pencil? If so, the path or graph is continuous at that point.

Continuity Conditions: (*IMPORTANT: KNOW THIS*)

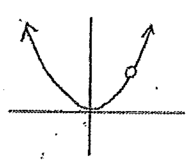
For a function, f, to be continuous at c, the following 3 conditions must be met.

1. $f(c)$ is defined — *point exists
2. $\lim_{x\to c} f(x)$ exists $\left(\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)\right)$ — *the limit exists
3. $\lim_{x\to c} f(x) = f(c)$ — * the limit exists at same location as point

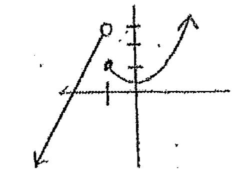
- When checking for discontinuity, step through each of the conditions above in order.

Types of Continuity:

1) **Removable Discontinuity** (hole in graph) – a graph with removable discontinuity can be made continuous by filling in a single point.



2) **Nonremovable Discontinuity** (step, jump discontinuity) – this is a discontinuity where the graph jumps from one connected piece of graph to another.



*Non-removable discontinuity fails the 2nd continuity condition:

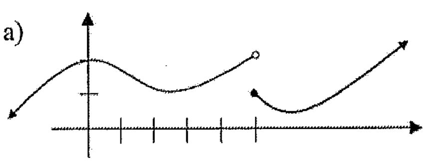
$\lim_{x \to -1^-} f(x) =$ $\lim_{x \to -1^+} f(x) =$ then $\lim_{x \to -1} f(x) =$

<u>Continuity Conditions revisited</u>

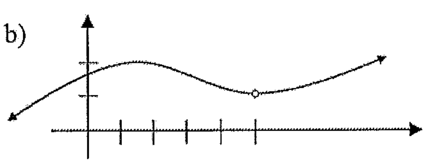
$i.\ f(c)$ *is defined*	*If first condition fails, function is not continuous at the point, but continue to test next condition(s) to categorize removable/nonremovable
$ii.\ \lim_{x \to c} f(x)$ exists ($\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$)	*If 2nd condition fails, then the limit does not exist, and this function must have **non-removable discontinuity** at that point *Test 3rd condition **<u>only</u>** if 2nd condition passes.
$iii.\ \lim_{x \to c} f(x) = f(c)$	*If 2nd condition passes, but 3rd condition fails, then this function must have **removable discontinuity** at that point *If all 3 condition passes, then the function is continuous at that point.

Class Example 2: Using continuity conditions, determine the reason why the following graphs are discontinuous. Then categorize as removable or nonremovable discontinuity

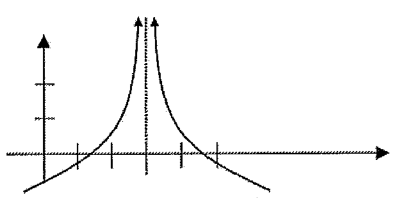
a)



b)



c)



d) Find the point (x-value) of discontinuity for the function $f(x) = \dfrac{x^2 - 9}{x - 3}$. Is it removable? If so, what would we need to set f(x) equal to at that value for the function to be continuous? (Step through continuity conditions to support your answer)

Calculus **Ch. 1.4b Notes** **Intermediate Value Theorem (IVT)**

Review continuity conditions:

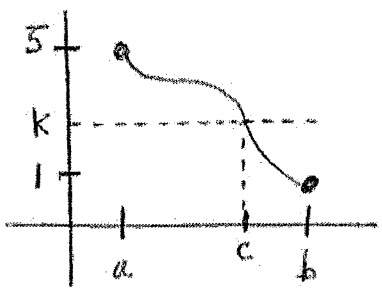
i) $f(c)$ is defined **ii)** $\lim_{x \to c} f(x)$ exists ($\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$) **iii)** $\lim_{x \to c} f(x) = f(c)$

Warm-up problem: Prove that the following is discontinuous at x = 2. Is it removable? If so, redefine f(2) to make the function continuous. (step through continuity conditions)

$$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x \neq 2 \\ x + 5 \ , & x = 2 \end{cases}$$

Continuity on a closed interval: If a function is continuous on an open interval (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$, then the function is continuous on the closed interval [a, b].

