

$$1. g(x) = \begin{cases} x + 5, & x < 2 \\ 11, & x = 2 \\ 2x - 1, & 2 < x < 5 \\ 20, & x = 5 \\ x + 4, & x > 5 \end{cases}$$

Find the following :

a)  $\lim_{x \rightarrow -\infty} g(x) =$

b)  $\lim_{x \rightarrow 2^-} g(x) =$

c)  $\lim_{x \rightarrow 2^+} g(x) =$

d)  $\lim_{x \rightarrow 2} g(x) =$

e)  $\lim_{x \rightarrow 5^-} g(x) =$

f)  $\lim_{x \rightarrow 5^+} g(x) =$

g)  $\lim_{x \rightarrow 5} g(x) =$

h)  $\lim_{x \rightarrow 6^+} g(x) =$

i)  $\lim_{x \rightarrow \infty} g(x) =$

Use Continuity conditions to justify whether  $f(x)$  is continuous or discontinuous . If discontinuous, identify type of discontinuity

$$2) f(x) = \begin{cases} x^2 + 5, & x \geq 3 \\ x^2 - 2, & x < 3 \end{cases}$$

3) Find the  $k$  value that will make the function  $f(x)$  continuous.

$$b) f(x) = \begin{cases} \frac{x^2+x-6}{x+3}, & x \neq -3 \\ k, & x = -3 \end{cases}$$

## PAST AP FREE-RESPONSE PROBLEMS COVERED BY THIS CHAPTER

Note: These and other questions can be found at  
 apcentral.com  
 2003 AB 6a  
 2006 BC 3c  
 2008 AB 6d

## MULTIPLE-CHOICE QUESTIONS

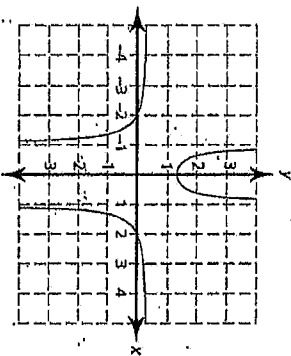
A calculator may not be used on the following questions.

- Evaluate the limit, if it exists:  $\lim_{x \rightarrow 6} \frac{x^2 + x - 6}{2 - x}$ .  
 (A) 5  
 (B) 3  
 (C) -3  
 (D) -5  
 (E) The limit does not exist.
- Evaluate the limit, if it exists:  $\lim_{x \rightarrow 9} \frac{\sqrt{x-5}-2}{x-9}$ .  
 (A)  $\frac{1}{4}$   
 (B)  $-\frac{1}{4}$   
 (C) 1  
 (D) 0  
 (E) The limit does not exist.
- Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2}$ .  
 (A)  $\frac{1}{4}$   
 (B)  $-\frac{1}{4}$   
 (C) 1  
 (D) -1  
 (E) The limit does not exist.

4)

For what value of  $k$  is the function  $f(x) = \frac{2x^2 + 5x - 3}{x^2 - 9}$ ,  $x \neq -3$  continuous at  $x = -3$ ?

- (A)  $-\frac{7}{5}$   
 (B)  $\frac{5}{6}$   
 (C) 0  
 (D)  $\frac{5}{6}$   
 (E)  $\frac{7}{6}$



5)

The function  $g(x)$  is shown in the graph above and is of the form  $g(x) = \frac{x^2 + a}{bx^2 - 3}$ . Which of the following could be the values of the

- constants  $a$  and  $b$ ?
- (A)  $a = -2$ ,  $b = -1$   
 (B)  $a = -2$ ,  $b = -3$   
 (C)  $a = -4$ ,  $b = 3$   
 (D)  $a = -4$ ,  $b = -3$   
 (E)  $a = 4$ ,  $b = 3$

6)

Identify the vertical asymptotes for  $f(x) = \frac{x^2 + 3x - 4}{x^2 + x - 2}$ .

- (A)  $x = -2$ ,  $x = 1$   
 (B)  $x = -2$   
 (C)  $x = 1$   
 (D)  $y = -2$ ,  $y = 1$   
 (E)  $y = -2$

1.  $g(x) = \begin{cases} x+5, & x < 2 \\ 11, & x = 2 \\ 2x-1, & 2 < x < 5 \\ 20, & x = 5 \\ x+4, & x > 5 \end{cases}$

Find the following:

a) $\lim_{x \rightarrow -\infty} g(x) = \boxed{-\infty}$	b) $\lim_{x \rightarrow 2^-} g(x) = \boxed{7}$	c) $\lim_{x \rightarrow 2^+} g(x) = \boxed{3}$
$\lim_{x \rightarrow -\infty} x+5 = \boxed{-\infty}$	$\lim_{x \rightarrow 2^-} x+5 = \boxed{7}$	$\lim_{x \rightarrow 2^+} 2x-1 = \boxed{3}$
$\lim_{x \rightarrow 2^-} g(x) = \boxed{DNE}$	$\lim_{x \rightarrow 2^+} g(x) = \boxed{9}$	$\lim_{x \rightarrow 2^+} g(x) = \boxed{9}$
$\lim_{x \rightarrow 2^-} g(x) = \boxed{9}$	$\lim_{x \rightarrow 5^-} g(x) = \boxed{10}$	$\lim_{x \rightarrow 5^+} g(x) = \boxed{+ \infty}$
$\lim_{x \rightarrow 5^-} g(x) = \boxed{9}$	$\lim_{x \rightarrow 5^-} 2x-1 = \boxed{9}$	$\lim_{x \rightarrow 5^+} x+4 = \boxed{9}$
$\lim_{x \rightarrow 5^+} g(x) = \boxed{+ \infty}$	$\lim_{x \rightarrow 5^+} g(x) = \boxed{+ \infty}$	$\lim_{x \rightarrow 5^+} x+4 = \boxed{+ \infty}$

Use Continuity conditions to justify whether  $f(x)$  is continuous or discontinuous. If discontinuous, identify type of discontinuity.

2)  $f(x) = \begin{cases} x^2+5, & x \geq 3 \\ x^2-2, & x < 3 \end{cases}$

$f(3) = 3^2+5 = 14$

$\lim_{x \rightarrow 3^-} f(x) = 3^2-2 = 7$   $\lim_{x \rightarrow 3^+} f(x) = 3^2+5 = 14$  Since  $\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$

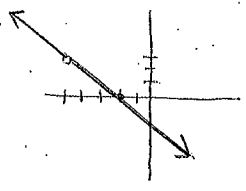
Nonremovable discontinuity since limit does not exist at  $x=3$   
 $\lim_{x \rightarrow 3} f(x) = DNE$

3) Find the k value that will make the function  $f(x)$  continuous.

b)  $f(x) = \begin{cases} \frac{x^2+x-6}{x+3}, & x \neq -3 \\ k, & x = -3 \end{cases}$  i)  $f(-3) = k$

ii)  $\lim_{x \rightarrow -3} \frac{x^2+x-6}{x+3} = \lim_{x \rightarrow -3} \frac{(x+3)(x-2)}{x+3} = -5$

iii)  $f(-3) = \lim_{x \rightarrow -3} f(x) = \boxed{k = -5}$



PAST AP FREE-RESPONSE PROBLEMS COVERED BY THIS CHAPTER

Note: These and other questions can be found at  
 apcentral.com  
 2003 AB 6a  
 2006 BC 3c  
 2008 AB 6d

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

1. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{x^2+x-6}{2-x}$ .  $\lim_{x \rightarrow 2} \frac{(x+3)(x-2)}{-(x-2)} = -1 = \boxed{-5}$

- (A) 5  
 (B) 3  
 (C) -3  
 (D) -5  
 (E) The limit does not exist.

2. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 9} \frac{\sqrt{x-5}-2}{x-9}$ .  $\lim_{x \rightarrow 9} \frac{(\sqrt{x-5}-2)(\sqrt{x-5}+2)}{(x-9)(\sqrt{x-5}+2)} = \lim_{x \rightarrow 9} \frac{x-5-4}{(x-9)(\sqrt{x-5}+2)} = \lim_{x \rightarrow 9} \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

3. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$ .  $\lim_{x \rightarrow 2} \frac{\frac{2-x}{2x} \cdot \frac{1}{x-2}}{x-2} = \lim_{x \rightarrow 2} \frac{-1}{2x(x-2)^2} = \boxed{-\frac{1}{4}}$

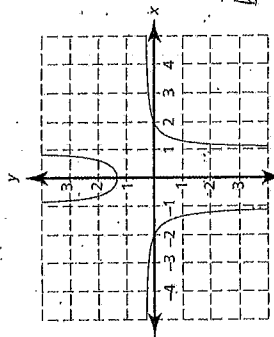
- (A)  $\frac{1}{4}$   
 (B)  $-\frac{1}{4}$   
 (C) 1  
 (D) -1  
 (E) The limit does not exist.

