General (Limit) Definition of a Derivative

This is the notation for finding derivative of a function or evaluating the derivative at a point.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Review problem:

1. Given: $f(x) = \sqrt[3]{x^2}$

a) $f'(x)$

b) Find $f'(8)$

2. Solve the previous problem again:

a) Express $f'(x)$ using limit definition

b) Express $f'(8)$ using limit definition

Application problem using limits definition:

3. $\lim_{h \to 0} \frac{\sqrt[3]{(8+h)^2} - 4}{h}$

4. $\lim_{h \to 0} \frac{(-2+h)^3 + 8}{h}$

5. Find $\lim_{h \to 0} \frac{(1+h)^4 - 1}{h}$

6. Find $\lim_{h \to 0} \frac{(2+h)^5 - 32}{h}$

7.

Express $f'(3)$ for $f(x) = x^3 + 2x^2$ using limit definition of derivative

8.

Which of the following gives the derivative of the function $f(x) = x^2$ at the point $(2, 4)$?

(A) $\lim_{h \to 0} \frac{(x+2)^2 - x^2}{4}$

(B) $\lim_{h \to \infty} \frac{(2+h)^2 - 2^2}{h}$

(C) $\frac{(2+h)^2 - 2^2}{h}$

(D) $\lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h}$

(E) $\lim_{h \to 0} \frac{(4+h)^2 - 4^2}{h}$

KEY

General (Limit) Definition of a Derivative

This is the notation for finding derivative of a function or evaluating the derivative at a point.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Review problem:

1. Given: f (x) = $\sqrt[3]{x^2}$ $= x^{2/3}$

a) f '(x) $= \frac{2}{3}x^{-1/3}$

$f'(x) = \frac{2}{3x^{1/3}}$

b) Find f '(8)

$f'(x) = \frac{2}{3x^{1/3}}$

$f'(8) = \frac{2}{3(8)^{1/3}}$

$f'(8) = \frac{\cancel{2}}{3(\cancel{2})} = \boxed{\frac{1}{3}}$

2. Solve the previous problem again:

a) Express f '(x) using limit definition

$f'(x) = \lim_{h\to 0}\frac{\sqrt[3]{(x+h)^2} - \sqrt[3]{x^2}}{h}$

b) Express f '(8) using limit definition

*plug 8 in for the x of f'(x)

$f'(8) = \lim_{h\to 0}\frac{\sqrt[3]{(8+h)^2} - \sqrt[3]{8^2}}{h}$

$f'(8) = \lim_{h\to 0}\frac{\sqrt[3]{(8+h)^2} - 4}{h} = \boxed{\frac{1}{3}}$

Application problem using limits definition:

3. $f'(8) = \lim_{h\to 0}\frac{\sqrt[3]{(8+h)^2} - 4}{h}$

*Find f'(8) for $f(x) = x^{2/3}$

$f'(x) = \frac{2}{3x^{1/3}}$

$f'(8) = \frac{1}{3}$

4. $\lim_{h\to 0}\frac{(-2+h)^3 + 8}{h}$

*find f'(-2) for $f(x) = x^3$

$f'(x) = 3x^2$

$f'(-2) = 3(-2)^2$

$\boxed{f'(-2) = 12}$

5. Find $\lim_{h\to 0}\frac{(1+h)^4-1}{h}$

Find $f'(1)$ for $f(x)=x^4$

$f'(x)=4x^3$

$f'(1)=4(1)^3$

$\boxed{f'(1)=4}$

6. Find $\lim_{h\to 0}\frac{(2+h)^5-32}{h}$

Find $f'(2)$ for $f(x)=x^5$

$f'(x)=5x^4$

$f'(2)=5(2)^4$

$\boxed{f'(2)=80}$

7.

Express $f'(3)$ for $f(x) = x^3 + 2x^2$ using limit definition of derivative

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^3+2(x+h)^2-(x^3+2x)}{h}$$

$$f'(3)=\lim_{h\to 0}\frac{(3+h)^3+2(3+h)^2-(3^3+6)}{h}$$

$$f'(3)=\lim_{h\to 0}\frac{(3+h)^3+2(3+h)^2-33}{h}$$

8.

Which of the following gives the derivative of the function $f(x)=x^2$ at the point $(2, 4)$?

(A) $\lim_{h\to 0}\frac{(x+2)^2-x^2}{4}$

(B) $\lim_{h\to\infty}\frac{(2+h)^2-2^2}{h}$

(C) $\frac{(2+h)^2-2^2}{h}$

$\boxed{\text{(D) } \lim_{h\to 0}\frac{(2+h)^2-2^2}{h}}$

(E) $\lim_{h\to 0}\frac{(4+h)^2-4^2}{h}$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

$$\boxed{f'(2)=\lim_{h\to 0}\frac{(2+h)^2-4}{h}}$$