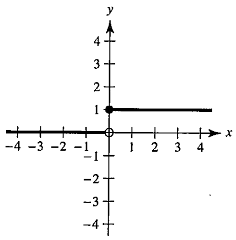
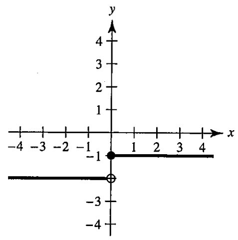
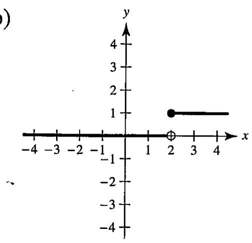
3.



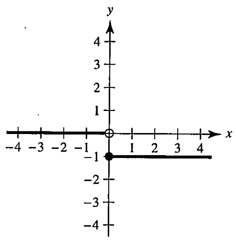
(a)



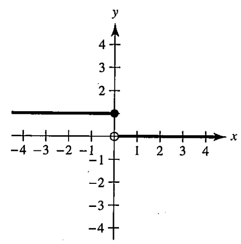
(b)



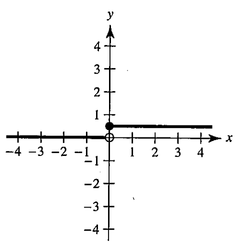
(c)



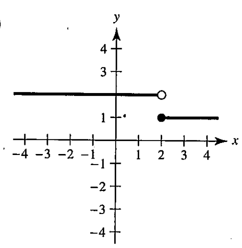
(d)



(e)

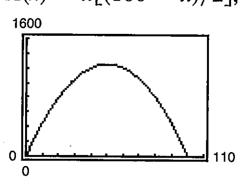


(f)



5. (a) $A(x) = x[(100 - x)/2]$; Domain: $(0, 100)$

(b)



Dimensions 50 m × 25 m yield maximum area of 1250 m^2.

(c) 50 m × 25 m; Area = 1250 m^2

7. $T(x) = \left[2\sqrt{4 + x^2} + \sqrt{(3 - x)^2 + 1}\right]/4$

9. (a) 5, less (b) 3, greater (c) 4.1, less
(d) $4 + h$ (e) 4; Answers will vary.

11. (a) Domain: $(-\infty, 1) \cup (1, \infty)$; Range: $(-\infty, 0) \cup (0, \infty)$

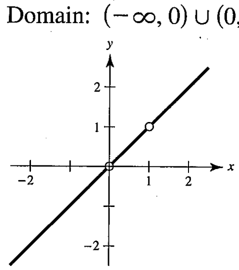
(b) $f(f(x)) = \dfrac{x - 1}{x}$

Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(c) $f(f(f(x))) = x$

Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

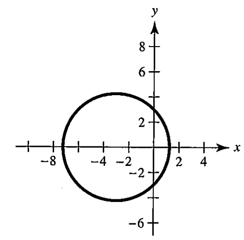
(d)



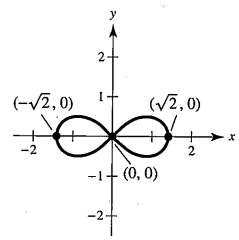
The graph is not a line because there are holes at $x = 0$ and $x = 1$.

13. (a) $x \approx 1.2426, -7.2426$

(b) $(x + 3)^2 + y^2 = 18$



15. Proof



Chapter 1

Section 1.1 *(page 47)*

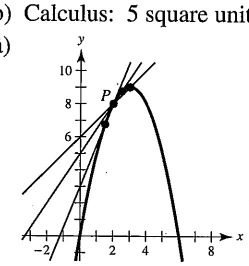
1. Precalculus: 300 ft

3. Calculus: Slope of the tangent line at $x = 2$ is 0.16.

5. (a) Precalculus: 10 square units

(b) Calculus: 5 square units

7. (a)



(b) $1; \frac{3}{2}; \frac{5}{2}$

(c) 2. Use points closer to P.

9. Area ≈ 10.417; Area ≈ 9.145; Use more rectangles.

Section 1.2 *(page 55)*

1.

x	3.9	3.99	3.999	4
$f(x)$	0.2041	0.2004	0.2000	?

x	4.001	4.01	4.1
$f(x)$	0.2000	0.1996	0.1961

$\lim_{x \to 4} \dfrac{x - 4}{x^2 - 3x - 4} \approx 0.2000 \left(\text{Actual limit is } \dfrac{1}{5}.\right)$

3.

x	−0.1	−0.01	−0.001	0
$f(x)$	0.5132	0.5013	0.5001	?

x	0.001	0.01	0.1
$f(x)$	0.4999	0.4988	0.4881

$\lim_{x \to 0} \dfrac{\sqrt{x + 1} - 1}{x} \approx 0.5000 \left(\text{Actual limit is } \dfrac{1}{2}.\right)$

5.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.9983	0.99998	1.0000	?

x	0.001	0.01	0.1
$f(x)$	1.0000	0.99998	0.9983

$\lim_{x\to 0} \frac{\sin x}{x} \approx 1.0000$ (Actual limit is 1.)

7.

x	0.9	0.99	0.999	1
$f(x)$	0.2564	0.2506	0.2501	?

x	1.001	1.01	1.1
$f(x)$	0.2499	0.2494	0.2439

$\lim_{x\to 1} \frac{x-2}{x^2+x-6} \approx 0.2500$ $\left(\text{Actual limit is } \frac{1}{4}.\right)$

9.

x	0.9	0.99	0.999	1
$f(x)$	0.7340	0.6733	0.6673	?

x	1.001	1.01	1.1
$f(x)$	0.6660	0.6600	0.6015

$\lim_{x\to 1} \frac{x^4-1}{x^6-1} \approx 0.6666$ $\left(\text{Actual limit is } \frac{2}{3}.\right)$

11.

x	-6.1	-6.01	-6.001	-6
$f(x)$	-0.1248	-0.1250	-0.1250	?

x	-5.999	-5.99	-5.9
$f(x)$	-0.1250	-0.1250	-0.1252

$\lim_{x\to -6} \frac{\sqrt{10-x}-4}{x+6} \approx -0.1250$ $\left(\text{Actual limit is } -\frac{1}{8}.\right)$

13.

x	-0.1	-0.01	-0.001	0
$f(x)$	1.9867	1.9999	2.0000	?

x	0.001	0.01	0.1
$f(x)$	2.0000	1.9999	1.9867

$\lim_{x\to 0} \frac{\sin 2x}{x} \approx 2.0000$ (Actual limit is 2.)