4) For what value of  $k$  is the function  $f(x) = \begin{cases} 2x^2 + 5x - 3 & x \neq -3 \\ k & x = -3 \end{cases}$  continuous at  $x = -3$ ?  $i) f(-3) = k$

$(A) \frac{7}{6}$     $(B) \frac{5}{6}$     $(C) 0$     $(D) \frac{5}{6}$     $(E) \frac{7}{6}$

$ii) \lim_{x \rightarrow -3} (2x-1)(x+3) = \frac{-6-1}{-3-3} = \frac{-7}{-6} = \frac{7}{6}$

$ii) f(-3) = \lim_{x \rightarrow -3} f(x), k = \frac{7}{6}$



$x$ -int:  $(-2, 0), (2, 0)$

V.A:  $x = -1, x = 1$

$x^2 + 4 = 0$  at  $x = 2$

$2 + 4 = 0$   $a = -4$

$6x^2 - 3 = 0$  at  $x = 1, x = -1$

$6(1)^2 - 3 = 0$   $b = 3$

5) The function  $g(x)$  is shown in the graph above and is of the form

$g(x) = \frac{x^2 + a}{bx^2 - 3}$ . Which of the following could be the values of the

constants  $a$  and  $b$ ?

(A)  $a = -2, b = -1$

(B)  $a = -2, b = 3$

(C)  $a = -4, b = 3$

(D)  $a = 4, b = 3$

(E)  $a = 4, b = 3$

$g(x) = \frac{x^2 - 4}{3x^2 - 3}$

6) Identify the vertical asymptotes for  $f(x) = \frac{x^2 + 3x - 4}{x^2 + x - 2}$ .

(A)  $x = -2, x = 1$

(B)  $x = -2$

(C)  $x = 1$

(D)  $y = -2, y = 1$

(E)  $y = -2$

$(x-1)(x+4)$   
 $(x-1)(x+2)$

$x = -2$

$$1. g(x) = \begin{cases} \frac{x+5}{2x-4}, & x < 2 \\ 12, & x = 2 \\ 2x - 5, & 2 < x < 5 \\ 20, & x = 5 \\ \frac{-x^2+4}{5-x}, & x > 5 \end{cases}$$

Find the following :

a) $\lim_{x \rightarrow -\infty} g(x) =$	b) $\lim_{x \rightarrow 2^-} g(x) =$	c) $\lim_{x \rightarrow 2^+} g(x) =$
d) $\lim_{x \rightarrow 2} g(x) =$	e) $\lim_{x \rightarrow 5^-} g(x) =$	f) $\lim_{x \rightarrow 5^+} g(x) =$
g) $\lim_{x \rightarrow 5} g(x) =$	h) $\lim_{x \rightarrow 3^+} g(x) =$	i) $\lim_{x \rightarrow \infty} g(x) =$

Use Continuity conditions to justify whether  $f(x)$  is continuous or discontinuous . If discontinuous, identify type of discontinuity

$$2) f(x) = \begin{cases} 2x^2 + 5, & x \geq 3 \\ 2^x, & x < 3 \end{cases}$$

3) Find the horizontal asymptote(s) of

$$f(x) = \frac{-2x-5}{\sqrt{6x^2+11x-30}}$$

$$4) \text{ Find } \lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3}$$

$$5) \text{ Let } f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

Which of the following is true? (Circle all that apply)

I.  $\lim_{x \rightarrow 3} f(x)$  does not exist

II.  $f$  is continuous at  $x = 3$

III. The line  $x = 3$  is a vertical asymptote:

6

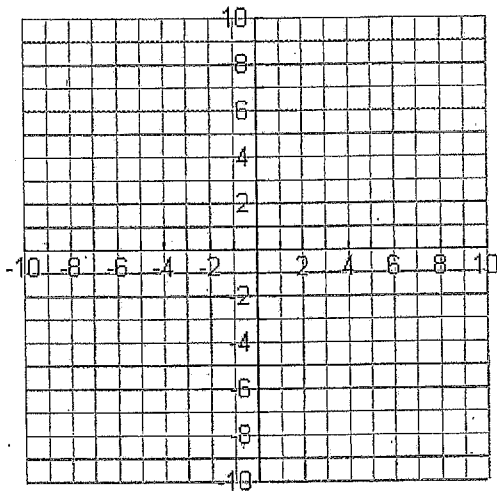
$$6. \text{ Let } f(x) = \begin{cases} \frac{x^2 - 7x + 10}{x^2 - 25}, & x^2 \neq 25 \\ A, & x = 5 \\ B, & x = -5 \end{cases}$$

- a) Are the lines  $x = 5$  and  $x = -5$  vertical asymptotes? Justify answer
- b) Identify all horizontal asymptotes. Justify answer
- c) Is there a value of  $A$  that makes  $f$  continuous at  $x = 5$ ?
- d) Is there a value of  $B$  that makes  $f$  continuous at  $x = -5$ ?

$x$	1	3	5	8
$f(x)$	-2	4	10	6

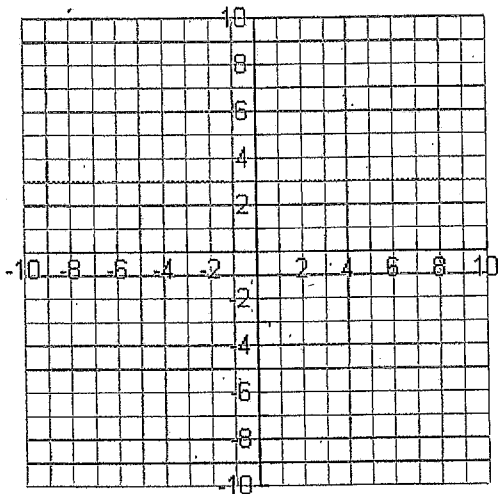
7. If  $f$  is continuous on  $[1, 8]$  and some values of  $f$  are given in the table above, which of the following must be true? Circle all that apply.

- I.  $f(x) = -3$  has a solution in  $[1, 8]$
- II.  $f(x) = 0$  has a solution in  $[1, 8]$
- III.  $f(x) = 9$  has a solution in  $[1, 8]$



8) Find value of given quantity

- a)  $\lim_{x \rightarrow -\infty} h(x) = -6$
- b)  $\lim_{x \rightarrow -4^-} h(x) = -3$
- c)  $\lim_{x \rightarrow -2} h(x) = 1$
- d)  $h(0) = 2$
- e)  $\lim_{x \rightarrow 0^-} h(x) = 2$
- f)  $\lim_{x \rightarrow 0} h(x) = \text{ONE}$
- g)  $\lim_{x \rightarrow 3^+} h(x) = -4$
- h)  $\lim_{x \rightarrow 4} h(x) = -\infty$
- i)  $\lim_{x \rightarrow 8} h(x) = 3$
- j)  $\lim_{x \rightarrow \infty} h(x) = 8$



9) Sketch graph of function satisfying the given values

- a)  $\lim_{x \rightarrow -\infty} f(x) = 8$
- b)  $f(-5) = 3$
- c)  $\lim_{x \rightarrow -5} f(x) = \text{DNE}$
- d)  $\lim_{x \rightarrow 1^-} f(x) = 3$
- e)  $\lim_{x \rightarrow 1^+} f(x) = -4$
- f)  $\lim_{x \rightarrow 3} f(x) = +\infty$
- g)  $\lim_{x \rightarrow +\infty} f(x) = -4$

$$1. g(x) = \begin{cases} \frac{x+5}{2x-4}, & x < 2 \\ 12, & x = 2 \\ 2x-5, & 2 < x < 5 \\ 20, & x = 5 \\ \frac{-x^2+4}{5-x}, & x > 5 \end{cases}$$

