# Ch. 2.1 Notes: The Derivative and Tangent Line Problem

Goal: To find a formula to calculate the slope of all tangent
lines to a curve. (steepness)
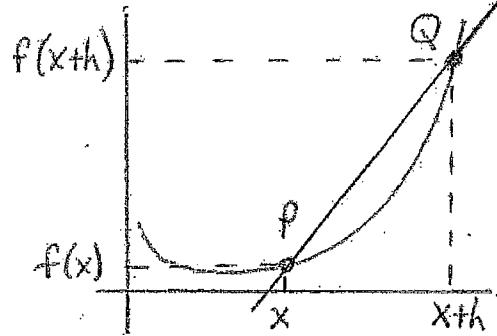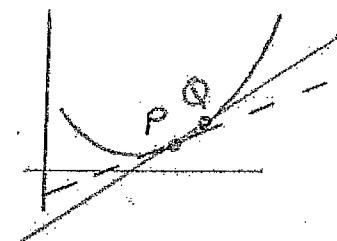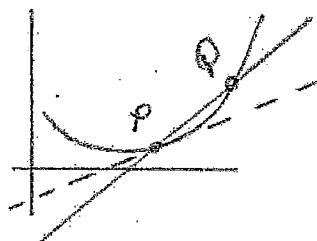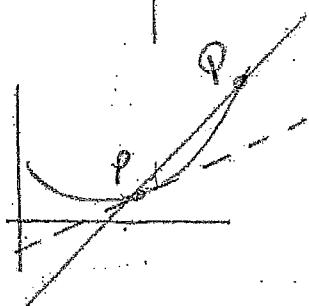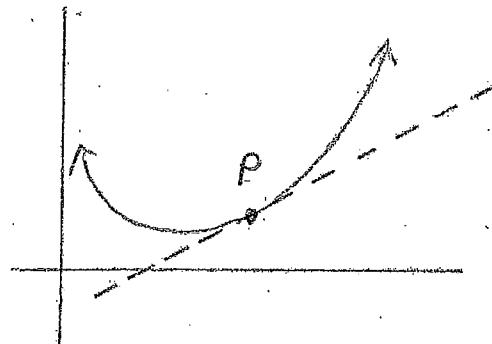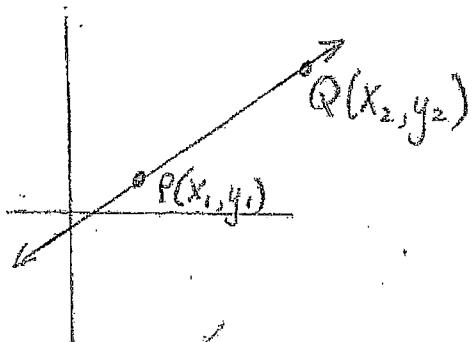
A. General (Limit) Definition of the Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

"f prime of x": This is the notation for the
derivative function

Derivative: the slope or steepness of a curve at
a single point.

* The Derivative is a slope-finding formula
for a curved function, where the slope
is ever-changing.

slope: $\dfrac{y_2 - y_1}{x_2 - x_1} =$

B. Alternative Derivative Definition

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Ex.1 Find the general derivative of $f(x)=x^2$. Then write the equation of the line tangent to $f(x)$ at $x=1$
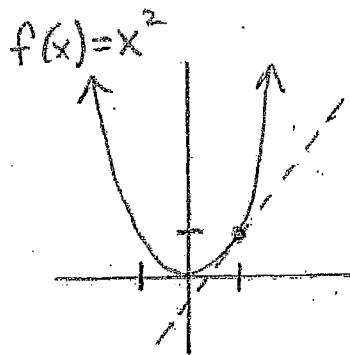
$f(x)=x^2$

$\underline{f(x)=x^2}$

- $f(x)$ is the height-finding formula
- Since $f(1)=1^2=1$, this tells us that when $x=1$, the height of graph has a y-value of 1

$\underline{f'(x)=2x}$

- $f'(x)$ is the slope-finding formula for the $f(x)$ graph
- Since $f'(1)=2(1)=2$, this tells us that when $x=1$ the slope of tangent line to $f(x)$ has slope of 2 (steepness)

* Therefore, the derivative (slope-finding formu for $f(x)=x^2$

Find Tangent-line equation:
* $y-y_1=m(x-x_1)$
point:
slope:

Ex.2 Find equation of tangent line to $f(x)=x^2$ at $x=-5$

Ex.3 ⓐ Find derivative of $f(x) = \sqrt{x}$.
ⓑ Find the slope of function at $x = 2$

Ex.4 Use the alternative derivative definition to find slope
of $f(x) = \sqrt{x}$ at $x = 2$

