Key

2.4 AP Practice Problems (p. 206) – Product & Quotient Rule & Higher order derivatives

1. What is the instantaneous rate of change at $x = -2$ of the function $f(x) = \dfrac{x-1}{x^2+2}$?

$\boxed{\text{(A)}}$ $-\frac{1}{6}$ (B) $\frac{1}{9}$ (C) $\frac{1}{2}$ (D) -1

*quotient Rule

$$f'(x) = \frac{\overbrace{(1)}^{f'}\overbrace{(x^2+2)}^{g} - \overbrace{(x-1)}^{f}\overbrace{(2x)}^{g'}}{\underbrace{(x^2+2)^2}_{g^2}}$$

$$f'(x) = \frac{x^2+2-2x^2+2x}{(x^2+2)^2}$$

$$f'(x) = \frac{-1x^2+2x+2}{(x^2+2)^2}$$

$$f'(-2) = \frac{-1(2)^2+2(-2)+2}{(2^2+2)^2}$$

$$f'(2) = \frac{-6}{36} = \boxed{\frac{-1}{6}}$$

2. An equation of the tangent line to the graph of $f(x) = \dfrac{5x-3}{3x-6}$ at the point (3, 4) is

(A) $7x + 3y = 37$ $\boxed{\text{(B)}}$ $7x + 3y = 33$
(C) $7x - 3y = 9$ (D) $13x + 3y = 51$

$$f'(x) = \frac{(5)(3x-6)-(5x-3)(3)}{(3x-6)^2} \rightarrow \frac{15x-30-15x+9}{(3x-6)^2}$$

$$f'(x) = \frac{-21}{(3x-6)^2}$$

$$f'(3) = \frac{-21}{(9-6)^2}$$

$$f'(3) = \frac{-21}{9} \rightarrow \frac{-7}{3}$$

point: (3,4)

slope: $m = -7/3$

$y - 4 = -7/3(x-3)$

$(y - 4 = -7/3 x + 7)3$

$3y - 12 = -7x + 21$

$\boxed{7x + 3y = 33}$

3. If f, g, and h are nonzero differentiable functions of x, then $\dfrac{d}{dx}\left(\dfrac{gh}{f}\right) =$

$\boxed{\text{(A)}}$ $\dfrac{fgh' + fg'h - f'gh}{f^2}$ (B) $\dfrac{g'h' - ghf'}{f^2}$

(C) $\dfrac{gh' + g'h}{f'}$ (D) $\dfrac{fgh' + fg'h + f'gh}{f^2}$

$$\frac{\overbrace{[(g')(h) + (g)(h')]}^{f'} \cdot \overbrace{f}^{g} - \overbrace{(gh)}^{f} \cdot \overbrace{f'}^{g'}}{\underbrace{f^2}_{g^2}}$$

$$\frac{(g'h + gh')f - f'gh}{f^2} \rightarrow \boxed{\frac{fg'h + fgh' - f'gh}{f^2}}$$

4. If $y = x^3e^x$, then $\dfrac{dy}{dx} =$

(A) $3x^2e^x$ (B) $3x^2 + e^x$
(C) $3x^2e^x(x+1)$ $\boxed{\text{(D)}}$ $x^2e^x(x+3)$

$$y' = \overbrace{3x^2}^{f'} \cdot \overbrace{e^x}^{g} + \overbrace{x^3}^{f} \cdot \overbrace{e^x}^{g'}$$

$$\boxed{y' = x^2e^x(3+x)}$$

5. $\dfrac{d}{dt}\left(t^2 - \dfrac{1}{t^2} + \dfrac{1}{t}\right)$ at $t = 2$ is

(A) $\frac{7}{2}$ (B) $\frac{9}{2}$ (C) $\frac{9}{4}$ $\boxed{\text{(D)}}$ 4

$f(t) = t^2 - t^{-2} + t^{-1}$

$f'(t) = 2t - (-2t^{-3}) - t^{-2}$

$f'(t) = 2t + \dfrac{2}{t^3} - \dfrac{1}{t^2}$

$f'(2) = 2(2) + \dfrac{2}{2^3} - \dfrac{1}{2^2}$

$f'(2) = 4 + \frac{1}{4} - \frac{1}{4}$

$\boxed{f'(2) = 4}$

6. The position of an object moving along a straight line at time t, in seconds, is given by $s(t) = 16t^2 - 5t + 20$ meters. What is the acceleration of the object when $t = 2$?

(A) 32 m/s (B) 0 m/s² $\boxed{\text{(C)}}$ 32 m/s² (D) 64 m/s²

$s'(t) = 32t - 5$

$s''(t) = 32$

$\boxed{s''(2) = 32\ m/s^2}$

7. If $y = \frac{x-3}{x+3}$, $x \neq -3$, the instantaneous rate of change of y with respect to x at $x = 3$ is

(A) $-\frac{1}{6}$ $\boxed{\text{(B)}}$ $\frac{1}{6}$ (C) $\frac{1}{36}$ (D) 1

* quotient Rule

$y' = \frac{\overbrace{(1)}^{f'}\overbrace{(x+3)}^{g} - \overbrace{(x-3)}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x+3)^2}_{g^2}}$

$y' = \frac{x+3-x+3}{(x+3)^2}$

$y' = \frac{6}{(x+3)^2}$

$y'(3) = \frac{6}{6^2} = \frac{1}{6}$

$\boxed{y'(3) = 1/6}$

8. Find an equation of the normal line to the graph of the function $f(x) = \frac{x^2}{x+1}$ at $x = 1$.

$\boxed{\text{(A)}}$ $8x + 6y = 11$ (B) $-8x + 6y = -5$

(C) $-3x + 4y = -1$ (D) $3x + 4y = 5$

$f'(x) = \frac{(2x)(x+1) - x^2(1)}{(x+1)^2}$

$f'(x) = \frac{2x^2 + 2x - x^2}{(x+1)^2}$

$f'(x) = \frac{x^2 + 2x}{(x+1)^2}$

$f'(1) = \frac{1+2}{2^2} = \frac{3}{4}$

slope of normal line:

$m_2 = -\frac{4}{3}$

$f(1) = \frac{1^2}{1+1} = \frac{1}{2}$

point: $(1, 1/2)$

slope: $m = -4/3$

$y - y_1 = m(x - x_1)$

$\left(y - \frac{1}{2} = -\frac{4}{3}(x-1)\right)6$

$6y - 3 = -8x + 8$

$\boxed{8x + 6y = 11}$

9. If $y = xe^x$, then the nth derivative of y is

(A) e^x $\boxed{\text{(B)}}$ $(x+n)e^x$ (C) ne^x (D) $x^n e^x$

$y' = 1e^x + xe^x = (x+1)e^x$

$y'' = (1)e^x + (x+1)e^x = (x+2)e^x$

$y''' = 1e^x + (x+2)e^x = (x+3)e^x$

$y^4(x) = 1e^x + (x+3)e^x \rightarrow (x+4)e^x$

$\boxed{y^n(x) = (x+n)e^x}$