Intermediate Value Theorem: If f is continuous on a closed interval [a, b] and k is any number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = k.



*In other words, if a function is continuous, then the graph has to touch all the y-values between the 2 endpoints (at least once)

Example 1: Use the IVT to show that there is a zero in the interval [0, 1] for the function $f(x) = x^3 + 2x - 1$.

14

Example 2: Verify the IVT applies to $f(x) = \frac{x^2+x}{x-1}$ on the interval $\left[\frac{5}{2}, 4\right]$ for f(c) = 6 and find c.

Additional Continuity Practice Problems:

Making a Function Continuous **In Exercises 61–66, find the constant *a*, or the constants *a* and *b*, such that the function is continuous on the entire real number line.**

Continuity conditions:

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists $\quad (\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x))$
3. $\lim_{x \to c} f(x) = f(c)$

61. $f(x) = \begin{cases} 3x^2, & x \geq 1 \\ ax - 4, & x < 1 \end{cases}$

66. $g(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & x \neq a \\ 8, & x = a \end{cases}$

Calculus **Ch. 1.5 Notes: Limits Approaching Infinity (Vertical Asymptotes)**

Infinite Limits: a limit where the function increases or decreases without bound (towards infinity) as x approaches c

*If the limit as x approaches c from either right or left is $\pm\infty$, then x = c is a vertical asymptote

* Rational Functions: $y = \frac{f(x)}{g(x)}$ If g(x) has no factors that cancel, then there is a vertical asymptote.

Example 1: Find all the vertical asymptotes of $f(x) = \frac{x^2-3x+2}{x^2-4}$

Finding One-Sided Limits approaching Vertical Asymptotes:

Steps:

1) Evaluate Limit using the argument (plug in the value)
2) If Limit is undefined ($\frac{nonzero}{zero}$) then there is a vertical asymptote
3) To further evaluate the one-sided limit (determining the direction of arrows as $+\infty$ or $-\infty$)
    a. Test decimals 0.1 to the left of the argument x-value
    b. Test decimal 0.1 to the right of the argument x-value
4) Determine if the resulting fraction is a positive or negative value
    a. A positive decimal value indicates the one-sided limit is $+\infty$
    b. A negative decimal value indicates the one-sided limit is $-\infty$

Example 2: Determine $\lim_{x\to 2} f(x)$ for $f(x) = \frac{x+1}{x-2}$

(16)

Algebraic Steps (for x approaching Real Number): **1)** Plug in x-value first (IGNORE one-sided limit) **2)** If result is a real number value, the value is the limit. **3)** If the result is $\frac{0}{0}$ (indeterminate form) then reduce by **i)** factoring **ii)** conjugate method **iii)** simplify complex fraction **4)** Re-evaluate the reduced Expression 4) If result is undefined, and it's a one-sided limit, then test using decimals.

Find the following:

3) $\lim_{x \to -3^-} \frac{9-x^2}{x-4} =$

4) $\lim_{x \to 0^-} \frac{5x-x^2}{x^2-x} =$

5) $\lim_{x \to -2^-} \frac{x^2+1}{x+2} =$

6) $\lim_{x \to 5} \frac{3x^2-1}{25-x^2} =$

7) $\lim_{x \to -3^+} \frac{2x^2+3x-9}{x+3}$

8) $\lim_{x \to -4^+} \frac{2x^2-1}{x^2-16}$

9) $\lim_{x \to 1^+} \frac{x^2-2}{x^2+2x+1} =$

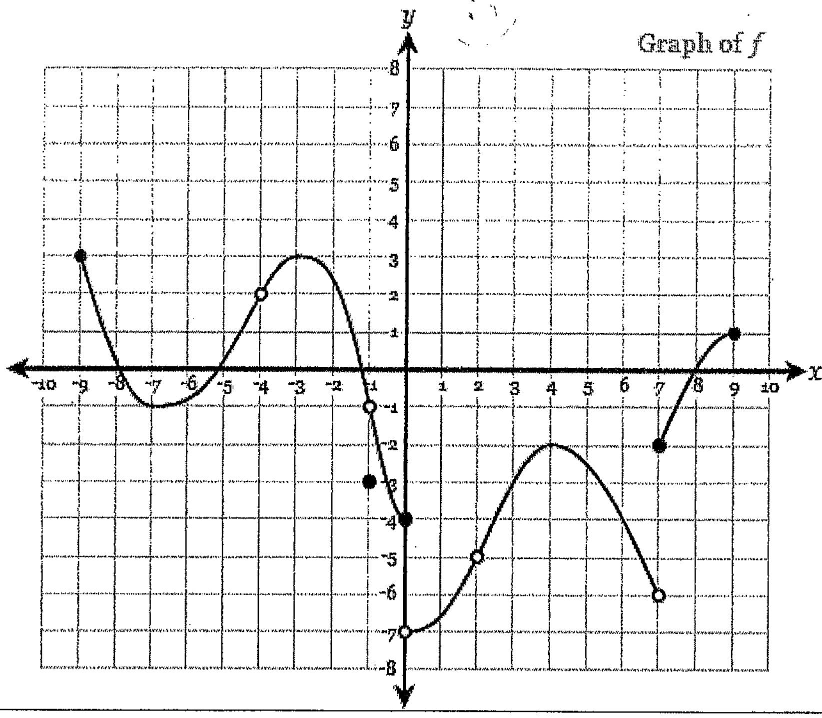
10) $\lim_{x \to 3^+} \frac{4x^2-14x+6}{x-3} =$

Non-AP Calculus **1.4-1.5 Continuity/IVT/Limits Classwork Problems**

Non-Removable discontinuity: point where graph is not continuous and Limit does not exist

Removable Discontinuity: point where graph is not continuous but the limit exists

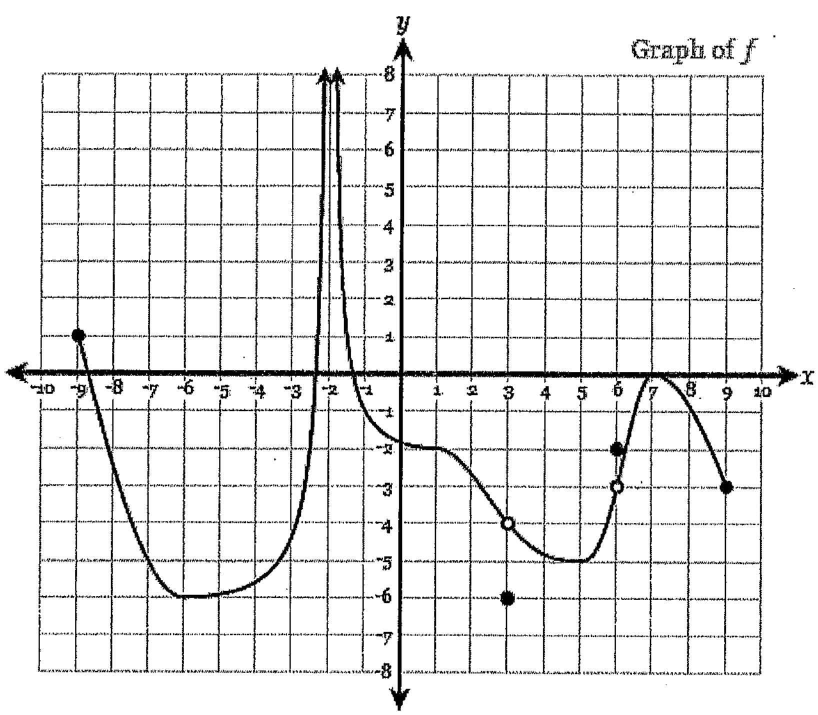
1) Identify values of x and determine the types of discontinuity for the below graph:



Non-Removable Discontinuity:

Removable Discontinuity:

2) Identify values of x and determine the types of discontinuity for the below graph:



Non-Removable Discontinuity:

Removable Discontinuity:

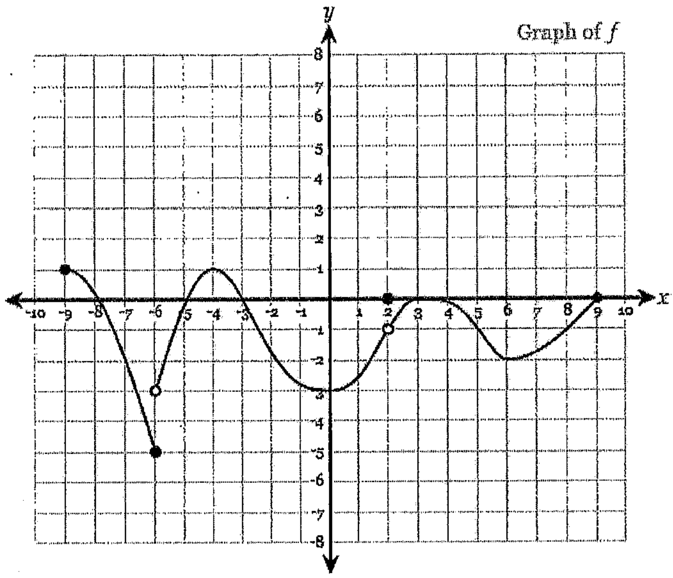
Continuity Conditions

i) f(c) is defined (point exists on the graph)

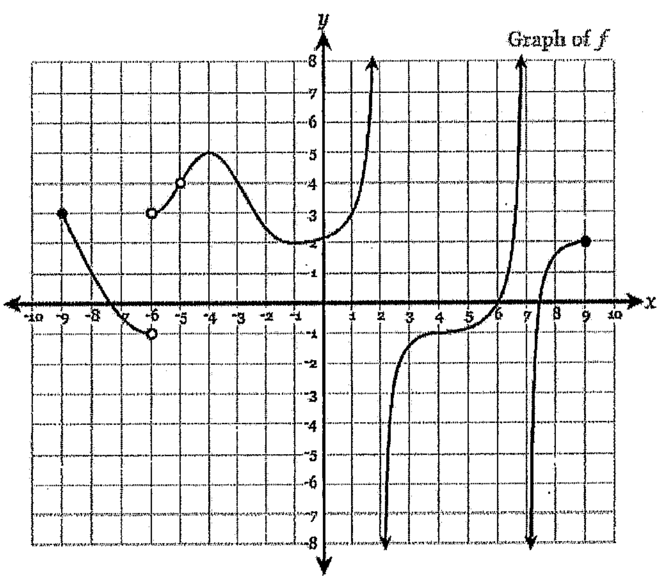
ii) The $\lim_{x \to c} f(x)$ exists $\left[\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)\right]$

iii) $f(c) = \lim_{x \to c} f(x)$

3) Use the definition of continuity to determine whether the function f(x) graphed below is continuous at x= 2.



4) Use the definition of continuity to determine whether the function f(x) graphed below is continuous at x= -6.



Continuity Conditions

i) f(c) is defined (point exists on the graph)

ii) The $\lim_{x \to c} f(x)$ exists $\left[\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)\right]$

iii) $f(c) = \lim_{x \to c} f(x)$

Use Continuity Conditions to show that f(x) is discontinuous at a point and state reason for discontinuity. Then determine if the discontinuity is removable or non-removable and state why.

5)

$$f(x) = \begin{cases} x^2 - 4x + 6, & x < 3 \\ -x^2 + 4x - 2, & x \geq 3 \end{cases}$$

---

6)

$$f(x) = \begin{cases} x, & x \leq 1 \\ 1 - x, & x > 1 \end{cases}$$

---

7)

$$f(x) = \begin{cases} \dfrac{x+2}{2}, & x \leq 3 \\ \dfrac{12 - 2x}{3}, & x > 3 \end{cases}$$

---

8)

$$f(x) = \begin{cases} x^2 - 4x + 6, & x < 3 \\ -x^2 + 4x - 2, & x \geq 3 \end{cases}$$

20

Using the Intermediate Value Theorem In Exercises 95–98, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem.

