Key

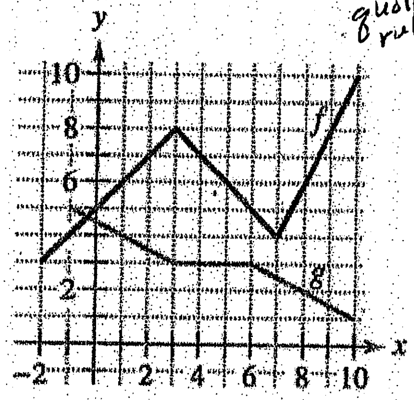
Ch. 2.3 **Product,Quotient Rule** HW Problems Evaluating Derivatives using graphs

Evaluating Derivatives In Exercises 81 and 82, use the graphs of f and g. Let $p(x) = f(x)g(x)$ and $q(x) = f(x)/g(x)$.

*product rule

81. (a) Find $p'(1)$.

(b) Find $q'(4)$.



a) $p'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$p'(1) = f'(1)\cdot g(1) + f(1)\cdot g'(1)$

$= (1)\cdot(4) + (6)\cdot(-\frac{1}{2}) = 4-3 = \boxed{1}$

quotient rule

b) $q'(x) = \dfrac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g(x)^2}$

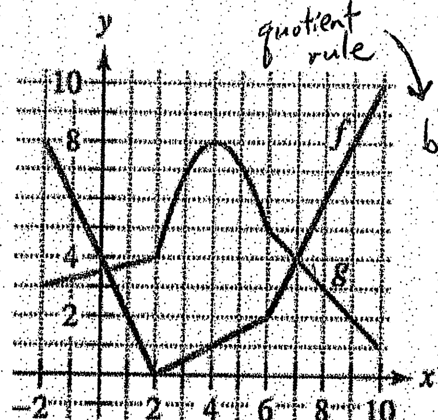
$q'(4) = \dfrac{f'(4)g(4) - f(4)g'(4)}{[g(4)]^2}$

$q'(4) = \dfrac{(-1)(3)-(7)(0)}{3^2}$

$q'(4) = \dfrac{-3}{3^2} = \boxed{-\frac{1}{3}}$

82. (a) Find $p'(4)$.

(b) Find $q'(7)$.



product Rule

a) $p'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$p'(4) = f'(4)\cdot g(4) + f(4)\cdot g'(4)$

$= (\frac{1}{2})\cdot(8) + (1)(0) = 4$

$\boxed{p'(4) = 4}$

quotient rule

b) $q'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$q'(7) = \dfrac{f'(7)g(7) - f(7)g'(7)}{g(7)^2}$

$q'(7) = \dfrac{(2)(4) - 4(-1)}{4^2} = \dfrac{8+4}{16} = \dfrac{12}{16} = \dfrac{3}{4}$ $\boxed{q'(7) = 3/4}$

Using Relationships In Exercises 103–106, use the given information to find $f'(2)$.

$g(2) = 3$ and $g'(2) = -2$

$h(2) = -1$ and $h'(2) = 4$

103. $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2g'(2) + h'(2)$

$= 2(-2) + 4 = 0$

$\boxed{f'(2) = 0}$

Apply quotient rule

105. $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$

$f'(2) = \dfrac{g'(2)h(2) - g(2)h'(2)}{h(2)^2}$

$f'(2) = \dfrac{(-2)(-1) - 3(4)}{(-1)^2}$

$f'(2) = \dfrac{2-12}{1} = -10$

$\boxed{f'(2) = -10}$

Ch.2.4 p139

[redacted]

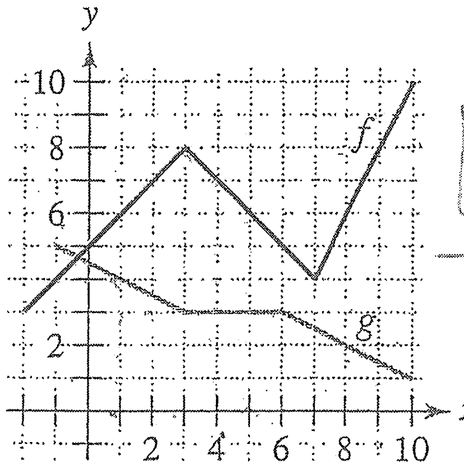
In Exercises 99 the graphs of f and g are shown. Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.

