

$$1. g(x) = \begin{cases} x + 5, & x < 2 \\ 11, & x = 2 \\ 2x - 1, & 2 < x < 5 \\ 20, & x = 5 \\ x + 4, & x > 5 \end{cases}$$

Find the following :

|  |                                      |   |
|--|--------------------------------------|---|
| a) $\lim_{x \rightarrow -\infty} g(x) =$ | b) $\lim_{x \rightarrow 2^-} g(x) =$ | c) $\lim_{x \rightarrow 2^+} g(x) =$    |
| d) $\lim_{x \rightarrow 2} g(x) =$       | e) $\lim_{x \rightarrow 5^-} g(x) =$ | f) $\lim_{x \rightarrow 5^+} g(x) =$    |
| g) $\lim_{x \rightarrow 5} g(x) =$       | h) $\lim_{x \rightarrow 6^+} g(x) =$ | i) $\lim_{x \rightarrow \infty} g(x) =$ |

Use Continuity conditions to justify whether  $f(x)$  is continuous or discontinuous . If discontinuous, identify type of discontinuity

$$2) f(x) = \begin{cases} x^2 + 5, & x \geq 3 \\ x^2 - 2, & x < 3 \end{cases}$$

3) Find the  $k$  value that will make the function  $f(x)$  continuous.

$$b) f(x) = \begin{cases} \frac{x^2 + x - 6}{x + 3}, & x \neq -3 \\ k, & x = -3 \end{cases}$$

**PAST AP FREE-RESPONSE PROBLEMS COVERED BY THIS CHAPTER**

Note: These and other questions can be found at  
 apcentral.com.  
 2003 AB 6a  
 2006 BC 3c  
 2008 AB 6d

**MULTIPLE-CHOICE QUESTIONS**

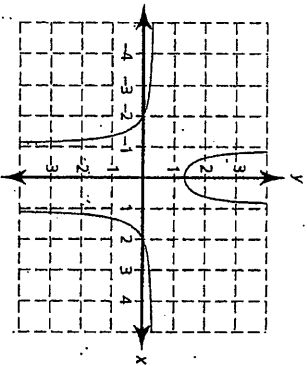
A calculator may not be used on the following questions.

- Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2 - x}$ .  
 (A) 5  
 (B) 3  
 (C) -3  
 (D) -5  
 (E) The limit does not exist.
- Evaluate the limit, if it exists:  $\lim_{x \rightarrow 9} \frac{\sqrt{x-5} - 2}{x-9}$ .  
 (A)  $\frac{1}{4}$   
 (B)  $-\frac{1}{4}$   
 (C) 1  
 (D) 0  
 (E) The limit does not exist.
- Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$ .  
 (A)  $\frac{1}{4}$   
 (B)  $-\frac{1}{4}$   
 (C) 1  
 (D) -1  
 (E) The limit does not exist.

4)

For what value of  $k$  is the function  $f(x) = \begin{cases} 2x^2 + 5x - 3, & x \neq -3 \\ k, & x = -3 \end{cases}$  continuous at  $x = -3$ ?

- (A)  $-\frac{7}{6}$   
 (B)  $-\frac{5}{6}$   
 (C) 0  
 (D)  $\frac{5}{6}$   
 (E)  $\frac{7}{6}$



5) The function  $g(x)$  is shown in the graph above and is of the form  $g(x) = \frac{x^2 + a}{bx^2 - 3}$ . Which of the following could be the values of the constants  $a$  and  $b$ ?

- (A)  $a = -2, b = -1$   
 (B)  $a = -2, b = -3$   
 (C)  $a = -4, b = 3$   
 (D)  $a = -4, b = -3$   
 (E)  $a = 4, b = 3$

6) Identify the vertical asymptotes for  $f(x) = \frac{x^2 + 3x - 4}{x^2 + x - 2}$ .