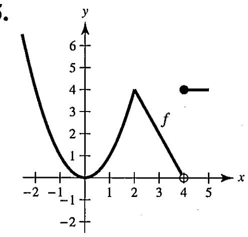
15. 1 **17.** 2

19. Limit does not exist. The function approaches 1 from the right side of 2, but it approaches -1 from the left side of 2.

21. Limit does not exist. The function oscillates between 1 and -1 as x approaches 0.

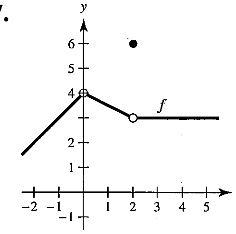
23. (a) 2
(b) Limit does not exist. The function approaches 1 from the right side of 1, but it approaches 3.5 from the left side of 1.
(c) Value does not exist. The function is undefined at $x = 4$.
(d) 2

25.



$\lim_{x\to c} f(x)$ exists for all points on the graph except where $c = 4$.

27.



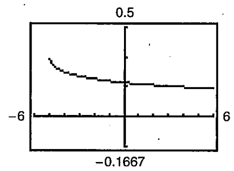
29. $\delta = 0.4$ **31.** $\delta = \frac{1}{11} \approx 0.091$

33. $L = 8$. Let $\delta = 0.01/3 \approx 0.0033$.

35. $L = 1$. Let $\delta = 0.01/5 = 0.002$. **37.** 6 **39.** -3

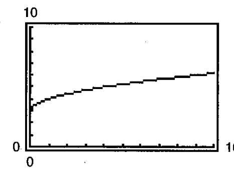
41. 3 **43.** 0 **45.** 10 **47.** 2 **49.** 4

51.



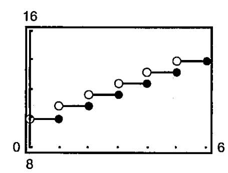
$\lim_{x\to 4} f(x) = \frac{1}{6}$
Domain: $[-5, 4) \cup (4, \infty)$
The graph has a hole at $x = 4$.

53.



$\lim_{x\to 9} f(x) = 6$
Domain: $[0, 9) \cup (9, \infty)$
The graph has a hole at $x = 9$.

55. (a)



(b)

t	3	3.3	3.4	3.5
C	11.57	12.36	12.36	12.36

t	3.6	3.7	4
C	12.36	12.36	12.36

$\lim_{t\to 3.5} C(t) = 12.36$

(c)

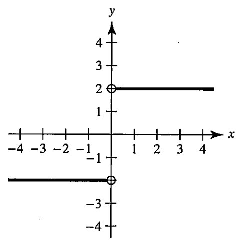
t	2	2.5	2.9	3
C	10.78	11.57	11.57	11.57

t	3.1	3.5	4
C	12.36	12.36	12.36

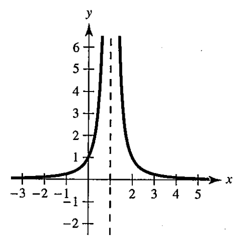
The limit does not exist because the limits from the right and left are not equal.

57. Answers will vary. Sample answer: As x approaches 8 from either side, $f(x)$ becomes arbitrarily close to 25.

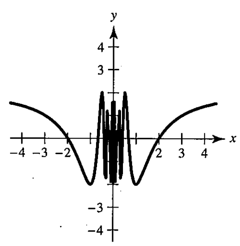
59. (i) The values of f approach different numbers as x approaches c from different sides of c.



(ii) The values of f increase or decrease without bound as x approaches c.



(iii) The values of f oscillate between two fixed numbers as x approaches c.



61. (a) $r = \frac{3}{\pi} \approx 0.9549$ cm

(b) $\frac{5.5}{2\pi} \le r \le \frac{6.5}{2\pi}$, or approximately $0.8754 < r < 1.0345$

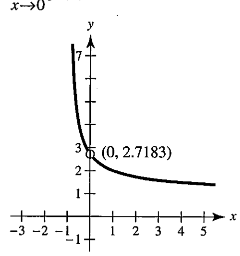
(c) $\lim_{r \to 3/\pi} 2\pi r = 6$; $\varepsilon = 0.5$; $\delta \approx 0.0796$

63.

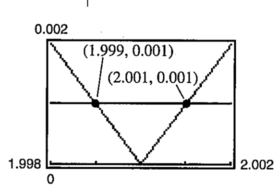
x	-0.001	-0.0001	-0.00001
$f(x)$	2.7196	2.7184	2.7183

x	0.00001	0.0001	0.001
$f(x)$	2.7183	2.7181	2.7169

$\lim_{x \to 0} f(x) \approx 2.7183$



65.



$\delta = 0.001$, $(1.999, 2.001)$

67. False. The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of the limit of $f(x)$ as $x \to c$.

69. False. See Exercise 17.

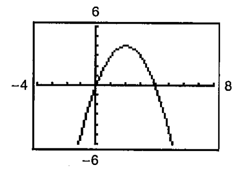
71. Yes. As x approaches 0.25 from either side, $\sqrt{x}$ becomes arbitrarily close to 0.5.

73. $\lim_{x \to 0} \frac{\sin nx}{x} = n$ **75–77.** Proofs

79. Putnam Problem B1, 1986

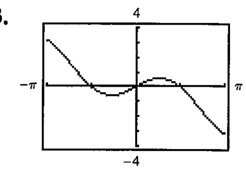
Section 1.3 *(page 67)*

1.



(a) 0 (b) -5

3.