95. $f(x) = x^2 + x - 1, \quad [0, 5], \quad f(c) = 11$

96. $f(x) = x^2 - 6x + 8, \quad [0, 3], \quad f(c) = 0$

97. $f(x) = x^3 - x^2 + x - 2, \quad [0, 3], \quad f(c) = 4$

21

Find the following:

1) $\lim_{x \to -3^-} \frac{9-x^2}{x-4} =$

2) $\lim_{x \to 0^-} \frac{5x-x^2}{x^2-x} =$

3) $\lim_{x \to -2^-} \frac{x^2+1}{x+2} =$

4) $\lim_{x \to 5} \frac{3x^2-1}{25-x^2} =$

5) $\lim_{x \to -3^+} \frac{2x^2+3x-9}{x+3}$

6) $\lim_{x \to -4^+} \frac{2x^2-1}{x^2-16}$

7) $\lim_{x \to 1^+} \frac{x^2-2}{x^2+2x+1} =$

8) $\lim_{x \to 3^+} \frac{4x^2-14x+6}{x-3} =$

Finding a One-Sided Limit In Exercises 33–48, find the one-sided limit (if it exists).

33. $\lim_{x \to -1^+} \frac{1}{x+1}$

34. $\lim_{x \to 1^-} \frac{-1}{(x-1)^2}$

35. $\lim_{x \to 2^+} \frac{x}{x-2}$

36. $\lim_{x \to 2^-} \frac{x^2}{x^2+4}$

37. $\lim_{x \to -3^-} \frac{x+3}{x^2+x-6}$

50. $f(x) = \frac{x^3-1}{x^2+x+1}$

$\lim_{x \to 1^-} f(x)$

51. $f(x) = \frac{1}{x^2-25}$

$\lim_{x \to 5^-} f(x)$

Calculus **Ch. 3.5 Notes** **Limits at Infinity (End behavior)**

A. Checking for Horizontal Asymptotes (H.A.) $\left(\lim_{x\to\infty} f(x) \quad or \quad \lim_{x\to-\infty} f(x)\right)$

If $f(x) = \frac{p(x)}{q(x)}$, then **compare the degrees between numerator and denominator**

i) If Numerator degree < Denominator degree, then the H.A. is y = 0

Example 1: $\lim_{x\to\infty} \frac{3x^2-7}{2x^3+1} =$

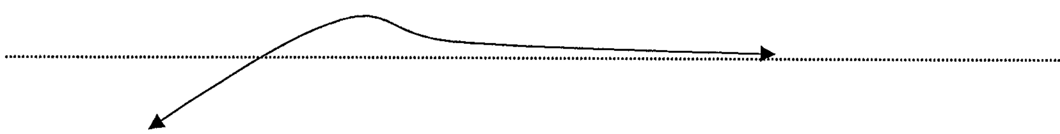
ii) If Denominator degree = Numerator degree, then H.A. is $y = \frac{numerator\ coefficient}{denominator\ coefficient}$

Example 2: $\lim_{x\to\infty} \frac{5x^2+3}{2x^2+4x-9} =$

iii) If Numerator degree > Denominator degree, then H.A. does not exist (limit is therefore $+\infty$ *or* $-\infty$)

Example 3: $\lim_{x\to\infty} \frac{2x^3+1}{7x^2+5x+10} =$

Note: a H.A. is a description of end behavior, not a boundary that the graph can't cross. A function can NEVER cross a vertical asymptote, but it might cross a horizontal asymptote.