- (A)  $x = -2, x = 1$   
 (B)  $x = -2$   
 (C)  $x = 1$   
 (D)  $y = -2, y = 1$   
 (E)  $y = -2$

$$1. g(x) = \begin{cases} x + 5, & x < 2 \\ 11, & x = 2 \\ 2x - 1, & 2 < x < 5 \\ 20, & x = 5 \\ x + 4, & x > 5 \end{cases}$$

Find the following :

|  |  |   |
|--|--|---|
| <p>a) <math>\lim_{x \rightarrow -\infty} g(x) =</math></p> $\lim_{x \rightarrow -\infty} \frac{x+5}{-1} = \boxed{-\infty}$                   | <p>b) <math>\lim_{x \rightarrow 2^-} g(x) =</math></p> $\lim_{x \rightarrow 2^-} x+5 = \boxed{7}$  | <p>c) <math>\lim_{x \rightarrow 2^+} g(x) =</math></p> $\lim_{x \rightarrow 2^+} 2x-1 = \boxed{3}$                      |
| <p>d) <math>\lim_{x \rightarrow 2} g(x) =</math></p> $\lim_{x \rightarrow 2^-} g(x) \neq \lim_{x \rightarrow 2^+} g(x)$ $\boxed{\text{DNE}}$ | <p>e) <math>\lim_{x \rightarrow 5^-} g(x) =</math></p> $\lim_{x \rightarrow 5^-} 2x-1 = \boxed{9}$ | <p>f) <math>\lim_{x \rightarrow 5^+} g(x) =</math></p> $\lim_{x \rightarrow 5^+} x+4 = \boxed{9}$                       |
| <p>g) <math>\lim_{x \rightarrow 5} g(x) =</math></p> $\boxed{9}$   | <p>h) <math>\lim_{x \rightarrow 6^+} g(x) =</math></p> $\lim_{x \rightarrow 6^+} x+4 = \boxed{10}$ | <p>i) <math>\lim_{x \rightarrow \infty} g(x) =</math></p> $\lim_{x \rightarrow \infty} \frac{x+4}{1} = \boxed{+\infty}$ |

Use Continuity conditions to justify whether f(x) is continuous or discontinuous. If discontinuous, identify type of discontinuity

$$2) f(x) = \begin{cases} x^2 + 5, & x \geq 3 \\ x^2 - 2, & x < 3 \end{cases}$$

i)  $f(3) = 3^2 + 5 = 14$

ii)  $\lim_{x \rightarrow 3^-} x^2 - 2 = 7$      $\lim_{x \rightarrow 3^+} x^2 + 5 = 14$     Since  $\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$

Nonremovable discontinuity  
since limit does not exist at  $x=3$

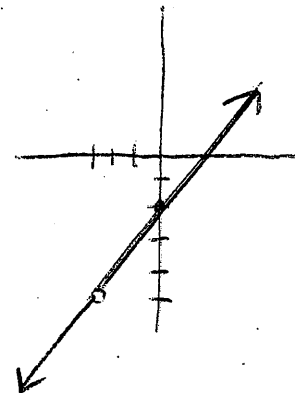
$\lim_{x \rightarrow 3} f(x) = \text{DNE}$

3) Find the k value that will make the function f(x) continuous.

b)  $f(x) = \begin{cases} \frac{x^2+x-6}{x+3}, & x \neq -3 \\ k, & x = -3 \end{cases}$     i)  $f(-3) = k$

ii)  $\lim_{x \rightarrow -3} \frac{x^2+x-6}{x+3} = \frac{0}{0} = \lim_{x \rightarrow -3} \frac{(x+3)(x-2)}{(x+3)} = -5$

iii)  $f(-3) = \lim_{x \rightarrow -3} f(x) = \boxed{k = -5}$



PAST AP FREE-RESPONSE PROBLEMS COVERED BY THIS CHAPTER

Note: These and other questions can be found at  
apcentral.com.