(a) 0 (b) About 0.52 or $\pi/6$

5. 8 **7.** -1 **9.** 0 **11.** 7 **13.** 2 **15.** 1
17. 1/2 **19.** 1/5 **21.** 7 **23.** (a) 4 (b) 64 (c) 64
25. (a) 3 (b) 2 (c) 2 **27.** 1 **29.** 1/2 **31.** 1
33. 1/2 **35.** -1 **37.** (a) 10 (b) 5 (c) 6 (d) 3/2
39. (a) 64 (b) 2 (c) 12 (d) 8

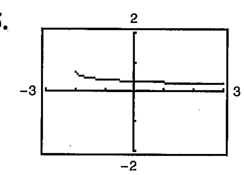
41. $f(x) = \frac{x^2 + 3x}{x}$ and $g(x) = x + 3$ agree except at $x = 0$.
$\lim_{x \to 0} f(x) = \lim_{x \to 0} g(x) = 3$

43. $f(x) = \frac{x^2 - 1}{x + 1}$ and $g(x) = x - 1$ agree except at $x = -1$.
$\lim_{x \to -1} f(x) = \lim_{x \to -1} g(x) = -2$

45. $f(x) = \frac{x^3 - 8}{x - 2}$ and $g(x) = x^2 + 2x + 4$ agree except at $x = 2$.
$\lim_{x \to 2} f(x) = \lim_{x \to 2} g(x) = 12$

47. -1 **49.** 1/8 **51.** 5/6 **53.** 1/6 **55.** $\sqrt{5}/10$
57. $-1/9$ **59.** 2 **61.** $2x - 2$ **63.** 1/5 **65.** 0
67. 0 **69.** 0 **71.** 1 **73.** 3/2

75.



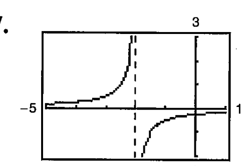
The graph has a hole at $x = 0$.

Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.358	0.354	0.354	0.354	0.353	0.349

$\lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x} \approx 0.354$; Actual limit is $\frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$.

77.



The graph has a hole at $x = 0$.

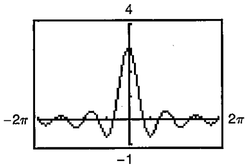
Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001
$f(x)$	-0.263	-0.251	-0.250

x	0.001	0.01	0.1
$f(x)$	-0.250	-0.249	-0.238

$\lim_{x \to 0} \frac{[1/(2 + x)] - (1/2)}{x} \approx -0.250$; Actual limit is $-\frac{1}{4}$.

79.



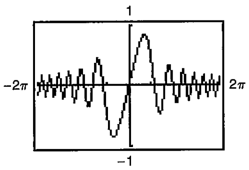
The graph has a hole at $t = 0$.

Answers will vary. Sample answer:

t	-0.1	-0.01	0	0.01	0.1
$f(t)$	2.96	2.9996	?	2.9996	2.96

$\lim_{t\to 0} \frac{\sin 3t}{t} \approx 3.0000$; Actual limit is 3.

81.



The graph has a hole at $x = 0$.

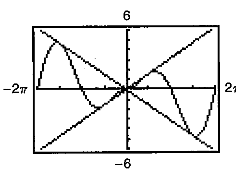
Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	-0.1	-0.01	-0.001	?	0.001	0.01	0.1

$\lim_{x\to 0} \frac{\sin x^2}{x} = 0$; Actual limit is 0.

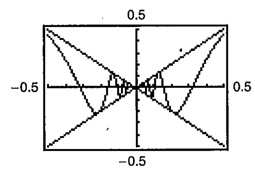
83. 3 **85.** $2x - 4$ **87.** $-1/(x + 3)^2$ **89.** 4

91.



0

93.



0

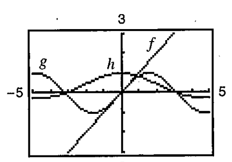
The graph has a hole at $x = 0$.

95. (a) f and g agree at all but one point if c is a real number such that $f(x) = g(x)$ for all $x \neq c$.

(b) Sample answer: $f(x) = \frac{x^2 - 1}{x - 1}$ and $g(x) = x + 1$ agree at all points except $x = 1$.

97. If a function f is squeezed between two functions h and g, $h(x) \leq f(x) \leq g(x)$, and h and g have the same limit L as $x \to c$, then $\lim_{x\to c} f(x)$ exists and equals L.

99.



The magnitudes of $f(x)$ and $g(x)$ are approximately equal when x is close to 0. Therefore, their ratio is approximately 1.

101. -64 ft/sec (speed = 64 ft/sec) **103.** -29.4 m/sec

105. Let $f(x) = 1/x$ and $g(x) = -1/x$.

$\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist. However,

$$\lim_{x\to 0} [f(x) + g(x)] = \lim_{x\to 0} \left[\frac{1}{x} + \left(-\frac{1}{x}\right)\right] = \lim_{x\to 0} 0 = 0$$

and therefore does exist.

107–111. Proofs

113. Let $f(x) = \begin{cases} 4, & x \geq 0 \\ -4, & x < 0 \end{cases}$.

$\lim_{x\to 0} |f(x)| = \lim_{x\to 0} 4 = 4$

$\lim_{x\to 0} f(x)$ does not exist because for $x < 0$, $f(x) = -4$ and for $x \geq 0$, $f(x) = 4$.

115. False. The limit does not exist because the function approaches 1 from the right side of 0 and approaches -1 from the left side of 0.

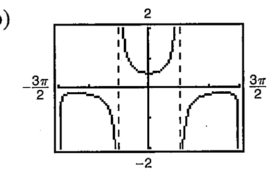
117. True.

119. False. The limit does not exist because $f(x)$ approaches 3 from the left side of 2 and approaches 0 from the right side of 2.

121. Proof

123. (a) All $x \neq 0, \frac{\pi}{2} + n\pi$

(b)



The domain is not obvious. The hole at $x = 0$ is not apparent from the graph.

(c) $\frac{1}{2}$ (d) $\frac{1}{2}$

Section 1.4 *(page 79)*

1. (a) 3 (b) 3 (c) 3; $f(x)$ is continuous on $(-\infty, \infty)$.

3. (a) 0 (b) 0 (c) 0; Discontinuity at $x = 3$

5. (a) -3 (b) 3 (c) Limit does not exist. Discontinuity at $x = 2$

7. $\frac{1}{16}$ **9.** $\frac{1}{10}$

11. Limit does not exist. The function decreases without bound as x approaches -3 from the left.

13. -1 **15.** $-1/x^2$ **17.** $5/2$ **19.** 2

21. Limit does not exist. The function decreases without bound as x approaches π from the left and increases without bound as x approaches π from the right.