Use Horizontal Asymptote Rules for the following:

4) $\lim_{x\to\infty} \frac{3x^2+1}{2x-5}$

5) $\lim_{x\to-\infty} \frac{3x^2+1}{2x-5}$

6) $\lim_{x\to-\infty} \frac{3x+1}{5-2x}$

7) $\lim_{x\to\infty} \frac{3x+1}{5-2x}$

8) $\lim_{x\to\infty} \frac{3x+1}{2x^2-5}$

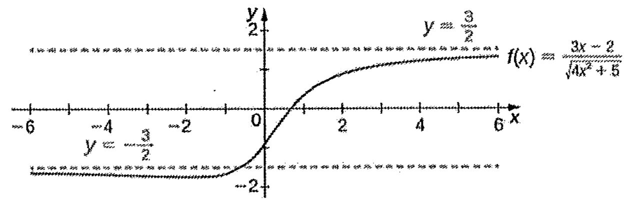
9) $\lim_{x\to-\infty} \frac{3x^3+1}{2x^2-5}$

24

B. Finding Horizontal Asymptotes with **Radicals in denominator**

Ex. 10: Find the Horizontal asymptotes for:

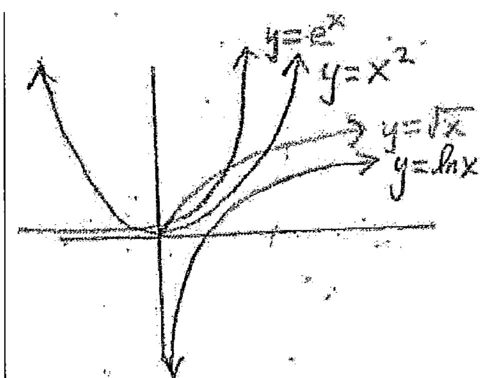
$$y = \frac{3x-2}{\sqrt{4x^2+5}}$$



C. Comparative Growth Rates

*Families of Functions grow at predictable rates in relations to each other as x **approaches** $+\infty$

***Logarithms < Radicals < Polynomial (Algebraic) < Exponential**
(slowest) (fastest)



*Note: Comparative Growth Rates relationship **only apply** when limit approaches infinity. (NOT $-\infty$)

Ex. 11 $\lim\limits_{x\to\infty} \frac{\sqrt{5000x+1000}}{x^2}$

Ex. 13 $\lim\limits_{x\to\infty} \frac{\ln(40000000x)}{2x}$

Ex. 12 $\lim\limits_{x\to\infty} \frac{-e^{2x}}{1000x^4+x^5}$

Ex. 14 $\lim\limits_{x\to\infty} \frac{-\sqrt{3000x-4}}{\ln(5x+1)}$

Ch. 1 Limits **Steps for Important Concepts**

I. Algebraic Steps Evaluating Limits Approaching a Real Number $\lim_{x\to c} f(x)$

1. Plug in argument x-value (Ignore one-sided limit for now)
2. Find the Limit (plug in/ reduce if $\frac{0}{0}$, re-evaluate)
3. If Limit DNE (does not exist), then evaluate further ONLY IF one-sided limit)
4. Choose between $+\infty$ and $-\infty$
5. Plug in the appropriate decimal value to determine $+\infty$ or $-\infty$

II. Algebraic Steps Evaluating Limits Approaching Infinity $\lim_{x\to\infty} f(x)$ *or* $\lim_{x\to-\infty} f(x)$

1. Compare degrees between numerator vs. denominator
   a. If Numerator < Denominator $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$
   b. If Numerator = Denominator $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) =$ ratio of coefficients
   c. If Numerator > Denominator, then $\lim_{x\to\infty} f(x) = \pm\infty$ *or* $\lim_{x\to-\infty} f(x) = \pm\infty$

   (Plug in a large positive or large negative value to help you determine the sign at infinity)

III. Continuity Conditions

1. f(c) is defined (point exists on the graph)
2. The $\lim_{x\to c} f(x)$ exists $\left[\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x)\right]$
3. f(c) $= \lim_{x\to c} f(x)$