Find the following:

correction

a) $\lim_{x \rightarrow -\infty} g(x) =$ $\boxed{\frac{1}{2}}$	b) $\lim_{x \rightarrow 2^-} g(x) =$ $\boxed{-\infty}$	c) $\lim_{x \rightarrow 2^+} g(x) =$ <del><math>\boxed{+\infty}</math></del> $\boxed{-1}$
d) $\lim_{x \rightarrow 2} g(x) =$ $\boxed{\text{ONE}}$	e) $\lim_{x \rightarrow 5^-} g(x) =$ $\boxed{5}$	f) $\lim_{x \rightarrow 5^+} g(x) =$ $\boxed{+\infty}$
g) $\lim_{x \rightarrow 5} g(x) =$ $\boxed{\text{ONE}}$	h) $\lim_{x \rightarrow 3^+} g(x) =$ $\boxed{1}$	i) $\lim_{x \rightarrow \infty} g(x) =$ $\boxed{+\infty}$

Use Continuity conditions to justify whether  $f(x)$  is continuous or discontinuous. If discontinuous, identify type of discontinuity

1)  $f(x) = \begin{cases} 2x^2 + 5, & x \geq 3 \\ 2x, & x < 3 \end{cases}$   
 i)  $f(3) = 2(3)^2 + 5 = 23$   
 ii)  $\lim_{x \rightarrow 3^-} 2x = 6$  and  $\lim_{x \rightarrow 3^+} 2x^2 + 5 = 23$  so  $\lim_{x \rightarrow 3} f(x) = \text{DNE}$   
 Nonremovable discontinuity at  $x=3$

3) Find the horizontal asymptote(s) of

$$f(x) = \frac{-2x-5}{\sqrt{6x^2+11x-30}}$$

$\lim_{x \rightarrow \infty} \frac{-2x-5}{\sqrt{6x^2+11x-30}} = \frac{-2}{\sqrt{6}}$   
 $\lim_{x \rightarrow -\infty} \frac{-2x-5}{-\sqrt{6x^2+11x-30}} = \frac{-2}{-\sqrt{6}} = \frac{2}{\sqrt{6}}$

$y = \pm \frac{2}{\sqrt{6}}$

4) Find  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$$\lim_{x \rightarrow 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \frac{1}{4}$$

5) Let  $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$

Which of the following is true? (Circle all that apply)

$\lim_{x \rightarrow 3} f(x)$  does not exist

$$\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)} \quad \lim_{x \rightarrow 3} x+3 = \boxed{6}$$

II.  $f$  is continuous at  $x = 3$

III. The line  $x = 3$  is a vertical asymptote:

i)  $f(3) = 6$   
 ii)  $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3} = 6$   
 iii)  $\lim_{x \rightarrow 3} f(x) = f(3)$

} All continuity condition passes.

8

6. Let  $f(x) = \begin{cases} \frac{x^2-7x+10}{x^2-25}, & x^2 \neq 25 \\ A, & x = 5 \\ B, & x = -5 \end{cases}$

$\frac{(x-5)(x-2)}{(x-5)(x+5)}$

At  $x=5$  is a hole since factor cancels in denominator  
 At  $x=-5$ , vertical asymptote

a) Are the lines  $x=5$  and  $x=-5$  vertical asymptotes? Justify answer

b) Identify all horizontal asymptotes. Justify answer  $\lim_{x \rightarrow -\infty} f(x) = 1$   $\lim_{x \rightarrow +\infty} f(x) = 1$

c) Is there a value of A that makes f continuous at  $x=5$ ? yes,  $\lim_{x \rightarrow 5} \frac{x-2}{x+5} = \frac{3}{10}$  so let  $A = \frac{3}{10}$

d) Is there a value of B that makes f continuous at  $x=-5$  No, since nonremovable discontinuity at  $x=-5$

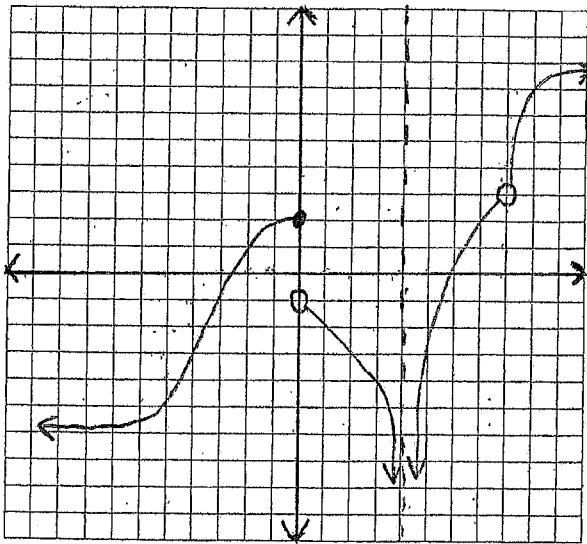
x	1	3	5	8
f(x)	-2	4	10	6

7. If f is continuous on  $[1,8]$  and some values of f are given in the table above, which of the following must be true? Circle all that apply.

i.  $f(x) = -3$  has a solution in  $[1,8]$

ii.  $f(x) = 0$  has a solution in  $[1,8]$   
 By IVT, since  $f(1) < 0 < f(8)$ , c exists in  $[1,8]$

iii.  $f(x) = 9$  has a solution in  $[1,8]$   
 By IVT, since  $f(1) < 9 < f(5)$ , c exists in  $[1,5]$ , therefore would exist in  $[1,8]$



8) Find value of given quantity

a)  $\lim_{x \rightarrow -\infty} h(x) = -6$

f)  $\lim_{x \rightarrow 0} h(x) = \text{ONE}$

b)  $\lim_{x \rightarrow 4^-} h(x) = -3$

g)  $\lim_{x \rightarrow 3^+} h(x) = -4$

c)  $\lim_{x \rightarrow -2} h(x) = 1$

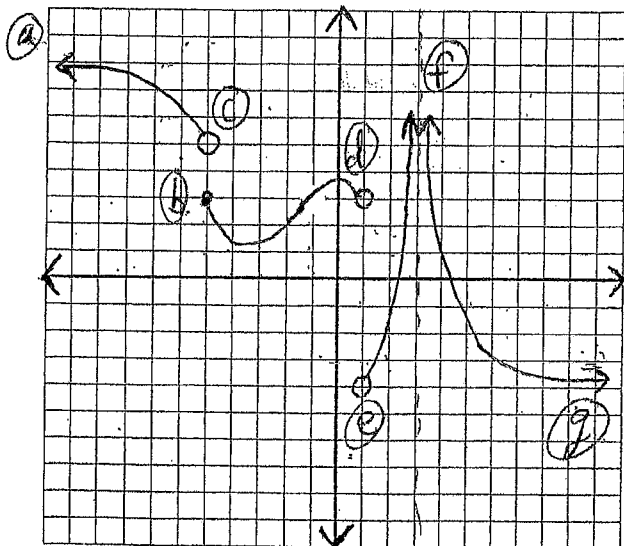
h)  $\lim_{x \rightarrow 4} h(x) = -\infty$

d)  $h(0) = 2$

i)  $\lim_{x \rightarrow 8} h(x) = 3$

e)  $\lim_{x \rightarrow 0^-} h(x) = 2$

j)  $\lim_{x \rightarrow \infty} h(x) = 8$



Sketch graph of function satisfying the given values.

a)  $\lim_{x \rightarrow -\infty} f(x) = 8$

e)  $\lim_{x \rightarrow 1^+} f(x) = -4$

b)  $f(-5) = 3$

f)  $\lim_{x \rightarrow 3} f(x) = +\infty$

c)  $\lim_{x \rightarrow -5} f(x) = \text{DNE}$

g)  $\lim_{x \rightarrow +\infty} f(x) = -4$

d)  $\lim_{x \rightarrow 1^-} f(x) = 3$



I. Algebraic Steps Evaluating Limits Approaching a Real Number  $\lim_{x \rightarrow c} f(x)$

1. Plug in argument x-value (Ignore one-sided limit for now)
2. Find the Limit (plug in/ reduce if  $\frac{0}{0}$ , re-evaluate)
3. If Limit DNE (does not exist), then evaluate further ONLY IF one-sided limit)
4. Choose between  $+\infty$  and  $-\infty$
5. Plug in the appropriate decimal value to determine  $+\infty$  or  $-\infty$

II. Algebraic Steps Evaluating Limits Approaching Infinity  $\lim_{x \rightarrow \infty} f(x)$  or  $\lim_{x \rightarrow -\infty} f(x)$

