Key

2.5 AP Practice Problems (p. 214) – Derivatives of Trig Functions

1. If $y = \underbrace{x}_{f}\underbrace{\sin x}_{g}$, then $\dfrac{dy}{dx} =$

$\boxed{\text{(A)}}$ $x\cos x + \sin x$ (B) $x\cos x - \sin x$

(C) $\cos x + \sin x$ (D) $(x+1)\cos x$

$y' = \underbrace{(1)}_{f'}\underbrace{\sin x}_{g} + \underbrace{(x)}_{f}\underbrace{\cos x}_{g'}$

$\boxed{y' = \sin x + x\cos x}$

2. What is $\lim\limits_{h\to 0} \dfrac{\cos\left(\frac{\pi}{3}+h\right) - \cos\frac{\pi}{3}}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ $\boxed{\text{(D)}}$ $-\frac{\sqrt{3}}{2}$

* Limit definition of derivative: $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$f'\left(\frac{\pi}{3}\right) = \lim\limits_{h\to 0} \dfrac{f\left(\frac{\pi}{3}+h\right) - f(\pi/3)}{h}$

$f(x) = \cos x$

$f'(x) = -\sin x$

$f'(\pi/3) = -\sin(\pi/3)$

$\boxed{f'(\pi/3) = -\frac{\sqrt{3}}{2}}$

3. If $f(x) = \tan x$, then $f'\left(\frac{\pi}{3}\right)$ equals

(A) $2\sqrt{3}$ $\boxed{\text{(B)}}$ 4 (C) 2 (D) $\frac{1}{4}$

$f'(x) = \sec^2 x$

$f'(\pi/3) = [\sec(\pi/3)]^2$

$f'(\pi/3) = \dfrac{1}{(\cos(\pi/3))^2} \rightarrow$

$f'(\pi/3) = \dfrac{1}{(1/2)^2} = \boxed{4}$

4. The position s (in meters) of an object moving along a horizontal line at time t, $0 \le t \le \frac{\pi}{2}$, (in seconds) is given by $s(t) = 6\sin t + \frac{3}{2}t^2 + 8$. What is the velocity of the object when its acceleration is zero?

(A) 6 m/s (B) $3 + \pi$ m/s

$\boxed{\text{(C)}}$ $\dfrac{6\sqrt{3}+\pi}{2}$ m/s (D) $\left(3\sqrt{3} - \frac{\pi}{2}\right)$ m/s

$s'(t) = 6\cos(t) + \frac{3}{2}\cdot 2t$

$s''(t) = -6\sin(t) + 3$

$0 = -6\sin(t) + 3$

$6\sin(t) = 3$

$\sin(t) = 1/2$

$t = \pi/6$

$v(t) = 6\cos(t) + 3t$

$v(\pi/6) = 6\cos(\pi/6) + 3(\pi/6)$

$v(\pi/6) = 6\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{2}$

$\boxed{v(\pi/6) = \dfrac{6\sqrt{3}+\pi}{2}}$

5. If $y = \sin x$, then $\dfrac{d^{50}}{dx^{50}} \sin x$ equals

(A) $\sin x$ $\boxed{\text{(B)}}$ $-\sin x$ (C) $\cos x$ (D) $-\cos x$

$50 \div 4 = 12$ R2 ($50 - 48 = 2$)

(R1) $y' = \cos x$

(R2) $y'' = -\sin x$

(R3) $y''' = -\cos x$

$y^4(x) = \sin x$ (R0)

$\boxed{y^{50}(x) = -\sin x}$

6. If $f(x) = \dfrac{x}{\cos x}$, find $f'\left(\dfrac{\pi}{3}\right)$.

(A) $2 - \dfrac{2\sqrt{3}}{3}\pi$ (B) $1 + \dfrac{\sqrt{3}}{3}\pi$

(C) $1 - \dfrac{\sqrt{3}}{3}\pi$ $\boxed{\text{(D)}}$ $2 + \dfrac{2\sqrt{3}}{3}\pi$

$f'(x) = \dfrac{(1)(\cos x) - x(-\sin x)}{\cos^2 x}$

$f'(\pi/3) = \dfrac{\cos(\pi/3) + \frac{\pi}{3}\sin(\pi/3)}{(\cos(\pi/3))^2} \rightarrow \dfrac{\frac{1}{2} + \frac{\pi}{3}\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)^2}$

$f'(\pi/3) = \dfrac{\frac{1}{2} + \frac{\pi\sqrt{3}}{6}}{1/4} \rightarrow \dfrac{\frac{3}{6} + \frac{\pi\sqrt{3}}{6}}{1/4} \rightarrow \dfrac{3 + \pi\sqrt{3}}{6} \cdot \dfrac{4}{1} \rightarrow \dfrac{(3 + \sqrt{3}\pi)2}{3} \rightarrow \dfrac{6}{3} + \dfrac{2\sqrt{3}\pi}{3}$

$\rightarrow \boxed{2 + \dfrac{2\sqrt{3}\pi}{3}}$

7. If $y = x - \tan x$, then $\dfrac{dy}{dx}$ equals

(A) $1 - \sec x \tan x$ $\boxed{\text{(B)}}$ $-\tan^2 x$

(C) $\tan^2 x$ (D) $-\sec^2 x$

$y' = 1 - \sec^2 x$

* trig identity: $1 + \tan^2 x = \sec^2 x$

$\sec^2 x = 1 + \tan^2 x$

$y' = 1 - (1 + \tan^2 x)$

$y' = 1 - 1 - \tan^2 x$

$\boxed{y = -\tan^2 x}$

8. If $g(x) = e^x \cos x + 2\pi$, then $g'(x) =$

(A) $e^x - \sin x$ (B) $e^x \cos x - e^x \sin x + 3\pi$

$\boxed{\text{(C)}}$ $e^x \cos x - e^x \sin x$ (D) $e^x \cos x + e^x \sin x$

$g'(\pi) = \underbrace{e^x}_{f'} \cdot \underbrace{\cos x}_{g} + \underbrace{e^x}_{f} \underbrace{(-\sin x)}_{g'} + 0$

$\boxed{g'(x) = e^x \cos x - e^x \sin x}$

9. At which of the following numbers x, $0 \le x \le 2\pi$, does the graph of $y = x + \cos x$ have a horizontal tangent line?

(A) 0 only $\boxed{\text{(B)}}$ $\dfrac{\pi}{2}$ only

(C) $\dfrac{3\pi}{2}$ only (D) 0 and $\dfrac{\pi}{2}$ only

$\boxed{x = \pi/2}$

* to find horizontal tangent → set $y'(x) = 0$

$y' = 1 + (-\sin x)$

$0 = 1 - \sin x$

$\sin x = 1$

10. An equation of the tangent line to the graph of $f(x) = \sin x$ at $x = \dfrac{2\pi}{3}$ is

(A) $3x + 6y = 4\pi - 3\sqrt{3}$ $\boxed{\text{(B)}}$ $3x + 6y = 2\pi + 3\sqrt{3}$

(C) $6y - 3x = 2\pi - 3\sqrt{3}$ (D) $6y - 3x = 4\pi - 3\sqrt{3}$

point: $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$f'(x) = \cos x$ $f'\left(\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}$

point: $\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2}\right)$ slope: $m = -1/2$

$\left[y - \frac{\sqrt{3}}{2} = \frac{-1}{2}\left(x - \frac{2\pi}{3}\right)\right]6$

$6y - 3\sqrt{3} = -3x + 2\pi$

$\boxed{3x + 6y = 2\pi + 3\sqrt{3}}$