23. 8

25. Limit does not exist. The function approaches 5 from the left side of 3 but approaches 6 from the right side of 3.

27. Discontinuities at $x = -2$ and $x = 2$

29. Discontinuities at every integer

31. Continuous on $[-7, 7]$ **33.** Continuous on $[-1, 4]$

35. Nonremovable discontinuity at $x = 0$

37. Continuous for all real x

39. Nonremovable discontinuities at $x = -2$ and $x = 2$

41. Continuous for all real x

43. Nonremovable discontinuity at $x = 1$
Removable discontinuity at $x = 0$

45. Continuous for all real x

47. Removable discontinuity at $x = -2$
Nonremovable discontinuity at $x = 5$

49. Nonremovable discontinuity at $x = -7$

51. Continuous for all real x

53. Nonremovable discontinuity at $x = 2$

55. Continuous for all real x

57. Nonremovable discontinuities at integer multiples of $\pi/2$

59. Nonremovable discontinuities at each integer

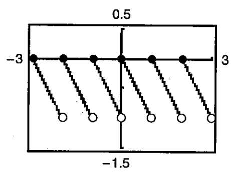
61. $a = 7$ **63.** $a = 2$ **65.** $a = -1,\ b = 1$

67. Continuous for all real x

69. Nonremovable discontinuities at $x = 1$ and $x = -1$

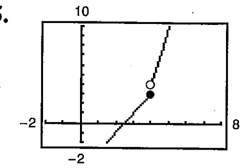
71. Continuous on the open intervals $\ldots, (-3\pi, -\pi), (-\pi, \pi), (\pi, 3\pi), \ldots$

73.



Nonremovable discontinuity at each integer

75.



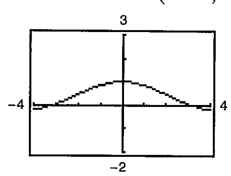
Nonremovable discontinuity at $x = 4$

77. Continuous on $(-\infty, \infty)$ **79.** Continuous on $[0, \infty)$

81. Continuous on the open intervals $\ldots, (-6, -2), (-2, 2), (2, 6), \ldots$

83. Continuous on $(-\infty, \infty)$

85.



The graph has a hole at $x = 0$. The graph appears to be continuous, but the function is not continuous on $[-4, 4]$. It is not obvious from the graph that the function has a discontinuity at $x = 0$.

87. Because $f(x)$ is continuous on the interval $[1, 2]$ and $f(1) = 37/12$ and $f(2) = -8/3$, by the Intermediate Value Theorem there exists a real number c in $[1, 2]$ such that $f(c) = 0$.

89. Because $f(x)$ is continuous on the interval $[0, \pi]$ and $f(0) = -3$ and $f(\pi) \approx 8.87$, by the Intermediate Value Theorem there exists a real number c in $[0, \pi]$ such that $f(c) = 0$.

91. 0.68, 0.6823 **93.** 0.56, 0.5636

95. $f(3) = 11$ **97.** $f(2) = 4$

99. (a) The limit does not exist at $x = c$.
(b) The function is not defined at $x = c$.
(c) The limit exists, but it is not equal to the value of the function at $x = c$.
(d) The limit does not exist at $x = c$.

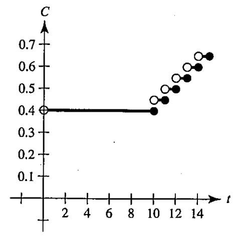
101. If f and g are continuous for all real x, then so is $f + g$ (Theorem 1.11, part 2). However, f/g might not be continuous if $g(x) = 0$. For example, let $f(x) = x$ and $g(x) = x^2 - 1$. Then f and g are continuous for all real x, but f/g is not continuous at $x = \pm 1$.

103. True

105. False. A rational function can be written as $P(x)/Q(x)$, where P and Q are polynomials of degree m and n, respectively. It can have, at most, n discontinuities.

107. The functions differ by 1 for non-integer values of x.

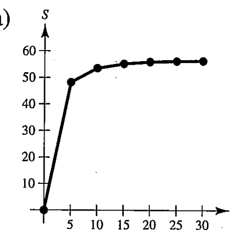
109. $C = \begin{cases} 0.40, & 0 < t \le 10 \\ 0.40 + 0.05[\![t - 9]\!], & t > 10,\ t \text{ is not an integer} \\ 0.40 + 0.05(t - 10), & t > 10,\ t \text{ is an integer} \end{cases}$



There is a nonremovable discontinuity at each integer greater than or equal to 10.

111–113. Proofs **115.** Answers will vary.

117. (a)

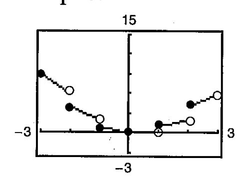


(b) There appears to be a limiting speed, and a possible cause is air resistance.

119. $c = (-1 \pm \sqrt{5})/2$

121. Domain: $[-c^2, 0) \cup (0, \infty)$; Let $f(0) = 1/(2c)$

123. $h(x)$ has a nonremovable discontinuity at every integer except 0.



125. Putnam Problem B2, 1988

Section 1.5 *(page 88)*

1. $\displaystyle\lim_{x\to-2^+} 2\left|\frac{x}{x^2-4}\right| = \infty, \quad \lim_{x\to-2^-} 2\left|\frac{x}{x^2-4}\right| = \infty$

3. $\displaystyle\lim_{x\to-2^+} \tan(\pi x/4) = -\infty, \quad \lim_{x\to-2^-} \tan(\pi x/4) = \infty$

5. $\displaystyle\lim_{x\to4^+} \frac{1}{x-4} = \infty, \quad \lim_{x\to4^-} \frac{1}{x-4} = -\infty$

7. $\displaystyle\lim_{x\to4^+} \frac{1}{(x-4)^2} = \infty, \quad \lim_{x\to4^-} \frac{1}{(x-4)^2} = \infty$

9.

x	-3.5	-3.1	-3.01	-3.001	-3
$f(x)$	0.31	1.64	16.6	167	?

x	-2.999	-2.99	-2.9	-2.5
$f(x)$	-167	-16.7	-1.69	-0.36

$\displaystyle\lim_{x\to-3^+} f(x) = -\infty; \quad \lim_{x\to-3^-} f(x) = \infty$

11.