1. Compare degrees between numerator vs. denominator
  - a. If Numerator < Denominator  $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$
  - b. If Numerator = Denominator  $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = \text{ratio of coefficients}$
  - c. If Numerator > Denominator, then  $\lim_{x \rightarrow \infty} f(x) = \pm\infty$  or  $\lim_{x \rightarrow -\infty} f(x) = \pm\infty$

(Plug in a large positive or large negative value to help you determine the sign at infinity)

III. Continuity Conditions

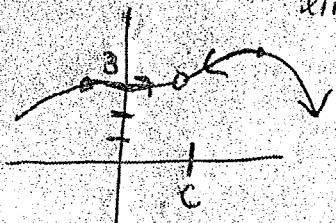
1.  $f(c)$  is defined (point exists on the graph)
  2. The  $\lim_{x \rightarrow c} f(x)$  exists  $\left[ \lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^-} f(x) \right]$
  3.  $f(c) = \lim_{x \rightarrow c} f(x)$
- If function passes all 3 conditions, the function has continuity at  $x = c$
  - If condition #2 FAILS, the function has **nonremovable** discontinuity at  $x = c$
  - If function PASSES condition #2 and FAILS condition #3, the function has **removable** discontinuity at  $x = c$

IV. Intermediate Value Theorem (IVT) Steps

1. Test and determine continuity on closed interval  $[a, b]$
2. Find the y-value at the endpoints,  $f(a)$  and  $f(b)$
3. Confirm that  $f(c)$  is between  $f(a)$  and  $f(b)$  [ example:  $f(a) < f(c) < f(b)$  ]
4. Find the c-value (find the x-value by plugging the y-value given at the start of problem into the function)

\*Make sure c-value(s) are inside the interval  $[a,b]$ . c-values that are outside the interval  $[a,b]$  are excluded.

If hole in graph, one-sided limits are same as the full limit

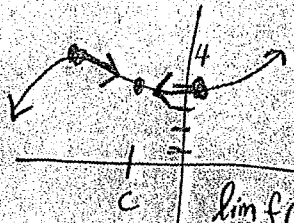


$$\lim_{x \rightarrow c^-} f(x) = 3$$

$$\lim_{x \rightarrow c^+} f(x) = 3$$

$$\lim_{x \rightarrow c} f(x) = 3$$

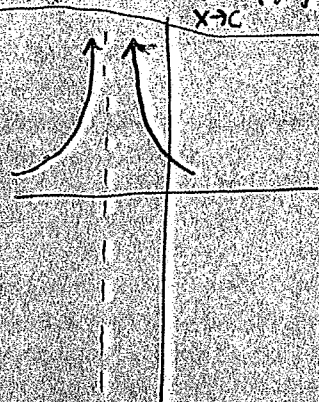
If function is continuous, one-sided limits are same as the full limit



$$\lim_{x \rightarrow c^-} f(x) = 4$$

$$\lim_{x \rightarrow c^+} f(x) = 4$$

$$\lim_{x \rightarrow c} f(x) = 4$$

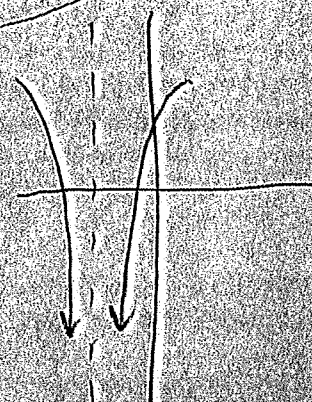


limit dne  
(+∞)

$$\lim_{x \rightarrow c^-} f(x) = +\infty$$

$$\lim_{x \rightarrow c^+} f(x) = +\infty$$

$$\lim_{x \rightarrow c} f(x) = \text{dne} \quad (+\infty)$$

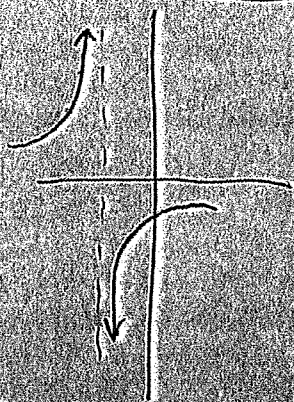


limit dne  
(-∞)

$$\lim_{x \rightarrow c^-} f(x) = -\infty$$

$$\lim_{x \rightarrow c^+} f(x) = -\infty$$

$$\lim_{x \rightarrow c} f(x) = \text{dne} \quad (-\infty)$$

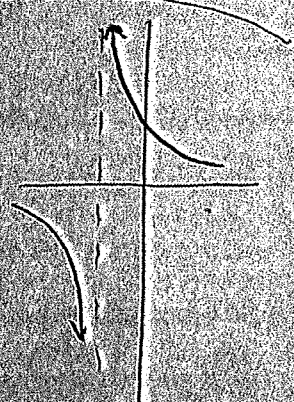


limit dne

$$\lim_{x \rightarrow c^-} f(x) = +\infty$$

$$\lim_{x \rightarrow c^+} f(x) = -\infty$$

$$\lim_{x \rightarrow c} f(x) = \text{dne}$$



limit dne

$$\lim_{x \rightarrow c^-} f(x) = -\infty$$

$$\lim_{x \rightarrow c^+} f(x) = +\infty$$

$$\lim_{x \rightarrow c} f(x) = \text{dne}$$

$$1. \quad g(x) = \begin{cases} \frac{9-x^2}{x^2-16}, & x < -4 \\ 2-x, & x = -4 \\ \frac{-x^2}{x+2}, & x > -4 \end{cases}$$

Given the piecewise function, find the following:

a.  $\lim_{x \rightarrow -4^-} g(x) =$       b.  $\lim_{x \rightarrow -\infty} g(x) =$       c.  $\lim_{x \rightarrow -4^+} g(x) =$       d.  $\lim_{x \rightarrow -4} g(x) =$       e.  $\lim_{x \rightarrow \infty} g(x) =$

1b) Step through continuity conditions for #1. Then state reason for discontinuity and identify the type of discontinuity. (at  $x = -4$ )

2. Find the horizontal asymptote(s) of  $f(x) = \frac{-x-5}{\sqrt{4x^2+12x-30}}$

3. Show whether Intermediate Value Theorem applies. Then find  $c$ . (Go through conditions!).

$f(x) = \frac{x}{x-3}; [4, 8] \quad f(c) = 2$

$c =$  \_\_\_\_\_

4. Find values for below:

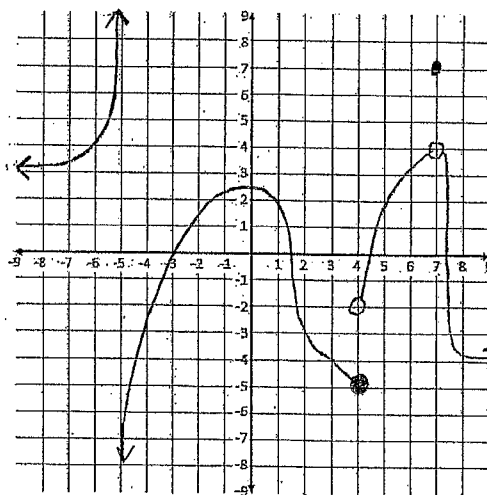
a.  $\lim_{x \rightarrow -5^-} g(x) =$

b.  $\lim_{x \rightarrow -\infty} g(x) =$

c.  $\lim_{x \rightarrow 4^+} g(x) =$

d.  $\lim_{x \rightarrow 7} g(x) =$

e.  $\lim_{x \rightarrow \infty} g(x) =$



$$1. g(x) = \begin{cases} \frac{9-x^2}{x^2-16}, & x < -4 \\ 2-x, & x = -4 \\ \frac{-x^2}{x+2}, & x > -4 \end{cases}$$

Given the piecewise function, find the following:

<p>a. <math>\lim_{x \rightarrow -4^-} g(x) =</math>  <math>\lim_{x \rightarrow -4^-} \frac{9-x^2}{x^2-16}</math>  <math>\lim_{x \rightarrow -4^-} \frac{9-x^2}{(x-4)(x+4)} = \frac{9-(-4)^2}{(-4-4)(-4+4)} = \frac{-7}{(-8)(0)} = -\infty</math></p>	<p>b. <math>\lim_{x \rightarrow -\infty} g(x) =</math>  <math>\lim_{x \rightarrow -\infty} \frac{9-x^2}{x^2-16} = -1</math></p>	<p>c. <math>\lim_{x \rightarrow -4^+} g(x) =</math>  <math>\lim_{x \rightarrow -4^+} \frac{-x^2}{x+2} = \frac{-16}{-2} = 8</math></p>	<p>d. <math>\lim_{x \rightarrow -4} g(x) =</math>  <math>\lim_{x \rightarrow -4} f(x) \neq \lim_{x \rightarrow -4^+} f(x)</math>  <b>DNE!</b></p>	<p>e. <math>\lim_{x \rightarrow \infty} g(x) =</math>  <math>\lim_{x \rightarrow \infty} \frac{-x^2}{x+2} = -\infty</math></p>
--	---	---	---	--

1b) Step through continuity conditions for #1. Then state reason for discontinuity and identify the type of discontinuity.

i)  $f(-4) = 6$   
 ii)  $\lim_{x \rightarrow -4^-} f(x) \neq \lim_{x \rightarrow -4^+} f(x)$  Nonremovable discontinuity since  $\lim_{x \rightarrow -4} f(x) = \text{DNE}$

2. Find the horizontal asymptote(s) of  $f(x) = \frac{-x-5}{\sqrt{4x^2+12x-30}}$

$$\lim_{x \rightarrow +\infty} \frac{-x-5}{\sqrt{4x^2+12x-30}} = \frac{-1}{\sqrt{4}} = -\frac{1}{2}$$

$$\lim_{x \rightarrow -\infty} \frac{-x-5}{\sqrt{4x^2+12x-30}} = \lim_{x \rightarrow -\infty} \frac{-x-5}{-\sqrt{4x^2+12x-30}} = \frac{-1}{-\sqrt{4}} = \frac{1}{2}$$

H.A. at  $y = -\frac{1}{2}$  and  $y = \frac{1}{2}$

3. Show whether Intermediate Value Theorem applies. Then find c. (Go through conditions!).

$$f(x) = \frac{x}{x-3}; [4, 8], k=2$$

i)  $f(x)$  continuous on interval  $[4, 8]$

$$ii) f(4) = 4$$

$$f(8) = \frac{8}{8-3} = \frac{8}{5} = 1.6$$

$$c = 6$$

$$\frac{x}{x-3} = \frac{2}{1} \quad 2(x-3) = x$$

$$2x-6 = x$$

$$x=6 \text{ so } c=6$$

Since  $f(8) < f(c) < f(4)$  IVT applies and c exists in interval.

4. Find values for below:

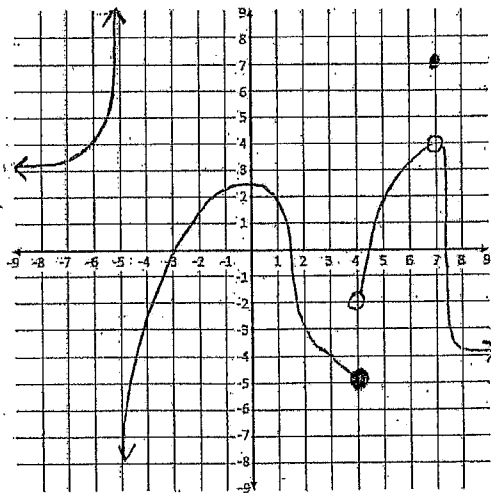
a.  $\lim_{x \rightarrow -5^-} g(x) = +\infty$

b.  $\lim_{x \rightarrow -\infty} g(x) = 3$

c.  $\lim_{x \rightarrow 4^+} g(x) = -2$

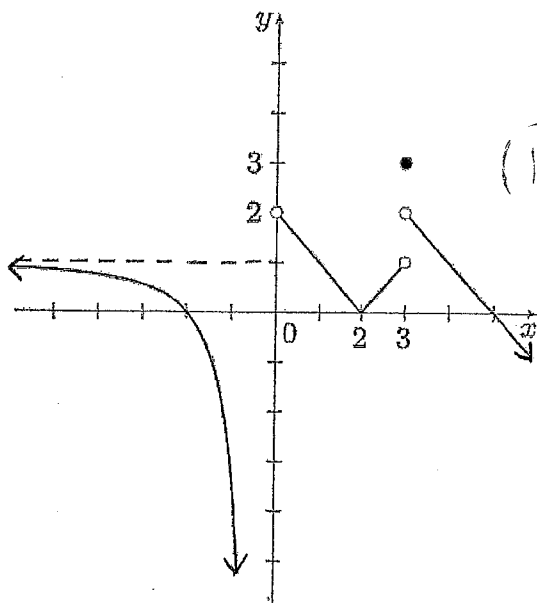
d.  $\lim_{x \rightarrow 7} g(x) = 4$

e.  $\lim_{x \rightarrow \infty} g(x) = -4$



Calculus AB 1.4-3.5 Morning Quiz Review

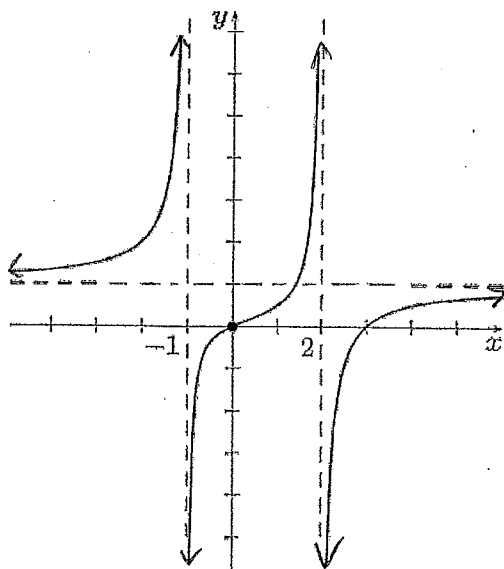
1. Use the graph of the function  $f(x)$  to answer each question.  
Use  $\infty$ ,  $-\infty$  or  $DNE$  where appropriate.



- (a)  $f(0) =$   
 (b)  $f(2) =$   
 (c)  $f(3) =$   
 (d)  $\lim_{x \rightarrow 0^-} f(x) =$   
 (e)  $\lim_{x \rightarrow 0} f(x) =$   
 (f)  $\lim_{x \rightarrow 3^+} f(x) =$   
 (g)  $\lim_{x \rightarrow 3} f(x) =$   
 (h)  $\lim_{x \rightarrow -\infty} f(x) =$

- 1b) Step through continuity conditions to determine if  $f(x)$  is continuous at  $x = 3$ . If discontinuous, determine type of discontinuity

2. Use the graph of the function  $f(x)$  to answer each question.  
Use  $\infty$ ,  $-\infty$  or  $DNE$  where appropriate.



- (a)  $f(0) =$   
 (b)  $f(2) =$   
 (c)  $f(3) =$   
 (d)  $\lim_{x \rightarrow -1^+} f(x) =$   
 (e)  $\lim_{x \rightarrow 0} f(x) =$   
 (f)  $\lim_{x \rightarrow 2^+} f(x) =$   
 (g)  $\lim_{x \rightarrow \infty} f(x) =$

14

3. Determine the horizontal asymptote(s) of  $y = \frac{-4x^{\frac{2}{3}} + 15}{\sqrt[3]{5x^2 - 23}}$

4. Step through continuity conditions. If discontinuous, Show work to determine type of discontinuity

a)  $f(x) = \begin{cases} -2x + 3 & x > -4 \\ 4x - 1 & x = -4 \\ \frac{5x-1}{2x} & x < -4 \end{cases}$  at  $x = -4$

b)  $g(x) = \begin{cases} \frac{x^2-4}{x+2} & x \neq -2 \\ 3x + 2 & x = -2 \end{cases}$

Answer the following questions for the piecewise defined function  $f(t)$  described on the right hand side.