<u>Differentiability</u> : In order for a function to be <u>differentiable</u> (smooth curve,
at a point, c, it must be continuous at that point,
cannot contain a sharp point, cannot have vertical tangent
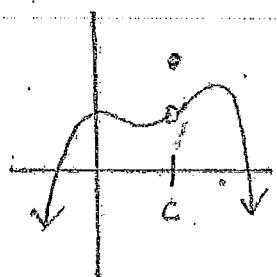
Graph not continuous	sharp point at $f(c)$	vertical tangent at $f(c)$
$f'(c) =$	$f'(c) =$	$f'(c) =$
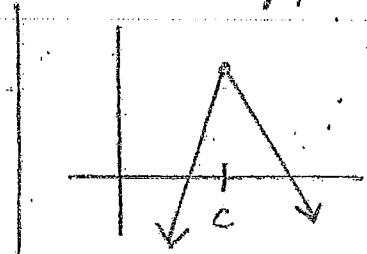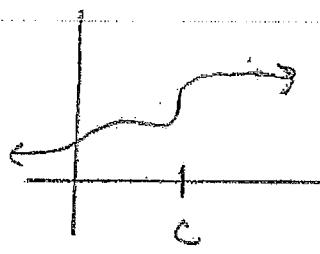
(4)

1) Use the Limit Definition of a derivative to find f'(x) if f(x) = $2x^2 - 3x + 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2) Use the Alternative definition of the derivative to find $f'(2)$ if $f(x) = \sqrt{2-x}$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

3) Use the Limit Definition of a Derivative to find f'(x) if $f(x) = \sqrt{2x-1}$

6 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

4) Use the Limit Definition of a derivative to find f'(3) if $f(x) = \dfrac{2}{5-x}$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2x - 3x^2$ at $x = -1$. $\quad y - y_1 = m(x - x_1)$

1) Use the Limit Definition of a derivative to find G'(x) if $G(x) = 3x^2 - 4x + 5$

2) Use the Alternative definition of the derivative to find $H'(2)$ if $H(x) = \sqrt{5-x}$

3) Use the Limit Definition of a Derivative to find H'(x) if $H(x) = \sqrt{x-3}$

(8)

4) Use the Limit Definition of a derivative to find $f'(3)$ if $f(x) = \dfrac{5}{x-2}$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 3x - 4x^2$ at $x = -1$.

Ch. 2.2a   Derivative Rules - Notes

1. <u>Constant Rule</u>: If $f(x)=c$, then $f'(x)=0$

   Ex. $f(x)=5 \rightarrow f'(x)=$
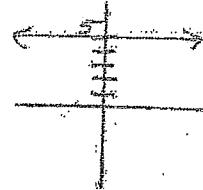
2. <u>Power Rule</u>: If $f(x)=x^n$, then $f'(x)=n \cdot x^{n-1}$

   steps: a) Bring exponent down, in front of variable

   b) subtract 1 from original exponent value

   ☀Important Note: Be sure function is in appropriate form before applying power rule.

   → convert any radicals to rational exponents

   → Move all variables from denominator to numerator (if necessary

   ┌─────┐
   │ Ex.1 │ Find derivatives of the following:
   └─────┘

   a) $y = x^7$

   b) $g(x) = \sqrt[3]{x}$

   c) $y = \dfrac{4}{x^5}$

   d) $y = 8x^{2/3} - \sqrt[5]{x} + \dfrac{2}{\sqrt{x}} + 0.875$

Ex.2 If $f(x) = x^{-2}$, find $f'(2)$

Ex.3 If $f(x) = \sqrt[3]{x^2}$, write tangent line equation to $f(x)$ at $x=8$

Ex.4 $f(x) = \dfrac{x^4 - 3x^2 + 4(\sqrt[3]{x})}{\sqrt{x}}$ find $f'(x)$

Ex.5 $f(x) = 3x(x+1)^2$ find $f'(x)$

2.2 Derivative Power Rule   Practice/Review Worksheet

Derivative Power Rule:

$$\frac{d}{dx}x^n = n * x^{n-1}$$

Power Rule Conditions:

i) All Radicals converted to Rational Exponents

ii) All denominator variables brought up to the numerator

iii) All parentheses resolved, all terms expanded

Finding a Derivative   use the rules of
differentiation to find the derivative of the function.