$h(x) = f(g(x))$

$s(x) = g(f(x))$

99. (a) Find $h'(1)$.

(b) Find $s'(5)$.



(a) $h'(x) = f'[g(x)] \cdot g'(x)$

$h'(1) = f'[g(1)] \cdot g'(1)$

$h'(1) = f'[4] \cdot (-\frac{1}{2})$

$= (-1)(-\frac{1}{2})$

$\boxed{h'(1) = \frac{1}{2}}$

$g(1) = 4$

$g'(1) = -\frac{1}{2}$

$f'(4) = -1$

$s'(x) = g'(f(x)) \cdot f'(x)$

$s'(5) = g'(f(5)) \cdot f'(5)$

$s'(5) = g'(6) \cdot (-1)$

$\boxed{s'(5) = DNE}$

$f(5) = 6$

$f'(5) = -1$

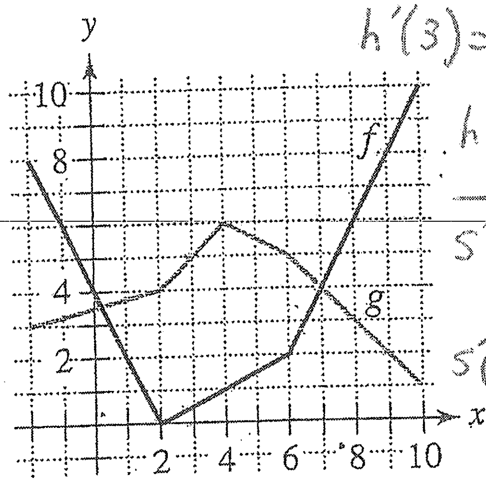
$g'(6) = DNE$

$h(x) = f(x)g(x)$

$s(x) = \frac{f(x)}{g(x)}$

100. (a) Find $h'(3)$.

(b) Find $s'(9)$.



$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(3) = f'(3)g(3) + f(3)g'(3)$

$h'(3) = (\frac{1}{2})(5) + (\frac{1}{2})(1)$

$h'(3) = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = \boxed{3}$

$f(3) = \frac{1}{2}$

$f'(3) = \frac{1}{2}$

$g(3) = 5$

$g'(3) = 1$

$s'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$s'(9) = \frac{f'(9)g(9) - f(9)g'(9)}{[g(9)]^2}$

$f(9) = 8$

$f'(9) = 2$

$g(9) = 2$

$g'(9) = -1$

$s'(9) = \frac{(2)(2) - (8)(-1)}{(2)^2} = \frac{4+8}{4} = \frac{12}{4} = 3$ $\boxed{s'(9) = 3}$

③ Key

Ch. 2.4 Chain Rule HW Problems #102, #115

102. Using Relationships Given that $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, and $h'(5) = -2$, find $f'(5)$ for each of the following, if possible. If it is not possible, state what additional information is required.

Recall: product rule: $\frac{d}{dx} f(x)g(x) = f'(x)(g(x)) + f(x)g'(x)$

quotient rule: $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Chain rule: $\frac{d}{dx} f[g(x)] = f'(g(x)) \cdot g'(x)$