(a)  $f(-3/2) =$

(b)  $f(2) =$

(c)  $f(1) =$

(d)  $\lim_{t \rightarrow -2} f(t) =$

(e)  $\lim_{t \rightarrow -1^+} f(t) =$

(f)  $\lim_{t \rightarrow 2} f(t) =$

(g)  $\lim_{t \rightarrow 0} f(t) =$

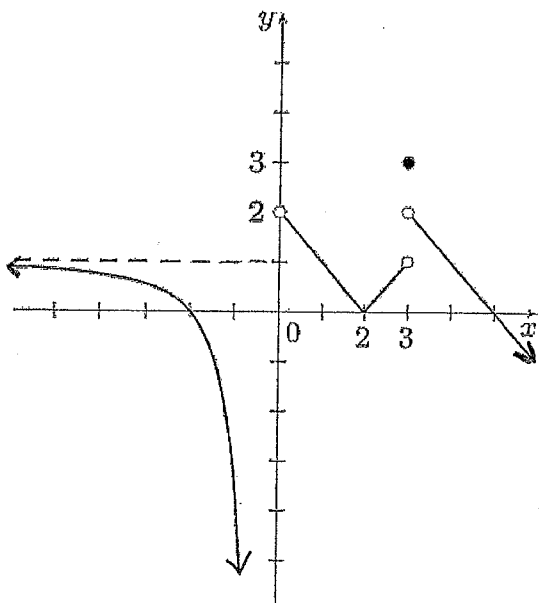
(h)  $\lim_{t \rightarrow 0^-} f(t) =$

$f(t) = \begin{cases} t^2 & \text{for } t < -2 \\ \frac{t+6}{t^2-t} & \text{for } -1 < t < 2 \\ 3t-2 & \text{for } t \geq 2 \end{cases}$
--

Calculus AB 1.4-3.5 Morning Quiz Review

Key (15)

1. Use the graph of the function  $f(x)$  to answer each question.  
Use  $\infty$ ,  $-\infty$  or  $DNE$  where appropriate.



- (a)  $f(0) = DNE$
- (b)  $f(2) = 0$
- (c)  $f(3) = 3$
- (d)  $\lim_{x \rightarrow 0^-} f(x) = -\infty$
- (e)  $\lim_{x \rightarrow 0} f(x) = DNE$
- (f)  $\lim_{x \rightarrow 3^+} f(x) = 2$
- (g)  $\lim_{x \rightarrow 3} f(x) = DNE$
- (h)  $\lim_{x \rightarrow -\infty} f(x) = 1$

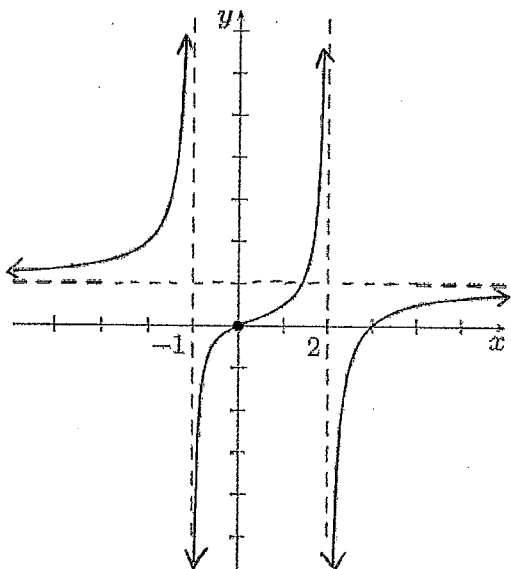
- 1b) Step through continuity conditions to determine if  $f(x)$  is continuous at  $x = 3$ . If discontinuous, determine type of discontinuity

i)  $f(3) = 3$   
 ii)  $\lim_{x \rightarrow 3^-} f(x) = 1$   
 $\lim_{x \rightarrow 3^+} f(x) = 2$

Since  $\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$ ,  $\lim_{x \rightarrow 3} f(x) = DNE$

nonremovable discontinuity at  $x=3$

2. Use the graph of the function  $f(x)$  to answer each question.  
Use  $\infty$ ,  $-\infty$  or  $DNE$  where appropriate.



- (a)  $f(0) = 0$
- (b)  $f(2) = DNE$
- (c)  $f(3) = 0$
- (d)  $\lim_{x \rightarrow -1^+} f(x) = -\infty$
- (e)  $\lim_{x \rightarrow 0} f(x) = 0$
- (f)  $\lim_{x \rightarrow 2^+} f(x) = -\infty$
- (g)  $\lim_{x \rightarrow \infty} f(x) = 1$

3. Determine the horizontal asymptote(s) of  $y = \frac{-4x^{\frac{2}{3}} + 15}{\sqrt[3]{5x^2 - 23}}$

$$\lim_{x \rightarrow \infty} \frac{-4x^{\frac{2}{3}}}{\sqrt[3]{5x^{\frac{2}{3}}}} = \frac{-4}{\sqrt[3]{5}}$$

$$\lim_{x \rightarrow -\infty} \frac{-4x^{\frac{2}{3}}}{\sqrt[3]{5x^{\frac{2}{3}}}} = -\left(\frac{-4x^{\frac{2}{3}}}{\sqrt[3]{5x^{\frac{2}{3}}}}\right) = \frac{4}{\sqrt[3]{5}}$$

$$\text{H.A. at } y = \frac{-4}{\sqrt[3]{5}}, y = \frac{4}{\sqrt[3]{5}}$$

4. Step through continuity conditions. If discontinuous, Show work to determine type of discontinuity

a)  $f(x) = \begin{cases} -2x + 3 & x > -4 \\ 4x - 1 & x = -4 \\ \frac{5x-1}{2x} & x < -4 \end{cases}$  at  $x = -4$

b)  $g(x) = \begin{cases} \frac{x^2-4}{x+2} & x \neq -2 \\ 3x+2 & x = -2 \end{cases}$  at  $x = -2$

i)  $f(-4) = -16 - 1 = -17$

i)  $g(-2) = -4$

ii)  $\lim_{x \rightarrow -4^-} \frac{5x-1}{2x} = \frac{-21}{-8} = 2\frac{1}{8}$

ii)  $\lim_{x \rightarrow -2} \frac{x^2-4}{x+2} = \lim_{x \rightarrow -2} \frac{(x+2)(x-2)}{(x+2)} = -4$

$\lim_{x \rightarrow -4^+} -2x + 3 = -5$

iii)  $g(-2) = \lim_{x \rightarrow -2} g(x) = -4$

Since  $\lim_{x \rightarrow -4^+} f(x) \neq \lim_{x \rightarrow -4^-} f(x)$ ,  
 $\lim_{x \rightarrow -4} f(x) = \text{DNE}$ , nonremovable discontinuity at  $x = -4$

All 3 conditions pass,  
 $g(x)$  continuous at  $x = -2$

Answer the following questions for the piecewise defined function  $f(t)$  described on the right hand side.

c)  $\frac{1+6}{1^2-1} = \frac{7}{0}$

(a)  $f(-3/2) = \text{DNE}$

(b)  $f(2) = 4$      $3(2) - 2 = 4$

(c)  $f(1) = \text{DNE}$

(d)  $\lim_{t \rightarrow -2} f(t) = \text{DNE}$

(e)  $\lim_{t \rightarrow -1^+} f(t) = 5/2$

(f)  $\lim_{t \rightarrow 2} f(t) = 4$

(g)  $\lim_{t \rightarrow 0} f(t) = \text{DNE}$

(h)  $\lim_{t \rightarrow 0^-} f(t) = +\infty$

$\lim_{t \rightarrow 0^-} \frac{t+6}{t^2-t} = \frac{6}{0} \rightarrow \begin{matrix} +\infty \\ \text{or} \\ -\infty \end{matrix}$

$$f(t) = \begin{cases} t^2 & \text{for } t < -2 \\ \frac{t+6}{t^2-t} & \text{for } -1 < t < 2 \\ 3t-2 & \text{for } t \geq 2 \end{cases}$$

f)  $\lim_{x \rightarrow 2^-} \frac{t+6}{t^2-t} = \frac{8}{2} = 4$   
 $\lim_{x \rightarrow 2^+} 3t-2 = 4$  }  $\lim_{x \rightarrow 2} f(x) = 4$

$\lim_{t \rightarrow 0^-} \frac{-0.1+6}{(-0.1)^2 - (-0.1)} = \frac{+}{+} = \boxed{+\infty}$