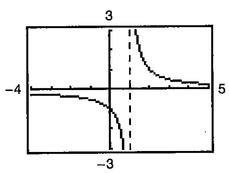
x	-3.5	-3.1	-3.01	-3.001	-3
$f(x)$	3.8	16	151	1501	?

x	-2.999	-2.99	-2.9	-2.5
$f(x)$	-1499	-149	-14	-2.3

$\lim_{x\to -3^+} f(x) = -\infty;\ \lim_{x\to -3^-} f(x) = \infty$

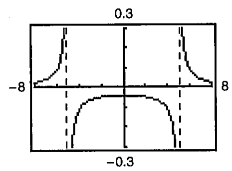
13. $x = 0$ **15.** $x = \pm 2$ **17.** No vertical asymptote
19. $x = -2, x = 1$ **21.** $x = 0, x = 3$
23. No vertical asymptote **25.** $x = n$, n is an integer.
27. $t = n\pi$, n is a nonzero integer.
29. Removable discontinuity at $x = -1$
31. Vertical asymptote at $x = -1$
33. ∞ **35.** ∞ **37.** $-\frac{1}{5}$ **39.** $-\infty$ **41.** $-\infty$
43. ∞ **45.** 0 **47.** ∞

49.



$\lim_{x\to 1^+} f(x) = \infty$

51.

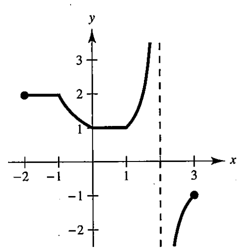


$\lim_{x\to 5^-} f(x) = -\infty$

53. Answers will vary.

55. Answers will vary. Sample answer: $f(x) = \dfrac{x-3}{x^2-4x-12}$

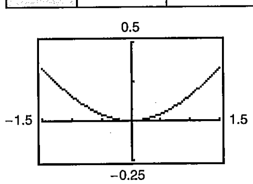
57.



59. (a)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.0411	0.0067	0.0017

x	0.01	0.001	0.0001
$f(x)$	≈ 0	≈ 0	≈ 0

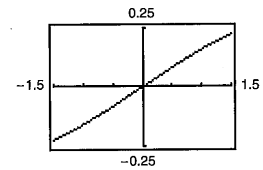


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x} = 0$

(b)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.0823	0.0333	0.0167

x	0.01	0.001	0.0001
$f(x)$	0.0017	≈ 0	≈ 0

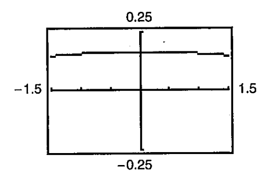


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^2} = 0$

(c)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.1646	0.1663	0.1666

x	0.01	0.001	0.0001
$f(x)$	0.1667	0.1667	0.1667

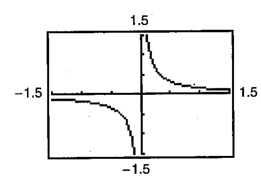


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^3} = 0.1667\ (1/6)$

(d)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.3292	0.8317	1.6658

x	0.01	0.001	0.0001
$f(x)$	16.67	166.7	1667.0



$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^4} = \infty$

For $n > 3$, $\lim_{x\to 0^+} \dfrac{x - \sin x}{x^n} = \infty$.

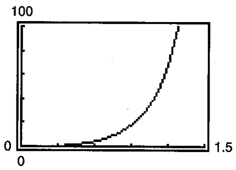
61. (a) $\dfrac{7}{12}$ ft/sec (b) $\dfrac{3}{2}$ ft/sec

(c) $\lim_{x\to 25^-} \dfrac{2x}{\sqrt{625 - x^2}} = \infty$

63. (a) $A = 50 \tan \theta - 50\theta$; Domain: $(0, \pi/2)$

(b)

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$	0.47	4.21	18.0	68.6	630.1



(c) $\lim_{\theta \to \pi/2} A = \infty$

65. False; let $f(x) = (x^2 - 1)/(x - 1)$

67. False; let $f(x) = \tan x$

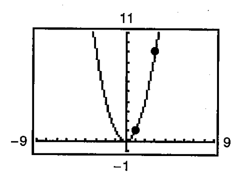
69. Let $f(x) = \dfrac{1}{x^2}$ and $g(x) = \dfrac{1}{x^4}$, and let $c = 0$. $\lim_{x\to 0} \dfrac{1}{x^2} = \infty$ and $\lim_{x\to 0} \dfrac{1}{x^4} = \infty$, but $\lim_{x\to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{x^4}\right) = \lim_{x\to 0} \left(\dfrac{x^2 - 1}{x^4}\right) = -\infty \neq 0.$

71. Given $\lim_{x\to c} f(x) = \infty$, let $g(x) = 1$. Then $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$ by Theorem 1.15.

73. Answers will vary.

Review Exercises for Chapter 1 *(page 91)*

1. Calculus



Estimate: 8.3

3.

x	2.9	2.99	2.999	3
$f(x)$	−0.9091	−0.9901	−0.9990	?

x	3.001	3.01	3.1
$f(x)$	−1.0010	−1.0101	−1.1111

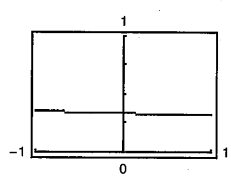
$\lim_{x\to 0} \dfrac{x - 3}{x^2 - 7x + 12} \approx -1.0000$

5. (a) 4 (b) 5 **7.** 5; Proof **9.** −3; Proof **11.** 36

13. $\sqrt{6} \approx 2.45$ **15.** 16 **17.** $\frac{4}{3}$ **19.** $-\frac{1}{4}$ **21.** $\frac{1}{2}$

23. −1 **25.** 0 **27.** $\sqrt{3}/2$ **29.** −3 **31.** −5

33.