2003 AB 6a

2006 BC 3c

2008 AB 6d

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

1. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2 - x}$ .  $\lim_{x \rightarrow 2} \frac{(x+3)(x-2)}{-(x-2)} = \frac{5}{-1} = \boxed{-5}$
- (A) 5  
(B) 3  
(C) -3  
(D) -5  
(E) The limit does not exist.
2. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 9} \frac{\sqrt{x-5}-2}{x-9}$ .  $\frac{(\sqrt{x-5}+2)}{(\sqrt{x-5}+2)} = \lim_{x \rightarrow 9} \frac{x-5-4}{(x-9)(\sqrt{x-5}+2)}$   
 $= \lim_{x \rightarrow 9} \frac{(x-9)}{(x-9)(\sqrt{x-5}+2)} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$
- (A)  $\frac{1}{4}$   
(B)  $-\frac{1}{4}$   
(C) 1  
(D) 0  
(E) The limit does not exist.
3. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2}$ .  $\frac{2-x}{2x} \cdot \frac{1}{x-2} = \lim_{x \rightarrow 2} \frac{-(x-2)}{2x(x-2)}$   
 $\frac{1}{x-2} \cdot \frac{1}{x-2} = \boxed{-\frac{1}{4}}$
- (A)  $\frac{1}{4}$   
(B)  $-\frac{1}{4}$   
(C) 1  
(D) -1  
(E) The limit does not exist.

- 4) For what value of  $k$  is the function  $f(x) = \begin{cases} \frac{2x^2+5x-3}{x^2-9}, & x \neq -3 \\ k, & x = -3 \end{cases}$

$$i) f(-3) = k$$

continuous at  $x = -3$ ?

(A)  $-\frac{7}{6}$

(B)  $-\frac{5}{6}$

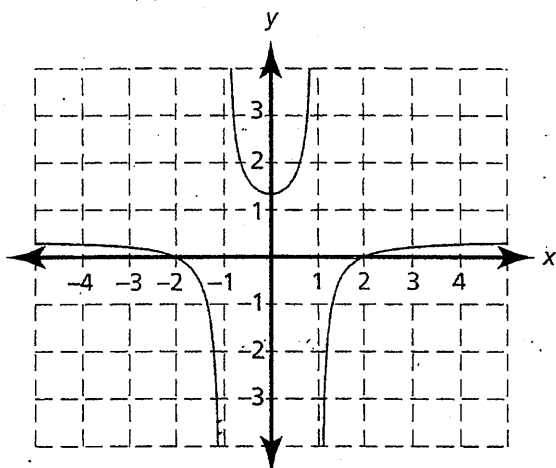
(C) 0

(D)  $\frac{5}{6}$

(E)  $\frac{7}{6}$

$$ii) \lim_{x \rightarrow -3} \frac{(2x-1)(x+3)}{(x+3)(x-3)} = \frac{-6-1}{-3-3} = \frac{-7}{-6} = \frac{7}{6}$$

$$iii) f(-3) = \lim_{x \rightarrow -3} f(x), \quad k = \frac{7}{6}$$



$$x\text{-int: } (-2, 0) (2, 0)$$

$$V.A: x = -1, x = 1$$

$$x^2 + a = 0 \text{ at } x = 2$$

$$2^2 + a = 0 \quad \boxed{a = -4}$$

$$bx^2 - 3 = 0: \text{ at } x = 1, x = -1$$

$$b(1)^2 - 3 = 0 \quad \boxed{b = 3}$$

- 5) The function  $g(x)$  is shown in the graph above and is of the form

$$g(x) = \frac{x^2 + a}{bx^2 - 3}. \text{ Which of the following could be the values of the}$$

constants  $a$  and  $b$ ?

(A)  $a = -2, b = -1$

(B)  $a = -2, b = -3$

(C)  $a = -4, b = 3$

(D)  $a = -4, b = -3$

(E)  $a = 4, b = 3$

$$g(x) = \frac{x^2 - 4}{3x^2 - 3}$$

- 6) Identify the vertical asymptotes for  $f(x) = \frac{x^2+3x-4}{x^2+x-2}$ .

(A)  $x = -2, x = 1$

(B)  $x = -2$

(C)  $x = 1$

(D)  $y = -2, y = 1$

(E)  $y = -2$

$$\frac{(x-1)(x+4)}{(x-1)(x+2)}$$

$$\boxed{x = -2}$$