# Limits - Piecewise Practice Problems

1)

$$f(x) = \begin{cases} \frac{x+2}{x^2-4} & x < -4 \\ x+3 & -4 \leq x < 3 \\ \frac{x^3}{x-5} & x \geq 3 \end{cases}$$

a)  $\lim_{x \rightarrow -\infty} f(x)$

b)  $\lim_{x \rightarrow -4^-} f(x)$

c)  $\lim_{x \rightarrow -4^+} f(x)$

d)  $\lim_{x \rightarrow 4} f(x)$

e)  $\lim_{x \rightarrow 5^+} f(x)$

f)  $\lim_{x \rightarrow \infty} f(x)$

g)  $\lim_{x \rightarrow -4} f(x)$

h)  $f(-4)$

18) Limits Piecewise problems

Use continuity conditions  
to determine if continuous.  
If discontinuous, determine  
if removable/nonremovable

$$2) f(x) = \begin{cases} x^2 - 3, & x \geq 4 \\ \frac{x+1}{x^2+1}, & x < 4 \end{cases}$$

$$3) g(x) = \begin{cases} \frac{x^2-1}{x+1}, & x \neq -1 \\ 3, & x = -1 \end{cases}$$

# Limits - Piecewise Practice Problems

1)

$$f(x) = \begin{cases} \frac{x+2}{x^2-4} & x < -4 \\ x+3 & -4 \leq x < 3 \\ \frac{x^3}{x-5} & x \geq 3 \end{cases}$$

a)  $\lim_{x \rightarrow -\infty} f(x)$

b)  $\lim_{x \rightarrow -4^-} f(x)$

c)  $\lim_{x \rightarrow -4^+} f(x)$

d)  $\lim_{x \rightarrow 4} f(x)$

e)  $\lim_{x \rightarrow 5^+} f(x)$

f)  $\lim_{x \rightarrow \infty} f(x)$

g)  $\lim_{x \rightarrow -4} f(x)$

h)  $f(-4)$

Key

(19)

a)  $\lim_{x \rightarrow -\infty} \frac{x+2}{x^2-4} = 0$

b)  $\lim_{x \rightarrow -4^-} \frac{x+2}{x^2-4} = \frac{-2}{12} = -\frac{1}{6}$

c)  $\lim_{x \rightarrow -4^+} x+3 = -1$

d)  $\lim_{x \rightarrow 4} \frac{x^3}{x-5} = \frac{64}{-1} = -64$

e)  $\lim_{x \rightarrow 5^+} \frac{x^3}{x-5} = \frac{125}{0} \rightarrow \begin{matrix} \nearrow +\infty \\ \searrow -\infty \end{matrix}$   
 $\frac{(5.1)^3}{5.1-5} = \frac{+}{+} = \boxed{+\infty}$

f)  $\lim_{x \rightarrow \infty} \frac{x^3}{x-5} = \boxed{+\infty}$

g)  $\lim_{x \rightarrow -4} f(x) = \boxed{\text{DNE}}$

$\hookrightarrow \lim_{x \rightarrow -4^-} f(x) = -\frac{1}{6}$

$\hookrightarrow \lim_{x \rightarrow -4^+} f(x) = -1$

$\lim_{x \rightarrow -4^-} f(x) \neq \lim_{x \rightarrow -4^+} f(x)$

h)  $f(-4) = \boxed{-1}$

## 20) Limits Piecewise problems

Use continuity conditions to determine if continuous. If discontinuous, determine if removable/nonremovable

$$2) f(x) = \begin{cases} x^2 - 3, & x \geq 4 \\ \frac{x+1}{x^2+1}, & x < 4 \end{cases}$$

$$3) g(x) = \begin{cases} \frac{x^2-1}{x+1}, & x \neq -1 \\ 3, & x = -1 \end{cases}$$

## KEY

$$2) i) f(4) = 16 - 3 = 13$$

$$ii) \lim_{x \rightarrow 4^-} \frac{x+1}{x^2+1} = \frac{5}{17}$$

$$\lim_{x \rightarrow 4^+} x^2 - 3 = 13$$

$$\text{Since } \lim_{x \rightarrow 4^-} f(x) \neq \lim_{x \rightarrow 4^+} f(x)$$

$$\lim_{x \rightarrow 4} f(x) = \text{DNE},$$

Nonremovable discontinuity at  $x = 4$

$$3) i) f(-1) = 3$$

$$ii) \lim_{x \rightarrow -1} \frac{x^2-1}{x+1} = \lim_{x \rightarrow -1} \frac{(x+1)(x-1)}{x+1} = -2$$

$$iii) f(-1) \neq \lim_{x \rightarrow -1} f(x)$$

Removable discontinuity at  $x = -1$

Limits Chapter 1 Morning Test Review #3

1. 
$$\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$$

2.

For  $x \neq 4$ , the function  $h(x)$  is equal to  $\frac{x^3 + x - 20}{x - 4}$ . What value should be assigned to  $h(4)$  to make  $h(x)$  continuous at  $x = 4$ ?

3.

Given 
$$f(x) = \begin{cases} x^2 - 2, & \text{if } x \leq 0 \\ 3x + k, & \text{if } x > 0 \end{cases}$$

Find the value for the constant  $k$  that will make the function continuous at  $x = 0$ .

4. (OMIT)

Determine the values of  $a$  and  $b$  so that  $f(x)$  is everywhere continuous. Justify your answer.

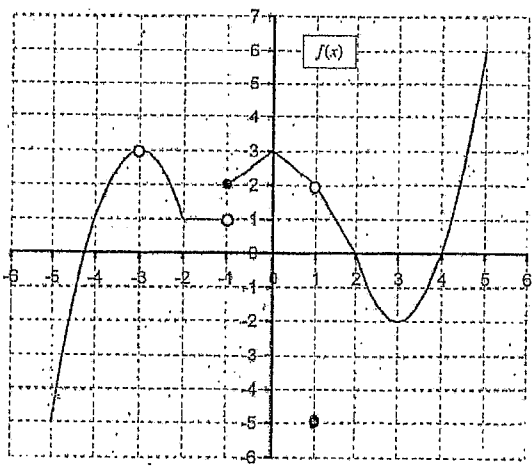
~~$$f(x) = \begin{cases} 5bx - 6a, & \text{if } x < -2 \\ -3b - 4ax, & \text{if } x = -2 \\ 5x - 1, & \text{if } x > -2 \end{cases}$$~~

5.

Let  $f$  be a continuous function. Selected values of  $f$  are given in the table below.

$x$	1	2	3	5	8
$f(x)$	-2	3	1	-5	7

What is the least number of solutions does the equation  $f(x) = \frac{1}{2}$  have on the closed interval  $[1, 8]$ ?



- $\lim_{x \rightarrow -3^-} f(x) =$       b)  $\lim_{x \rightarrow 3^+} f(x) =$       c)  $\lim_{x \rightarrow -3} f(x) =$   
 $\lim_{x \rightarrow -1} f(x) =$       e)  $\lim_{x \rightarrow 1^+} f(x) =$       f)  $\lim_{x \rightarrow 1} f(x) =$

Use continuity conditions to justify if graph is continuous at  $x = 1$ . Determine the type of discontinuity.

7. Find the value of  $k$  that makes  $f(x)$  continuous for all real numbers if:

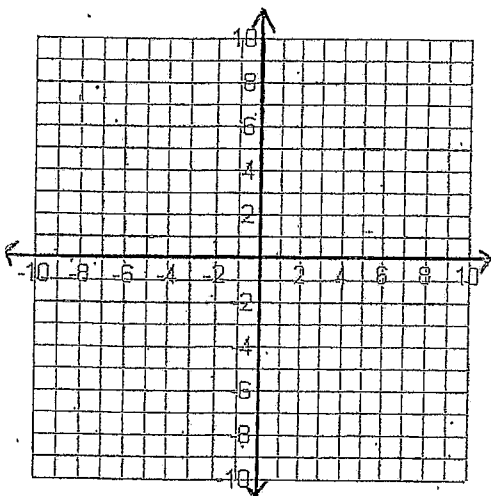
$$g(x) = \begin{cases} x^2 + x - 6, & x \neq -3 \\ x + 3, & x = -3 \\ k, & x = -3 \end{cases}$$

8.

Find  $A = \lim_{x \rightarrow -3^+} \frac{x(x+1)}{3-x}$  and  $B = \lim_{x \rightarrow \infty} \frac{(x^2+2)(3x^2-5)}{x^4+6}$ .