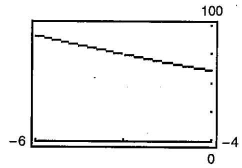
The graph has a hole at $x = 0$.

x	−0.1	−0.01	−0.001	0
$f(x)$	0.3352	0.3335	0.3334	?

x	0.001	0.01	0.1
$f(x)$	0.3333	0.3331	0.3315

$\lim_{x\to 0} \dfrac{\sqrt{2x + 9} - 3}{x} \approx 0.3333$; Actual limit is $\dfrac{1}{3}$.

35.



The graph has a hole at $x = -5$.

x	−5.1	−5.01	−5.001	−5
$f(x)$	76.51	75.15	75.02	?

x	−4.999	−4.99	−4.9
$f(x)$	74.99	74.85	73.51

$\lim_{x\to -5} \dfrac{x^3 + 125}{x + 5} \approx 75.00$; Actual limit is 75.

37. −39.2 m/sec **39.** $\frac{1}{6}$ **41.** $\frac{1}{4}$ **43.** 0

45. Limit does not exist. The limit as t approaches 1 from the left is 2, whereas the limit as t approaches 1 from the right is 1.

47. 3 **49.** Continuous for all real x

51. Nonremovable discontinuity at $x = 5$

53. Nonremovable discontinuities at $x = -1$ and $x = 1$
Removable discontinuity at $x = 0$

55. $c = -\frac{1}{2}$ **57.** Continuous for all real x

59. Continuous on $[4, \infty)$

61. Removable discontinuity at $x = 1$
Continuous on $(-\infty, 1) \cup (1, \infty)$

63. Proof **65.** (a) −4 (b) 4 (c) Limit does not exist.

67. $x = 0$ **69.** $x = \pm 3$ **71.** $x = \pm 8$ **73.** $-\infty$

75. $\frac{1}{3}$ **77.** $-\infty$ **79.** $\frac{4}{5}$ **81.** ∞

83. (a) \$14,117.65 (b) \$80,000.00 (c) \$720,000.00
(d) ∞

P.S. Problem Solving *(page 93)*

1. (a) Perimeter $\triangle PAO = 1 + \sqrt{(x^2 - 1)^2 + x^2} + \sqrt{x^4 + x^2}$
Perimeter $\triangle PBO = 1 + \sqrt{x^4 + (x - 1)^2} + \sqrt{x^4 + x^2}$

(b)

x	4	2	1
Perimeter $\triangle PAO$	33.0166	9.0777	3.4142
Perimeter $\triangle PBO$	33.7712	9.5952	3.4142
$r(x)$	0.9777	0.9461	1.0000

x	0.1	0.01
Perimeter $\triangle PAO$	2.0955	2.0100
Perimeter $\triangle PBO$	2.0006	2.0000
$r(x)$	1.0475	1.0050

1

3. (a) Area (hexagon) $= (3\sqrt{3})/2 \approx 2.5981$
Area (circle) $= \pi \approx 3.1416$
Area (circle) − Area (hexagon) ≈ 0.5435

(b) $A_n = (n/2) \sin(2\pi/n)$

(c)

n	6	12	24	48	96
A_n	2.5981	3.0000	3.1058	3.1326	3.1394

3.1416 or π

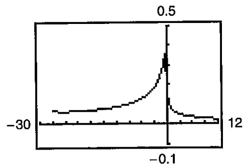
5. (a) $m = -\frac{12}{5}$ (b) $y = \frac{5}{12}x - \frac{169}{12}$

(c) $m_x = \dfrac{-\sqrt{169 - x^2} + 12}{x - 5}$

(d) $\frac{5}{12}$; It is the same as the slope of the tangent line found in (b).

7. (a) Domain: $[-27, 1) \cup (1, \infty)$

(b)

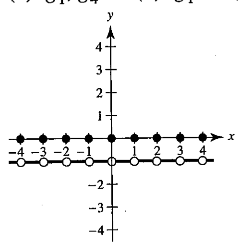


(c) $\frac{1}{14}$ (d) $\frac{1}{12}$

The graph has a hole at $x = 1$.

9. (a) g_1, g_4 (b) g_1 (c) g_1, g_3, g_4

11.



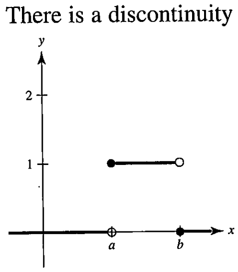
The graph jumps at every integer.

(a) $f(1) = 0,\ f(0) = 0,\ f(\frac{1}{2}) = -1,\ f(-2.7) = -1$

(b) $\lim_{x\to 1^-} f(x) = -1,\ \lim_{x\to 1^+} f(x) = -1,\ \lim_{x\to 1/2} f(x) = -1$

(c) There is a discontinuity at each integer.

13. (a)



(b) (i) $\lim_{x\to a^+} P_{a,b}(x) = 1$

(ii) $\lim_{x\to a^-} P_{a,b}(x) = 0$

(iii) $\lim_{x\to b^+} P_{a,b}(x) = 0$

(iv) $\lim_{x\to b^-} P_{a,b}(x) = 1$

(c) Continuous for all positive real numbers except a and b

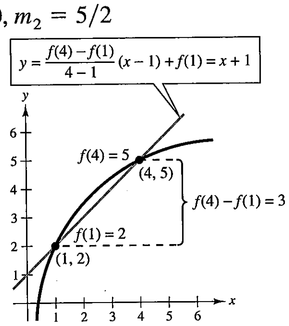
(d) The area under the graph of U and above the x-axis is 1.