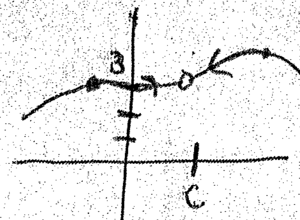
- If function passes all 3 conditions , the function has continuity at x = c
- If condition #2 FAILS, the function has **nonremovable** discontinuity at x = c
- If function PASSES condition #2 and FAILS condition #3, the function has **removable** discontinuity at x = c

IV. Intermediate Value Theorem (IVT) Steps

1. Test and determine continuity on closed interval [a, b]
2. Find the y-value at the endpoints , f(a) and f(b)
3. Confirm that f(c) is between f(a) and f(b) [example: f(a) < f(c) < f(b)]
4. Find the c-value (find the x-value by plugging the y-value given at the start of problem into the function)

   *Make sure c-value(s) are inside the interval [a,b]. c-values that are outside the interval [a,b] are excluded.

If hole in graph, one-sided limits are same as the full limit

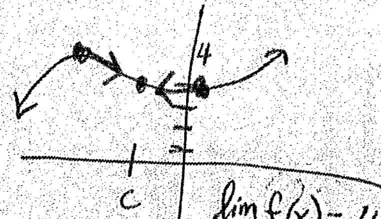


$\lim_{x \to c^-} f(x) = 3$

$\lim_{x \to c^+} f(x) = 3$

$\lim_{x \to c} f(x) = 3$

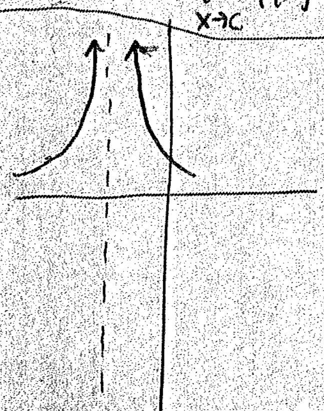
If function is continuous, one-sided limits are same as the full limit



$\lim_{x \to c^-} f(x) = 4$

$\lim_{x \to c^+} f(x) = 4$

$\lim_{x \to c} f(x) = 4$

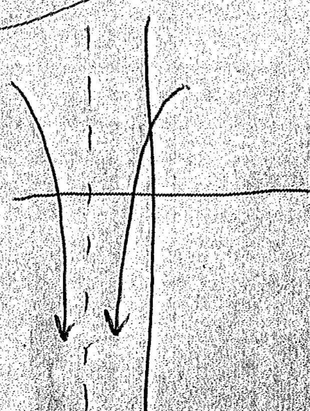


limit dne ($+\infty$)

$\lim_{x \to c^-} f(x) = +\infty$

$\lim_{x \to c^+} f(x) = +\infty$

$\lim_{x \to c} f(x) = \text{dne}$ ($+\infty$)

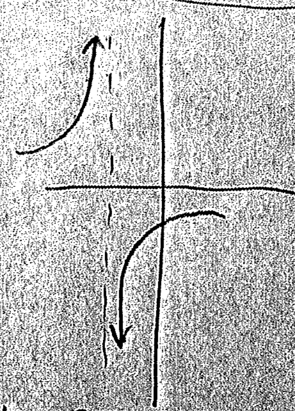


limit dne ($-\infty$)

$\lim_{x \to c^-} f(x) = -\infty$

$\lim_{x \to c^+} f(x) = -\infty$

$\lim_{x \to c} f(x) = \text{dne}$ ($-\infty$)

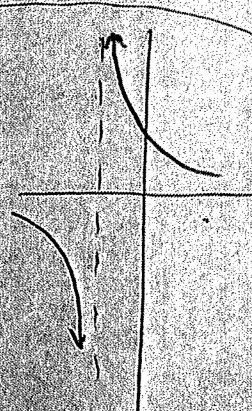


limit dne

$\lim_{x \to c^-} f(x) = +\infty$

$\lim_{x \to c^+} f(x) = -\infty$

$\lim_{x \to c} f(x) = \text{dne}$



limit dne

$\lim_{x \to c^-} f(x) = -\infty$

$\lim_{x \to c^+} f(x) = +\infty$

$\lim_{x \to c} f(x) = \text{dne}$