- a)  $A = -\infty$  and  $B = -\frac{5}{3}$
- b)  $A = 0$  and  $B = 0$
- c)  $A = \infty$  and  $B = \infty$
- d)  $A = -\infty$  and  $B = 3$

9. Evaluate  $\lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{x}$



9) Sketch graph satisfying the given values

- a)  $\lim_{x \rightarrow -\infty} h(x) = -\infty$
- b)  $\lim_{x \rightarrow -4^-} h(x) = -3$
- c)  $\lim_{x \rightarrow 4^+} h(x) = 1$
- d)  $h(6) = \text{undefined}$
- e)  $\lim_{x \rightarrow 0^-} h(x) = -\infty$
- f)  $\lim_{x \rightarrow 0} h(x) = \text{DNE}$
- g)  $\lim_{x \rightarrow 3^+} h(x) = -4$
- h)  $\lim_{x \rightarrow 4} h(x) = -\infty$
- i)  $\lim_{x \rightarrow 6} h(x) = 3$
- j)  $\lim_{x \rightarrow \infty} h(x) = -2$

Limits Chapter 1 Morning Test Review #3

$\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} = \frac{\sqrt{2} - \sqrt{2}}{0} = \frac{0}{0}$

$\lim_{x \rightarrow 0} \frac{(\sqrt{2+x} - \sqrt{2})(\sqrt{2+x} + \sqrt{2})}{(x)(\sqrt{2+x} + \sqrt{2})} = \frac{2 - 2}{(x)(\sqrt{2+x} + \sqrt{2})} = \frac{0}{(x)(\sqrt{2+x} + \sqrt{2})}$

For  $x \neq 0$ , the function  $h(x)$  is equal to  $\frac{1}{\sqrt{2+x} + \sqrt{2}}$ . What value should be assigned to  $h(0)$  to make  $h(x)$  continuous at  $x=0$ ? *continuity conditions*

i)  $h(0) = K$   
 ii)  $\lim_{x \rightarrow 0} \frac{1}{\sqrt{2+x} + \sqrt{2}} = \frac{1}{\sqrt{2} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$   
 iii)  $h(0) = \lim_{x \rightarrow 0} h(x) \rightarrow K = \frac{1}{2\sqrt{2}}$

Given  $f(x) = \begin{cases} x^2 - 2, & \text{if } x \leq 0 \\ 3x + k, & \text{if } x > 0 \end{cases}$   $c=0$

Find the value for the constant  $k$  that will make the function continuous at  $x=0$ .

i)  $f(0) = 0^2 - 2 = -2$   
 ii)  $\lim_{x \rightarrow 0^+} 3x + k = 3(0) + k = k$

$K = -2$

4. OMIT

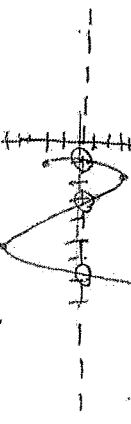
Determine the values of  $a$  and  $b$  so that  $f(x)$  is everywhere continuous. Justify your answer.

$f(x) = \begin{cases} 5bx - 6a, & \text{if } x < -2 \\ -3b - 4ax, & \text{if } x = -2 \\ 5x - 1, & \text{if } x > -2 \end{cases}$

5. Let  $f$  be a continuous function. Selected values of  $f$  are given in the table below.

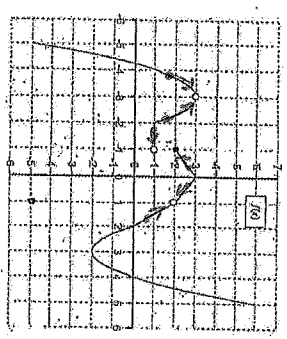
$x$	1	2	3	5	8
$f(x)$	-2	3	1	-5	7

What is the least number of solutions does the equation  $f(x) = \frac{1}{2}$  have on the closed interval  $[1, 8]$ ?



3 times for IVT

Key



7. Find the value of  $K$  that makes  $f(x)$  continuous for all real numbers if:

$f(x) = \begin{cases} x^2 + x - 6, & x \neq -3 \\ K, & x = -3 \end{cases}$

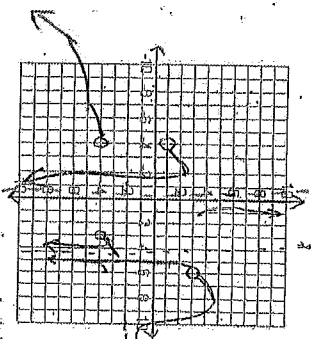
i)  $g(-3) = K$   
 ii)  $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3} = \lim_{x \rightarrow -3} \frac{(x+3)(x-2)}{x+3} = -3 - 2 = -5$   
 iii)  $g(-3) = \lim_{x \rightarrow -3} g(x) = -5$   
 $K = -5$

8. Find  $A = \lim_{x \rightarrow 3} \frac{x(x+1)}{3-x}$  and  $B = \lim_{x \rightarrow 3} \frac{(x^2+2)(3x^2-5)}{x^2+16}$

- a)  $A = -\infty$  and  $B = -\frac{5}{3}$
- b)  $A = 0$  and  $B = 0$
- c)  $A = \infty$  and  $B = \infty$
- d)  $A = -\infty$  and  $B = 3$

\* WSWK shoun on next page

9. Evaluate  $\lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{x}$   
 $\lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{x} = \frac{3 - \sqrt{9}}{0} = \frac{0}{0}$   
 $\lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{x} = \lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{(3 + \sqrt{x+9})} = \lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{3 + \sqrt{x+9}} = \frac{3 - 3}{3 + 3} = \frac{0}{6} = 0$



- 9) Sketch graph satisfying the given values
- a)  $\lim_{x \rightarrow 2} f(x) = -\infty$
  - b)  $\lim_{x \rightarrow 4} f(x) = 1$
  - c)  $\lim_{x \rightarrow 6} f(x) = -\infty$
  - d)  $\lim_{x \rightarrow 2} f(x) = -3$
  - e)  $\lim_{x \rightarrow 4} f(x) = -4$
  - f)  $\lim_{x \rightarrow 6} f(x) = 3$
  - g)  $\lim_{x \rightarrow 2} f(x) = -\infty$
  - h)  $\lim_{x \rightarrow 4} f(x) = -\infty$
  - i)  $\lim_{x \rightarrow 6} f(x) = -2$

a) \*step thru continuity conditions

i)  $k(4) = k$

ii)  $\lim_{x \rightarrow 4} \frac{x^2 + x - 20}{x - 4} \xrightarrow{\frac{0}{0}} \lim_{x \rightarrow 4} \frac{(x+5)(x-4)}{(x-4)} = 4+5 = \boxed{9}$

iii)  $h(4) = \lim_{x \rightarrow 4} f(x) = \boxed{k=9}$

8)  $\lim_{x \rightarrow 3} \frac{x(x+1)}{3-x} + \frac{3(3+1)}{3-3} = \frac{12}{0} \rightarrow \text{VA} \rightarrow \text{limit does not exist}$

test  $x=3.1$   $\left| \frac{3.1(3.1+1)}{3-3.1} \right| = \frac{+}{-} = \boxed{-\infty}$

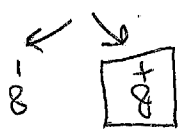
b)  $\lim_{x \rightarrow \infty} \frac{(x^2+2)(3x^2-5)}{x^4+6} \rightarrow \lim_{x \rightarrow \infty} \frac{3x^4 + |x^2 - 10|}{x^4+6} = \boxed{3}$

10)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x+1000}}{3^x}$

$\frac{\text{Rad}}{\text{Exponent}} = \boxed{0}$

$\frac{3^x}{\sqrt{x+1000}} = \frac{\text{Exp}}{\text{Rad}}$

test  $x=100$



Comparative growth rates  
 $L < R < P < E$



**Unit 1 Limits Test Topics**

- 1) Evaluate Limits Algebraically (Real #'s, Approaching Infinity, One-Sided limits)
- 2) Evaluating Limits Algebraic with hole in graph: Apply methods of Factoring, Conjugates, and/or Common Denominator
- 3) Identify limits and order pairs from a graph
- 4) Sketch a function graph given limit properties.
- 5) Use Continuity conditions with a piecewise function to determine continuity/discontinuity (and categorize the type of discontinuity)
- 6) Apply Intermediate Value Theorem to guarantee a value
- 7) Find Horizontal Asymptotes (Radical in denominator) ex.  $f(x) = \frac{3x}{\sqrt{5x^2-4}}$
- 8) Comparative Growth Rate: example:  $\lim_{n \rightarrow \infty} \frac{e^{2x}}{\sqrt{2x+1000}}$