Chapter 2

Section 2.1 *(page 103)*

1. $m_1 = 0, m_2 = 5/2$

3. (a)–(c)



5. $m = -5$

7. $m = 4$

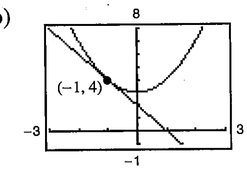
9. $m = 3$ **11.** $f'(x) = 0$ **13.** $f'(x) = -10$

15. $h'(s) = \frac{2}{3}$ **17.** $f'(x) = 2x + 1$ **19.** $f'(x) = 3x^2 - 12$

21. $f'(x) = \dfrac{-1}{(x - 1)^2}$ **23.** $f'(x) = \dfrac{1}{2\sqrt{x + 4}}$

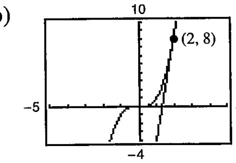
25. (a) Tangent line: $y = -2x + 2$

(b)



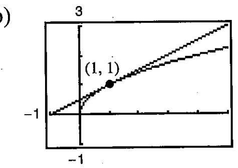
27. (a) Tangent line: $y = 12x - 16$

(b)



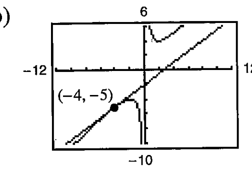
29. (a) Tangent line: $y = \frac{1}{2}x + \frac{1}{2}$

(b)



31. (a) Tangent line: $y = \frac{3}{4}x - 2$

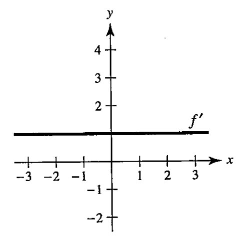
(b)



33. $y = 2x - 1$ **35.** $y = 3x - 2; y = 3x + 2$

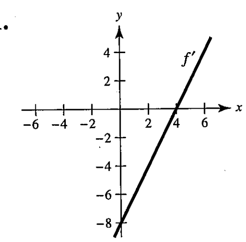
37. $y = -\frac{1}{2}x + \frac{3}{2}$

39.



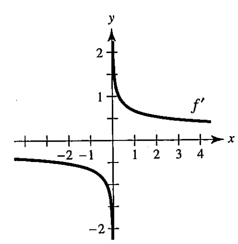
The slope of the graph of f is 1 for all x-values.

41.



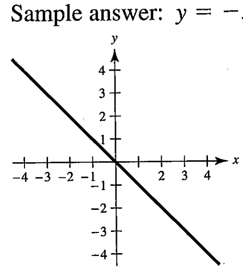
The slope of the graph of f is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.

43.



The slope of the graph of f is negative for $x < 0$ and positive for $x > 0$. The slope is undefined at $x = 0$.

45. Answers will vary.
Sample answer: $y = -x$



47. $g(4) = 5; g'(4) = -\frac{5}{3}$

49. $f(x) = 5 - 3x$
$c = 1$

51. $f(x) = -x^2$
$c = 6$

61. $f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$

63. (a) $f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$ (b) Two miles from touchdown

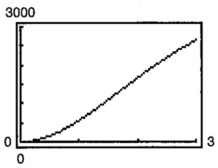
65. $x = 100$ units

67. (a)

t	0.5	1	1.5	2	2.5	3
S	151.5	555.6	1097.6	1666.7	2193.0	2647.1

$1.5 < t < 2$

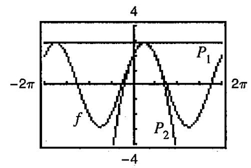
(b)



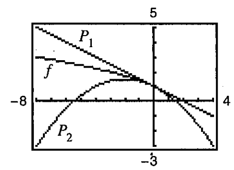
$t \approx 1.5$

(c) About 1.633 yr

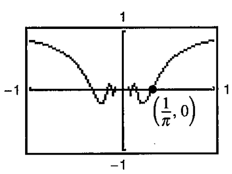
69. $P_1(x) = 2\sqrt{2}$
$P_2(x) = 2\sqrt{2} - \sqrt{2}(x - \pi/4)^2$
The values of f, P_1, and P_2 and their first derivatives are equal when $x = \pi/4$. The approximations worsen as you move away from $x = \pi/4$.



71. $P_1(x) = 1 - x/2$
$P_2(x) = 1 - x/2 - x^2/8$
The values of f, P_1, and P_2 and their first derivatives are equal when $x = 0$. The approximations worsen as you move away from $x = 0$.



73.



75. True

77. False. f is concave upward at $x = c$ if $f''(c) > 0$.

79. Proof

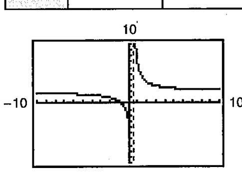
Section 3.5 *(page 202)*

1. f **2.** c **3.** d **4.** a **5.** b **6.** e

7.

x	10^0	10^1	10^2	10^3
$f(x)$	7	2.2632	2.0251	2.0025

x	10^4	10^5	10^6
$f(x)$	2.0003	2.0000	2.0000

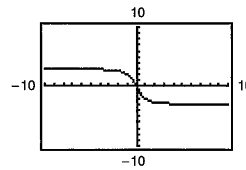


$$\lim_{x\to\infty} \frac{4x + 3}{2x - 1} = 2$$

9.

x	10^0	10^1	10^2	10^3
$f(x)$	-2	-2.9814	-2.9998	-3.0000

x	10^4	10^5	10^6
$f(x)$	-3.0000	-3.0000	-3.0000

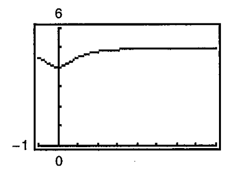


$$\lim_{x\to\infty} \frac{-6x}{\sqrt{4x^2 + 5}} = -3$$

11.

x	10^0	10^1	10^2	10^3
$f(x)$	4.5000	4.9901	4.9999	5.0000

x	10^4	10^5	10^6
$f(x)$	5.0000	5.0000	5.0000



$$\lim_{x\to\infty} \left(5 - \frac{1}{x^2 + 1}\right) = 5$$

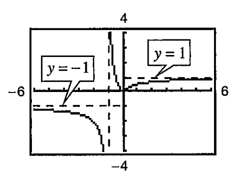
13. (a) ∞ (b) 5 (c) 0 **15.** (a) 0 (b) 1 (c) ∞

17. (a) 0 (b) $-\frac{2}{3}$ (c) $-\infty$ **19.** 4 **21.** $\frac{2}{3}$ **23.** 0

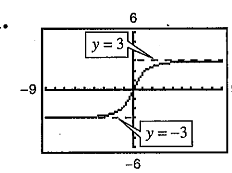
25. $-\infty$ **27.** -1 **29.** -2 **31.** $\frac{1}{2}$ **33.** ∞

35. 0 **37.** 0

39.