1) $y = x^7$

2) $y = \frac{1}{x^5}$

3) $y = \frac{3}{x^7}$

4) $f(x) = \sqrt[5]{x}$

5) $f(t) = -2t^2 + 3t - 6$

6) $y = \frac{5}{2x^2}$

7) $y = \frac{3}{2x^4}$

8) $y = \frac{6}{(5x)^3}$

Derivative Power Rule:

$$\frac{d}{dx}x^n = n * x^{n-1}$$

Power Rule Conditions:

i)   All Radicals converted to Rational Exponents
ii) All denominator variables brought up to the numerator
iii) All parentheses resolved, all terms expanded

Find the derivative of the functions below:

9) $g(t) = t^2 - \dfrac{4}{t^3}$

10) $f(x) = \dfrac{4x^3 + 3x^2}{x}$

11) $f(x) = \dfrac{2x^4 - x}{x^3}$

12) $y = x^2(2x^2 - 3x)$

13) $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

14) $f(t) = t^{2/3} - t^{1/3} + 4$

Finding an Equation of a Tangent Line In Exercises (a) find an equation of the tangent line to the graph of f at the given point.

Equation of tangent line:

i)   Find ordered pair $((x_1, y_1)$ using $f(x)$
ii) Find slope m using $f'(x)$
iii) $y - y_1 = m(x - x_1)$

15) $y = x^4 - 3x^2 + 2$ $(1, 0)$

16) $y = x^3 - 3x$ $(2, 2)$

2.2 Derivative Power Rule   Practice/Review Worksheet #2

Derivative Power Rule:

$$\frac{d}{dx}x^n = n * x^{n-1}$$

Power Rule Conditions:
i)   All Radicals converted to Rational Exponents
ii)   All denominator variables brought up to the numerator
iii)   All parentheses resolved, all terms expanded

Finding a Derivative   In Exercises 3–24, use the rules of differentiation to find the derivative of the function.

1) $f(x) = 3x^5 - 4x + 156$

2) $f(x) = \dfrac{5}{3x^6}$

3) $g(x) = 3\sqrt{x^9}$

4) $f(x) = \dfrac{\sqrt{x^9}}{3}$

5) $h(t) = \dfrac{7}{5(2t)^3}$

6) $f(t) = \dfrac{7}{5(2t)^3}$

7) $f(x) = \dfrac{7}{x\sqrt{x}}$

8) $f(x) = 5\sqrt{x} - 3x^2(2 - 5x)$

(14)

Power Rule Conditions:

i)   All Radicals converted to Rational Exponents

ii)   All denominator variables brought up to the numerator

iii)   All parentheses resolved, all terms expanded

Find the derivative of the functions below:

9) $f(x) = x(2 - 5x)^2$

10) $f(x) = \dfrac{5x^4 - 3x + 1}{x^2}$

11) $f(x) = \dfrac{3x^4 - 2x + 1}{\sqrt{x}}$

12) $f(x) = \dfrac{2x^3 - 4x^2 + 5}{\sqrt{x}}$

Finding an Equation of a Tangent Line In Exercises 53–56, (a) find an equation of the tangent line to the graph of f at the given point.

Equation of tangent line:

i)   Find ordered pair $((x_1, y_1)$ using $f(x)$

ii)   Find slope m using $f'(x)$

iii) $y - y_1 = m(x - x_1)$

13) $f(x) = \dfrac{2}{\sqrt[4]{x^3}}$ $(1, 2)$

14) $y = (x - 2)(x^2 + 3x)$ $(1, -4)$

AP Calculus Ch. 2.2   <u>Notes: Position-Velocity-Acceleration(PVA)</u>

Instantaneous velocity, v(t), of the object is the derivative of the position function s(t) with respect to time

Acceleration, a(t), is the derivative of velocity with respect to time

AVERAGE rate of change of f(x) from a to b = slope of secant = $\dfrac{f(b) - f(a)}{b - a}$

INSTANTANEOUS rate of change of f(x) at x = c = slope of tangent = $f'(c)$

Speed = |velocity|

Displacement = how far you are from where you started
Distance = total amount you have traveled

Ex) If I travel 10 feet to the right and then turn around and travel 3 feet back to the left, my distance is 13 feet but my displacement is 7 feet.

Speed is **increasing** when velocity and acceleration have the **same** sign.
Speed is **decreasing** when velocity and acceleration have **opposite** signs.

Particle Motion

Particle motion (linear motion) describes the object moving along a line (usually along a horizontal line)

x(t) = Position function
v(t) = velocity function
a(t) = acceleration function

Positive velocity indicates _____

Negative velocity indicates _____

When v(t) = 0, this indicates _____

A.P. Calculus PVA   **Worksheet 2.2a**

A ball is thrown vertically upwards from the edge of a building and it eventually hits the ground next to the building. If the height of the ball at any given time, $t \geq 0$ (seconds), is
$h(t) = -16t^2 + 64t + 80$ (feet), answer the following:

1. Sketch a diagram and label values at important places

2. How tall is the building?

3. When does the ball reach maximum height?

4. What is the maximum height?

5. How long does it take to hit the ground?

6. What was the initial velocity?

7. What is the velocity at $t = 1$ second? At $t = 2$ seconds?

8. What is the height at $t = 3$ seconds?

9. What is the <u>speed</u> when it hits the ground?

10. What is the acceleration at $t = 1$ second? At $t = 2$ seconds?

11. Find the average velocity in $[0, 2]$

12. Find the average acceleration in $[1, 2]$

13. Is the speed increasing or decreasing at $t = 1$ seconds?

14. Is the velocity increasing or decreasing at $t = 3$ seconds?

A.P. Calculus PVA   Worksheet 2-2b   Linear Motion Problems

1. An object is traveling at 20 m/sec to the left. What is its speed and velocity?

2. Which has the greater speed and velocity: object A with a velocity of -20 m/sec or object B with a velocity of -10 m/sec?

3. A billiard ball is hit and travels in a straight line. If x centimeters is the distance of the ball from its initial position at t seconds, then $x(t) = 5t^2 - 4t$. If the ball hits a cushion that is 12 cm from its initial position, at what velocity does it hit the cushion?

4. If a particle moves along a line according to the equation $s(t) = t^5 - 5t^4$ for all real numbers, t, then how many times does the particle reverse its direction?

5. The position in meters of a particle moving on the x-axis is given by $x(t) = 2t^3 - 2t + 1$ at all times t, $t > 0$. Find the acceleration when the velocity is 4 m/sec.

6. If $x(t) = \dfrac{t}{t^2 + 5}$ is the position function of a moving particle for $t > 0$, at what instant of time will the particle start to reverse its direction of motion, and where is it at that instant?

7. The position function of a particle moving on a coordinate line is given by: $x(t) = 2t^3 - 21t^2 + 60t + 3$, where x is in feet and t is in seconds.

a) When is the particle at rest?	b) When does the particle reverse direction?
c) What is the velocity when the acceleration is zero?	d) What is the speed when the acceleration is 6 ft/sec?
e) What is the displacement from $t = 1$ to $t = 3$?	f) What is the total distance moved from $t = 1$ to $t = 3$?

8. If $v(t) = (t - 5)(t - 3)^2(t - 1)$ represents the velocity of a particle moving along a line,
   a) When will the particle be at rest?
   b) When will the particle move to the left?
   c) When will the particle change direction?

9. A ball is thrown vertically upwards from the edge at the top of a building 160 ft tall with an initial velocity of 24 ft/sec. If the height of the ball (measured from the ground) is given by the function: $h(t) = -16t^2 + bt + c$,
a) Find the values of b and c.

b) How long does it take the ball to reach its maximum height?

c) What is the maximum height of the ball?

d) How long before the ball passes the top of the building on the way down?

e) How long does it take for the ball to hit the ground?

f) What is the speed of the ball when it hits the ground?

g) What is the speed of the ball at $t = 1$ second?

Vertical Motion In Exercises 97 and 98, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.

97. A silver dollar is dropped from the top of a building that is 1362 feet tall.

    (a) Determine the position and velocity functions for the coin.

    (b) Determine the average velocity on the interval $[1, 2]$.

    (c) Find the instantaneous velocities when $t = 1$ and $t = 2$.

    (d) Find the time required for the coin to reach ground level.

    (e) Find the velocity of the coin at impact.

Vertical Motion In Exercises 97 and 98, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.

98. A ball is thrown straight down from the top of a 220-foot building with an initial velocity of -22 feet per second. What is its velocity after 3 seconds? What is its velocity after falling 108 feet?

Vertical Motion In Exercises 99 and 100, use the position function $s(t) = -4.9t^2 + v_0 t + s_0$ for free-falling objects.

99. A projectile is shot upward from the surface of Earth with an initial velocity of 120 meters per second. What is its velocity after 5 seconds? After 10 seconds?

Vertical Motion In Exercises 99 and 100, use the position function $s(t) = -4.9t^2 + v_0 t + s_0$ for free-falling objects.

100. To estimate the height of a building, a stone is dropped from the top of the building into a pool of water at ground level. The splash is seen 5.6 seconds after the stone is dropped. What is the height of the building?

Particle Motion Problem Steps (Motion along a line)

1) Find $v(t)$ by taking the derivative of the position function $x(t)$ (use power rule)

2) Find times when object is motionless $\left(v(t)=0\right)$. Solve for t. (ex: $t = 4, 8$)

3) Create velocity sign line $v(t)$
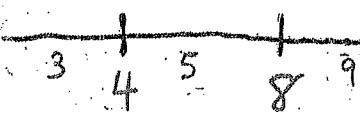
4) Pick a value in each interval to plug into $v(t)$

Ex. $V(3) = -6$ (moves left)

$V(5) = 1$ (moves right)

$V(9) = 3$ (moves right) $V(t)$
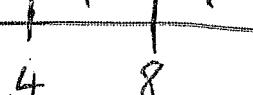
5) Find when object change directions $\left(\begin{array}{c}\text{count the number of}\\ \text{sign changes}\end{array}\right)$

object changes direction at $t=4$ b/c $v(t)$ changes signs.

