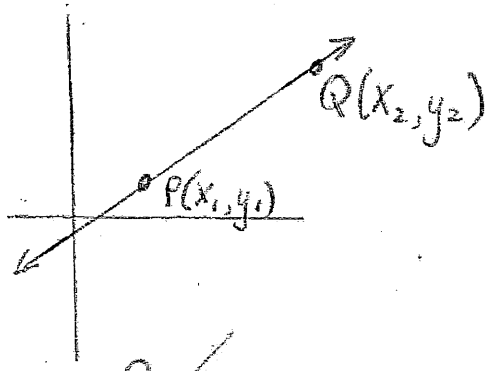
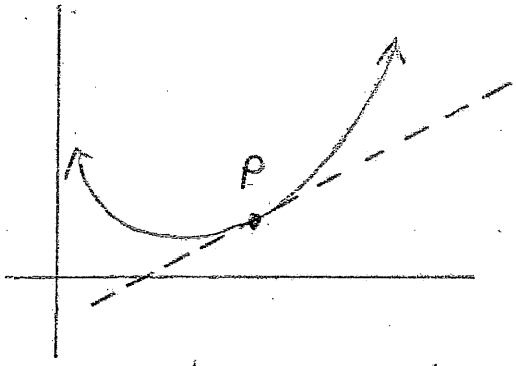
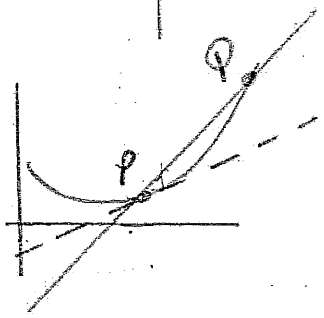
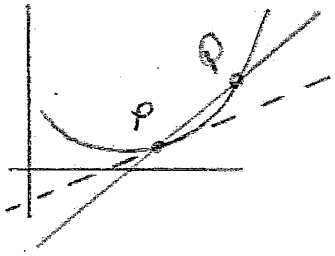
Ch. 2.1 Notes: The Derivative and Tangent Line Problem

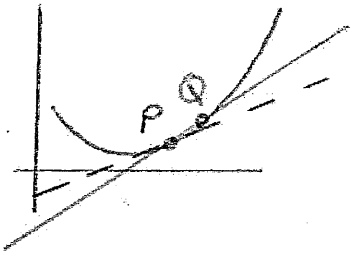
Goal: To find a formula to calculate the slope (steepness) of all tangent lines to a curve.

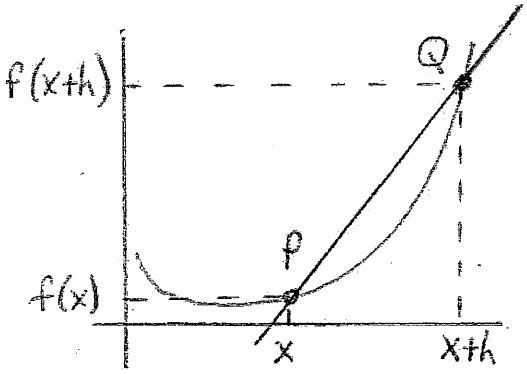












slope: $\frac{y_2 - y_1}{x_2 - x_1} =$

A. General (Limit) Definition of the Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

"f prime of x": This is the notation for the derivative function

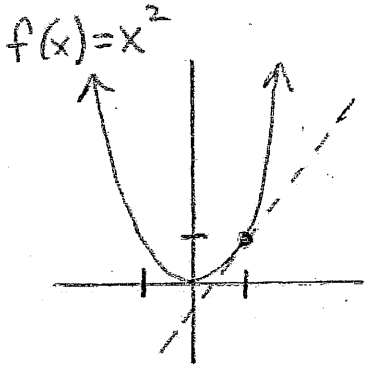
Derivative: the slope or steepness of a curve at a single point.

* The Derivative is a slope-finding formula for a curved function, where the slope is ever-changing.

B. Alternative Derivative Definition

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Ex. 1 Find the general derivative of $f(x)=x^2$. Then write the equation of the line tangent to $f(x)$ at $x=1$



$f(x)=x^2$

- $f(x)$ is the height-finding formula
- Since $f(1)=1^2=1$, this tells us that when $x=1$, the height of graph has a y-value of 1

* Therefore, the derivative (slope-finding formula for $f(x)=x^2$

$f'(x)=2x$

- $f'(x)$ is the slope-finding formula for the $f(x)$ graph
- Since $f'(1)=2(1)=2$ this tells us that when $x=1$ the slope of tangent line to $f(x)$ has slope of 2 (steepness)

Find Tangent-line equation:

* $y-y_1=m(x-x_1)$

point:

slope:

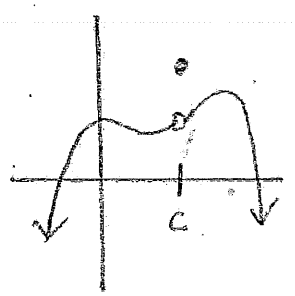
Ex. 2 Find equation of tangent line to $f(x)=x^2$ at $x=-5$

Ex. 3 ④ Find derivative of $f(x) = \sqrt{x}$.

① Find the slope of function at $x = 2$

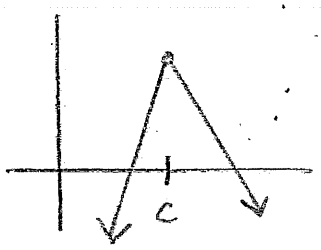
Ex. 4 Use the alternative derivative definition to find slope of $f(x) = \sqrt{x}$ at $x = 2$

Differentiability: In order for a function to be differentiable (smooth curve) at a point, c, it must be continuous at that point, cannot contain a sharp point, cannot have vertical tangent



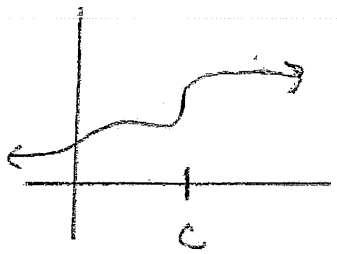
Graph not continuous

$f'(c) =$



sharp point at $f(c)$

$f'(c) =$



vertical tangent at $f(c)$

$f'(c) =$