Key

2.3 AP Practice Problems (p. 193) – Derivative Power Rule & exponential e^x

1. If $g(x) = x$, then $g'(7) =$

(A) 0 $\boxed{\text{(B)}}$ 1 (C) 7 (D) $\frac{49}{2}$

$g(x) = x$

$g'(x) = 1$

$\boxed{g'(7) = 1}$

2. The line $x + y = k$, where k is a constant, is a tangent line to the graph of the function $f(x) = x^2 - 5x + 2$. What is the value of k?

(A) -1 (B) 2 $\boxed{\text{(C)}}$ -2 (D) -4

* find slope of the line, set equal to $f'(x)$

$x + y = k$

$y = -1x + k$

$m = -1$

$f'(x) = 2x - 5$

$-1 = 2x - 5$

$4 = 2x$

$x = 2$

set equations equal at $x = 2$

$-1x + k = x^2 - 5x + 2$

$-2 + k = 2^2 - 10 + 2$

$-2 + k = -4$

$\boxed{k = -2}$

3. An object moves along the x-axis so that its position at time t is $x(t) = 3t^2 - 9t + 7$. For what time t is the velocity of the object zero?

(A) -3 (B) 3 $\boxed{\text{(C)}}$ $\frac{3}{2}$ (D) 7

$x(t) = 3t^2 - 9t + 7$

$v(t) = 6t - 9$

$0 = 6t - 9$

$9 = 6t$

$6t = 9$

$t = 9/6 = 3/2$

$\boxed{t = 3/2}$

4. If $f(x) = e^x$, then $\ln(f'(3)) =$

$\boxed{\text{(A)}}$ 3 (B) 0 (C) e^3 (D) $\ln 3$

$f'(x) = e^x$ $f'(3) = e^3$

$\ln(f'(3))$

$\downarrow$

$\ln(e^3) \rightarrow 3\ln e = \boxed{3}$

5. An equation of the normal line to the graph of $g(x) = x^3 + 2x^2 - 2x + 1$ at the point where $x = -2$ is

(A) $x + 2y = 12$ (B) $x - 2y = 8$

(C) $2x + y = -9$ $\boxed{\text{(D)}}$ $x + 2y = 8$

$g'(x) = 3x^2 + 4x - 2$

$g'(-2) = 3(-2)^2 + 4(-2) - 2 = 2$

* slope of tangent line is $m = 2$

* slope of normal line (perpendicular) is $m_2 = \frac{-1}{2}$

$g(-2) = (-2)^3 + 2(-2)^2 - 2(-2) + 1$

$g(-2) = -8 + 8 + 4 + 1 = 5$

point: $(-2, 5)$

slope: $m = -1/2$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-1}{2}(x + 2)$

$2(y - 5 = \frac{-1}{2}x - 1)$

$2y - 10 = -1x - 2$

$\boxed{x + 2y = 8}$

6. The line $9x - 16y = 0$ is tangent to the graph of $f(x) = 3x^3 + k$, where k is a constant, at a point in the first quadrant. Find k.

$\boxed{\text{(A)}}$ $\frac{3}{32}$ (B) $\frac{3}{16}$ (C) $\frac{3}{64}$ (D) $\frac{9}{64}$

$9x - 16y = 0$

$-16y = -9x$

$y = \frac{9}{16}x$

slope is $9/16$

$f'(x) = 9x^2 + 0$

$\frac{9}{16} = 9x^2$

$\frac{1}{16} = x^2$

$x = \sqrt{\frac{1}{16}} = 1/4$

$9(1/4) - 16y = 0$

$\frac{9}{4} = 16y$

$y = 9/64$

$y = 3x^3 + k$

$9/64 = 3(1/4)^3 + k$

$\frac{9}{64} = \frac{3}{64} + k$

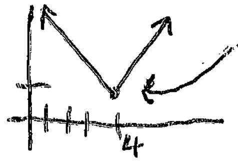
$\boxed{k = \frac{6}{64} = \frac{3}{32}}$

7. If $f(x) = 1 + |x - 4|$, find $f'(4)$.

(A) -1 (B) 0 (C) 1 $\boxed{\text{(D)}}$ $f'(4)$ does not exist.

$y = |x-4| + 1$

vertex at $(4,1)$



$f(x)$ not differentiable at $x = 4$

8. The cost C (in dollars) of manufacturing x units of a product is $C(x) = 0.3x^2 + 4.02x + 3500$.
What is the rate of change of C when $x = 1000$ units?

(A) 307.52 (B) 0.60402 $\boxed{\text{(C)}}$ 604.02 (D) 1020

$C'(x) = 2(0.3)x + 4.02 + 0$

$C'(x) = 0.6x + 4.02$

$C'(1000) = 0.6(1000) + 4.02$

$C'(1000) = 600 + 4.02 = 604.02$

9. $\frac{d}{dx}(5 \ln x) =$

(A) $\frac{1}{5x}$ (B) $5e^x$ (C) $-\frac{5}{\ln x}$ $\boxed{\text{(D)}}$ $\frac{5}{x}$

$* \frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} 5 \ln x \to 5\left(\frac{1}{x}\right) \to \boxed{\frac{5}{x}}$

10. For the function $f(x) = x^2 + 4$
(a) Find $f'(1)$.
(b) Find an equation of the tangent line to the graph of f at $x = 1$.
(c) Find $f'(-4)$.
(d) Find an equation of the tangent line to the graph of f at $x = -4$.
(e) Find the point of intersection of the two tangent lines found in (b) and (d).

a) 2
b) $2x+3$
c) -8
d) $-8x+12$
e) $(-3/2, 0)$

a) $f'(x) = 2x \to f'(1) = 2(1) = 2$

c) $f'(-4) = 2(-4) = -8$

d) $f(-4) = (-4)^2 + 4 = 20$ $\boxed{y - 20 = -8(x+4)}$

point: $(-4, 20)$ slope: $m = -8$

b) $f(1) = (1)^2 + 4 = 5$

point: $(1, 5)$

slope: $m = 2$

$\boxed{y - 5 = 2(x-1)}$ or $y = 2x + 3$

11. Which is an equation of the tangent line to the graph of $f(x) = x^4 + 3x^2 + 2$ at the point where $f'(x) = 2$?

(A) $y = 2x + 2$ (B) $y = 2x + 2.929$

$\boxed{\text{(C)}}$ $y = 2x + 1.678$ (D) $y = 2x - 2.929$

$*$set $f'(x) = 2$

$f'(x) = 4x^3 + 6x + 0$

$2 = 4x^3 + 6x$

$0 = 4x^3 + 6x + 2$

$0 = 2(2x^3 + 3x + 1)$

$0 = 2x^3 + 3x + 1$