6) Determine the time intervals of object moving left and right

based on above example

moving left: $(-\infty, 4)$ because $v(t) < 0$

moving right: $(4, 8), (8, \infty)$ because $v(t) > 0$

PVA Quiz Review Problem

Given:

$s(2) = 40$ m

$s(4) = 10$ m

$v(2) = -15$ m/s

$v(4) = -12$ m/s

$v(6) = -4$ m/s

$a(4) = 3$ m/s^2

$a(6) = 7$ m/s^2

Find the following:

a) Average velocity on $[2,4]$

b) Instantaneous velocity at $t=4$

c) Is velocity positive or negative at $t=4$?

d) Is velocity increasing or decreasing at $t=4$?

e) Is speed increasing or decreasing at $t=4$?

f) Find average acceleration ~~at~~ on $[4,6]$

a) Avg. velocity $= \dfrac{\text{change in position}}{\text{change in time}} = \dfrac{s(4)-s(2)}{4-2} = \dfrac{10-40}{4-2} = \dfrac{-30}{2} = -15$ m/s

b) $v(4) = -12$ m/s

c) $v(4) < 0$, negative

d) velocity is increasing at $t=4$ because $a(4) > 0$

e) speed is decreasing at $t=4$ because velocity and acceleration have opposite signs: $v(4) < 0$ and $a(4) > 0$.

f) avg. acceleration $= \dfrac{\text{change in velocity}}{\text{change in time}} = \dfrac{v(6)-v(4)}{6-4}$

$= \dfrac{-4-(-12)}{6-4} = \dfrac{8}{2} = \boxed{4 \text{ m/s}^2}$

Ch. 2.3 Notes   Product and Quotient Rules

Product Rule : formula used to find the derivatives of
products of two or more functions

$$* \frac{d}{dx}\left[f(x) \cdot g(x)\right] = f'(x) g(x) + f(x) g'(x)$$

"f prime g plus f g prime"

Ex. 1 $y = \underbrace{(3x - 2x^2)}_{f(x)} \underbrace{(5 + 4x)}_{g(x)}$ Find $\frac{dy}{dx}$

Quotient Rule : formula for finding derivative of function that
is the quotient of two other functions.

$$* \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) g(x) - f(x) g'(x)}{[g(x)]^2}$$

Ex. 2 $y = \frac{3x - 2x^2}{5 + 4x}$ Find y'

Higher order derivatives

Ex.3 $y = 2x^5 + x^4 - 3x^3 - 8x^2 + 10x - 12$. Find y''''

$y' =$

$y'' =$

$y''' =$

$y'''' =$

$y''''' =$

※ Notations

Notations for 1st derivative: $f'(x)$, $g'(x)$, y', $\dfrac{dy}{dx}$

Notation for 2nd derivative: $f''(x)$, $y''(x)$, y'', $\dfrac{d^2 y}{dx^2}$

※Note: This means "2nd derivative" NOT "square the 1st derivative"

Notation for 3rd derivative: $f'''(x)$, y''', $\dfrac{d^3 y}{dx^3}$

2.2-2.3 Review WS #2 (Asynchronous Wednesday)

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(x-1) - \frac{3x^2}{11} + 4\pi x - 5\pi^4 + \sqrt[5]{x^4} + \frac{5}{\sqrt{x^7}}$

2. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

3) Find the derivative of f(x) and then evaluate the slope of the graph at x = 1
$f(x) = (3x^5 - 4\sqrt{x})(2x - 5\pi + 9)$

4. Particle moves along the x-axis so that its position at time t is given $x(t) = t^3 - 9t^2 + 15t - 7$ where $x(t)$ is in feet per second and $t \geq 0$. Use this to answer the questions below. **Include units with your answers**

a) Find the velocity and acceleration function

b) What is its velocity at $t = 2$ seconds?

c) What is its acceleration at $t = 4$ seconds?

d) Find the average velocity of particle in [3, 8]

e) When is the particle at rest?

f) When is the particle moving right? When does particle change directions? (Create Sign Line) Give justification.

g) What is displacement of particle from $t = 2$ to $t = 6$? Show work.

h) What is the total distance of particle from $t = 2$ to $t = 6$? Show work.

i) Is the speed increasing or decreasing at $t = 4$? Justify.

j) Is velocity increasing or decreasing at $t = 2$? Justify.