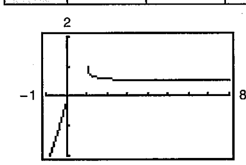
41.



43. 1 **45.** 0 **47.** $\frac{1}{6}$

49.

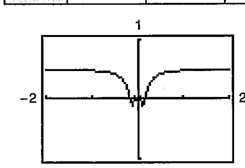
x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1.000	0.513	0.501	0.500	0.500	0.500	0.500



$$\lim_{x\to\infty} \left[x - \sqrt{x(x - 1)}\right] = \frac{1}{2}$$

51.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	0.479	0.500	0.500	0.500	0.500	0.500	0.500



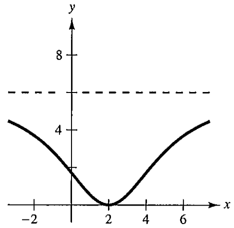
The graph has a hole at $x = 0$.

$$\lim_{x\to\infty} x \sin\frac{1}{2x} = \frac{1}{2}$$

53. As x becomes large, $f(x)$ approaches 4.

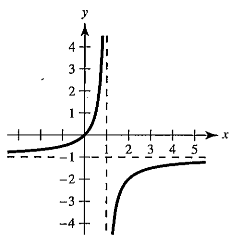
55. Answers will vary. Sample answer: Let

$$f(x) = \frac{-6}{0.1(x-2)^2+1} + 6.$$

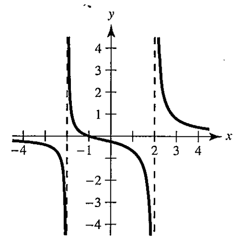


57. (a) 5 (b) -5

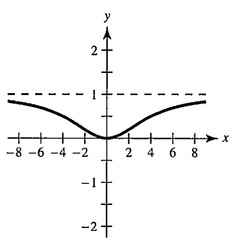
59.



61.



63.



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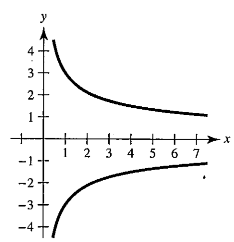

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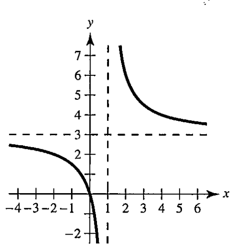

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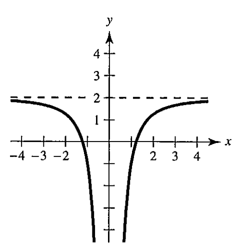

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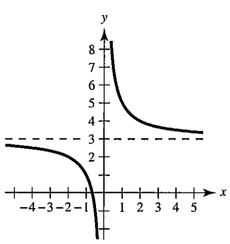

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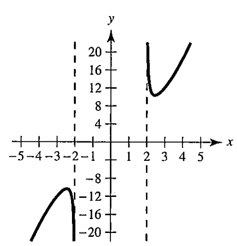

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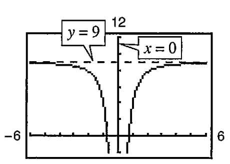

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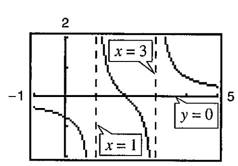

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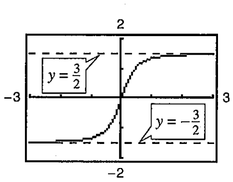
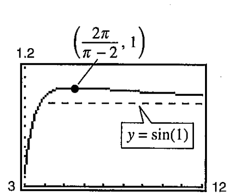
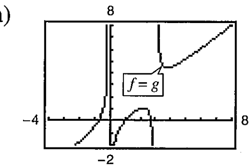


81.

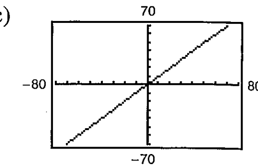


83. (a)



(b) Proof

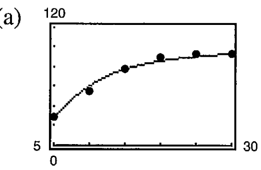
(c)



Slant asymptote: $y = x$

85. 100% **87.** $\lim_{t\to\infty} N(t) = +\infty$; $\lim_{t\to\infty} E(t) = c$

89. (a)



(b) Yes. $\lim_{t\to\infty} S = \frac{100}{1} = 100$

91. (a) $\lim_{x\to\infty} f(x) = 2$

(b) $x_1 = \sqrt{\frac{4-2\varepsilon}{\varepsilon}}$, $x_2 = -\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$

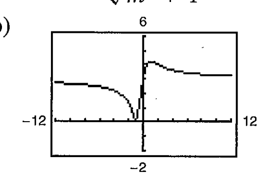
(c) $\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$ (d) $-\sqrt{\frac{4-2\varepsilon}{\varepsilon}}$

93. (a) Answers will vary. $M = \frac{5\sqrt{33}}{11}$ **95–97.** Proofs

(b) Answers will vary. $M = \frac{29\sqrt{177}}{59}$

99. (a) $d(m) = \frac{|3m+3|}{\sqrt{m^2+1}}$

(b)



(c) $\lim_{m\to\infty} d(m) = 3$;
$\lim_{m\to-\infty} d(m) = 3$;
As m approaches $\pm\infty$, the distance approaches 3

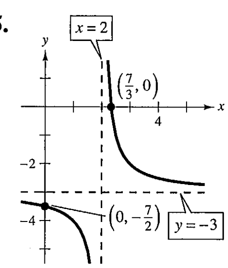
101. Proof

103. False. Let $f(x) = \frac{2x}{\sqrt{x^2+2}}$. $f'(x) > 0$ for all real numbers.

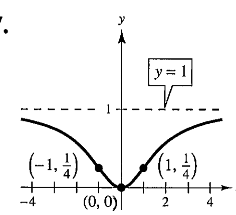
Section 3.6 *(page 212)*

1. d **2.** c **3.** a **4.** b

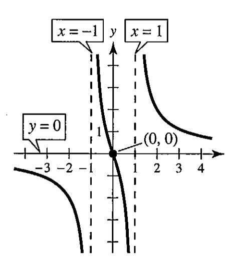
5.



7.



9.



11.