2.2- 2.3 Quiz Review

1. The velocity of a function is described by the function $v(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 2$.

   a) Find the time(s) when acceleration is zero

   b) Find the velocity when acceleration is zero

2. The position function of a particle moving in a straight line is $x(t) = t^3 - 9t^2 + 24t - 2$ meters for $t > 0$ seconds.

   a. When is the particle at rest?

   b. During what time interval is particle moving to the right?

   c. During what time interval is the particle moving to the left?

3. Given function $f(x) = \frac{x}{x-3}$

a. Find the equation of tangent line to the curve at $x = 4$

b. Find the equation of the tangent line to the curve where the slope is equal to $-\frac{3}{4}$

4. Given $f(x) = \sqrt[3]{x}\,(1 - x^3)$.   Find $f'(x)$

5. Given $f(x) = \frac{2}{\sqrt{x}} - 5\sqrt[4]{x} + 12x^3 - 4\pi + 6.5x$ Find $f'(x)$

(28)

KEY

1. The velocity of a function is described by the function $v(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 2$.

   a) Find the time(s) when acceleration is zero

   $a(t) = t^2 - 4t + 3$

   $0 = (t-3)(t-1)$

   $\boxed{t = 1, 3}$ $t = 1, 3$

   b) Find the velocity when acceleration is zero

   $V(3) = \frac{1}{3}(3)^3 - 2(3)^2 + 3(3) + 2 =$

   $= 9 - 18 + 9 + 2 = \boxed{2}$

   $V(1) = \frac{1}{3} - 2 + 3 + 2 = \boxed{\frac{16}{3}}$

2. The position function of a particle moving in a straight line is $x(t) = t^3 - 9t^2 + 24t - 2$ meters for $t > 0$ seconds.

   a. When is the particle at rest?

 $V(t) = 3t^2 - 18t + 24$

   $V(t) = 3(t^2 - 6t + 8)$

   $V(t) = 3(t-4)(t-2)$ $\boxed{t = 2, 4 \text{ secs.}}$

   $0 = 3(t-4)(t-2)$

   b. During what time interval is particle moving to the right?

   $(0, 2) \cup (4, \infty)$

   (sign chart: + from 0 to 2, − from 2 to 4, + after 4)
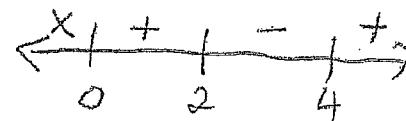
   c. During what time interval is the particle moving to the left?

   $(2, 4)$

3. Given function $f(x) = \frac{x}{x-3}$ $f(4) = \frac{4}{4-3} = 4$ point $(4, 4)$ $\boxed{y - 4 = -3(x-4)}$

   a. Find the equation of tangent line to the curve at $x = 4$ slope: $m = -3$

   $f'(x) = \frac{1(x-3) - x(1)}{(x-3)^2}$ $f'(x) = \frac{-3}{(x-3)^2}$

   $= \frac{x-3-x}{(x-3)^2} = \frac{-3}{(x-3)^2}$ $f'(4) = \frac{-3}{(4-3)^2} = \frac{-3}{1} = -3$

   b. Find the equation of the tangent line to the curve where the slope is equal to $-\frac{3}{4}$

   $\frac{-3}{4} = \frac{-3}{(x-3)^2}$ $(x-3)^2 = 4$ $x = 1, 5$ $y + \frac{1}{2} = -\frac{3}{4}(x-1)$

   $x^2 - 6x + 9 = 4$ $f(1) = \frac{1}{1-3} = -\frac{1}{2}$

   $-3(x-3)^2 = -3(4)$ $x^2 - 6x + 5 = 0$ $f(5) = \frac{5}{5-3} = \frac{5}{2}$ $y - \frac{5}{2} = -\frac{3}{4}(x-5)$

   $(x-5)(x-1) = 0$

4. Given $f(x) = \sqrt[3]{x}(1-x^3)$ Find $f'(x)$

   $f(x) = x^{1/3}(1-x^3)$

   $f'(x) = \frac{1}{3}x^{-2/3}(1-x^3) + x^{1/3}(-3x^2)$

   $= \frac{1-x^3}{3x^{2/3}} - 3x^{7/3}$

   $\boxed{f'(x) = \frac{1-x^3}{3\sqrt[3]{x^2}} - 3\sqrt[3]{x^7}}$

5. Given $f(x) = \frac{2}{\sqrt{x}} - 5\sqrt[4]{x} + 12x^3 - 4\pi + 6.5x$ Find $f'(x)$

   $f(x) = 2x^{-1/2} - 5x^{1/4} + 12x^3 - 4\pi + 6.5x$

   $f'(x) = 2(-\frac{1}{2})x^{-3/2} - 5(\frac{1}{4})x^{-3/4} + 36x^2 + 0 + 6.5$

   $= \frac{-1}{x^{3/2}} - \frac{5}{4x^{3/4}} + 36x^2 + 6.5$

   $\boxed{= \frac{-1}{\sqrt{x^3}} - \frac{5}{4\sqrt[4]{x^3}} + 36x^2 + 6.5}$

Ch. 2.4 Notes: The Chain Rule

<u>Chain Rule</u>: Method of computing the derivative of the <u>composition</u> of 2 or more functions (function within a function)

* Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

Steps:

1) Take the derivative of the outside while keeping the inside portion unchanged

2) Then multiply by the derivative of the inside function.

Ex.1 $f(x) = (3x^2 + 2)^5$

$f'(x) =$

Ex.2 Find all values of x of $f(x) = \sqrt[3]{(x^2-1)^2}$ for which $f'(x) = 0$ and where $f'(x)$ does not exist

Ex.3 $y = \dfrac{4}{(x+2)^2}$ find equation of tangent line to y at $x = -3$

Ex.4 $y = \left(\dfrac{x-1}{x^2-4}\right)^3$

Ex.5 $y = \dfrac{x}{\sqrt{x^2-1}}$

Ch. 2.5 Notes   Implicit Differentiation

Explicit equations: Equations where x's and y's are $\overset{\text{separated}}{\text{on}}$ different
(solved for y)   sides of the equation. (example: $y = 9x^2 + 4\sqrt{x} + 3$)

Implicit equations: Equations where x's and y's are mixed
(not solved for y)   together on same side(s) of the equation
                    (example: $y^2 = xy - x^2$)

Explicit Differentiation | Implicit Differentiation

$y = 3x^2 - 9x^3 + 5$ | $y^2 - 5x = 4$

Steps:
1) Take derivative of each term with respect to x
2) If variable is y, find derivative and attach $\frac{dy}{dx}$ to the derivative
3) Move all terms containing $\frac{dy}{dx}$ to left side of equation.
4) Move all other terms to right side of equation.
5) Factor out $\frac{dy}{dx}$ on left side of equation
6) Solve for $\frac{dy}{dx}$

Ex.1 $x^2 - 2y^3 + 4y = 2$ Find $\frac{dy}{dx}$

Ex.2 $3xy^3 - 2y = 7$ Find $\frac{dy}{dx}$ or y'

Ex.3 Differentiate $y^2 = 5x$ with respect to t

Ch. 2.5 Implicit Differentiation Worksheet #1

Finding a Derivative In Exercises 1–16, find dy/dx by implicit differentiation.

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$$

1. $x^2 + y^2 = 9$

2. $x^2 - y^2 = 25$

4. $2x^3 + 3y^3 = 64$

5. $x^3 - xy + y^2 = 7$

6. $x^2y + y^2x = -2$

7. $x^3y^3 - y = x$

(34)

21. $xy = 6$, $(-6, -1)$

22. $y^3 - x^2 = 4$, $(2, 2)$

24. $x^{2/3} + y^{2/3} = 5$, $(8, 1)$

25)

$(x^2 + 4)y = 8$

Point: $(2, 1)$

Find Equation of tangent line:

51. $\sqrt{x} + \sqrt{y} = 5$, $(9, 4)$

/

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( )

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

5. Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$

$\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

$2\begin{cases} 1: \text{implicit differentiation} \\ \\ 1: \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) When $x = 1$, $y^2 - y = 6$

$\qquad\qquad y^2 - y - 6 = 0$

$\qquad\qquad (y - 3)(y + 2) = 0$

$\qquad\qquad y = 3,\ y = -2$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9 - 9}{6 - 1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6 - 4}{-4 - 1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x - 1)$

$4\begin{cases} 1: y^2 - y = 6 \\ 1: \text{solves for } y \\ 2: \text{tangent lines} \end{cases}$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x\left(2y - x^2\right) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$\qquad\qquad -\dfrac{1}{4}x^5 = 6$

$\qquad\qquad x = \sqrt[5]{-24}$

$3\begin{cases} 1: \text{sets denominator of } \dfrac{dy}{dx} \text{ equal to 0} \\ 1: \text{substitutes } y = \dfrac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \qquad \text{into the equation for the curve} \\ 1: \text{solves for } x\text{-coordinate} \end{cases}$

Ch. 2.3 **Product,Quotient Rule** HW Problems   Evaluating Derivatives using graphs

Evaluating Derivatives In Exercises 81 and 82, use the graphs of f and g. Let $p(x) = f(x)g(x)$ and $q(x) = f(x)/g(x)$.

81. (a) Find $p'(1)$.

   (b) Find $q'(4)$.
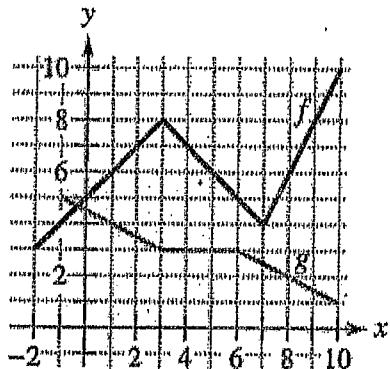
82. (a) Find $p'(4)$.

   (b) Find $q'(7)$.
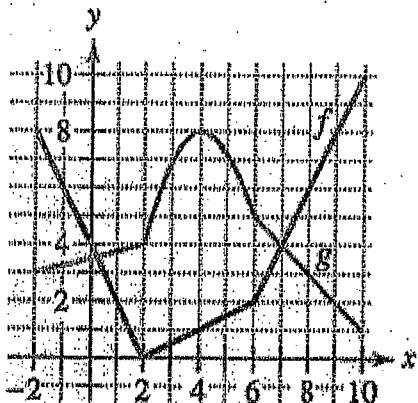
Using Relationships In Exercises 103–106, use the given information to find $f'(2)$.

$g(2) = 3$ and $g'(2) = -2$

$h(2) = -1$ and $h'(2) = 4$

103. $f(x) = 2g(x) + h(x)$

105. $f(x) = \dfrac{g(x)}{h(x)}$

Ch.2+ p.139

In Exercises 99 and 100, the graphs of f and g are shown. Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.

99. (a) Find $h'(1)$.

(b) Find $s'(5)$.
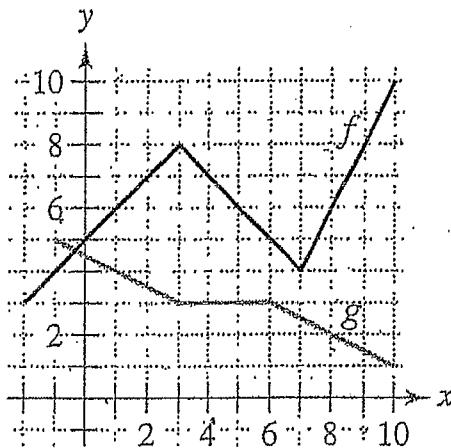
100. (a) Find $h'(3)$.

(b) Find $s'(9)$.
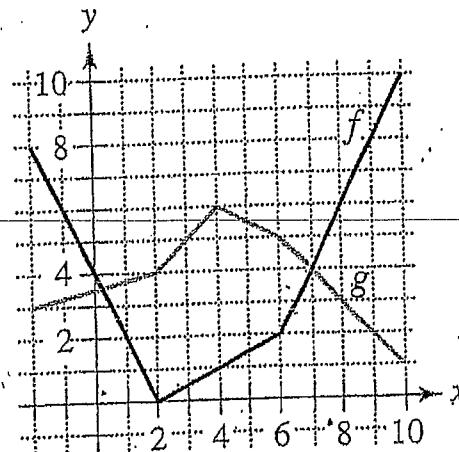
Ch. 2.4 Chain Rule   HW Problems #102, #115

102. Using Relationships   Given that $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, and $h'(5) = -2$, find $f'(5)$ for each of the following, if possible. If it is not possible, state what additional information is required.

Recall: Product Rule: $\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$ Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx} f[g(x)] = f'[g(x)] * g'(x)$

(a) $f(x) = g(x)h(x)$

(b) $f(x) = g(h(x))$

(c) $f(x) = \dfrac{g(x)}{h(x)}$

(d) $f(x) = [g(x)]^3$

115. Think About It   Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$, where f and g are shown in the figure. Find (a) $r'(1)$ and (b) $s'(4)$.
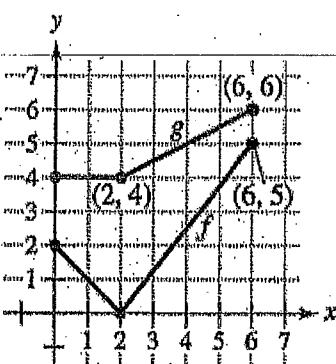
Vertical and Horizontal Tangent Lines   In Exercises 57 and 58, find the points at which the graph of the equation has a vertical or horizontal tangent line.

*Find **Horizontal Tangent** lines by setting **numerator of derivative** equal to zero, solve for x

*Find **Vertical Tangent** lines by setting **denominator of derivative** equal to zero, solve for x

57. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

58. $4x^2 + y^2 - 8x + 4y + 4 = 0$