(a) $f(x) = g(x)h(x)$ *product rule

$f'(x) = g'(x)h(x) + g(x)h'(x)$

$f'(5) = g'(5)h(5) + g(5)h'(5)$

$f'(5) = 6(3) + (-3)(-2)$

$f'(5) = 18 + 6 = 24$ $\boxed{f'(5) = 24}$

(b) $f(x) = g(h(x))$ *chain rule

$f'(x) = g'[h(x)] \cdot h'(x)$

$f'(5) = g'[h(5)] \cdot h'(5)$

$= g'(3) \cdot h'(5)$

$= g'(3) \cdot -2$

$\boxed{f'(5) = -2g'(3)}$

(c) $f(x) = \dfrac{g(x)}{h(x)}$ *quotient rule

$f'(x) = \dfrac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$

$f'(5) = \dfrac{g'(5)h(5) - g(5)h'(5)}{h(5)^2}$

$= \dfrac{6(3) - (-3)(-2)}{3^2}$

$= \dfrac{18-6}{9} = \dfrac{12}{9} = 4/3$

$\boxed{f'(5) = 4/3}$

(d) $f(x) = [g(x)]^3$ *chain rule

$f'(x) = 3[g(x)]^2 \cdot g'(x)$

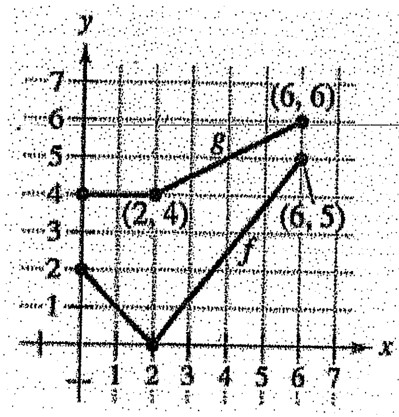
$f'(5) = 3[g(5)]^2 \cdot g'(5)$

$= 3[-3]^2 \cdot 6$

$= 3(9)(6) = 162$

$\boxed{f'(5) = 162}$

115. Think About It Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$, where f and g are shown in the figure. Find (a) $r'(1)$ and (b) $s'(4)$. ← Apply chain rule



a) $r'(x) = f'[g(x)] \cdot g'(x)$

$r'(1) = f'[g(1)] \cdot g'(1)$

$r'(1) = f'[4] \cdot 0$

$r'(1) = \frac{5}{4}(0) = \boxed{0}$

$\boxed{r'(1) = 0}$

b) $s'(x) = g'[f(x)] \cdot f'(x)$

$s'(4) = g'[f(4)] \cdot f'(4)$

$= g'[5/2] \cdot (5/4)$

$= (\frac{1}{2})(5/4) = 5/8$

$\boxed{s'(4) = 5/8}$

4 Key

Ch.2.5 **Implicit Differentiation** Vertical, Horizontal Tangent Lines HW Problems #57, #58

Vertical and Horizontal Tangent Lines **In Exercises 57 and 58, find the points at which the graph of the equation has a vertical or horizontal tangent line.**

(dy/dx)

*Find **Horizontal Tangent** lines by setting **numerator of derivative** equal to zero, solve for x

*Find **Vertical Tangent** lines by setting **denominator of derivative** equal to zero, solve for x

57. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$ ← Apply implicit differentiation

$50x + 32y\left(\frac{dy}{dx}\right) + 200 - 160\left(\frac{dy}{dx}\right) + 0 = 0$

$32y\left(\frac{dy}{dx}\right) - 160\left(\frac{dy}{dx}\right) = -50x - 200$

$\frac{dy}{dx}\left[32y - 160\right] = -50x - 200$

$\frac{dy}{dx} = \frac{-50x-200}{32y-160}$

horiz. tangent: $-50x - 200 = 0$

$-50x = +200$

plug into equation ← $\boxed{x = -4}$

$25(-4)^2 + 16y^2 + 200(-4) - 160y + 400 = 0$

$400 + 16y^2 - 800 - 160y + 400 = 0$

$16y^2 - 160y = 0$ $16y(y-10) = 0$ $y = 0, 10$

$\boxed{\text{Horiz. tangents: } (-4,0) \text{ and } (-4,10)}$

vertical tangent: $32y - 160 = 0$ $32y = 160$ $y = 5$

$25x^2 + 16(5)^2 + 200x - 160(5) + 400 = 0$

$25x^2 + 400 + 200x - 800 + 400 = 0$

$25x^2 + 200x = 0$

$25x(x+8) = 0$

$x = 0, -8$

$\boxed{\text{vertical tangents: } (0,5) \text{ and } (-8,5)}$

58. $4x^2 + y^2 - 8x + 4y + 4 = 0$

$8x + 2y\left(\frac{dy}{dx}\right) - 8 + 4\left(\frac{dy}{dx}\right) + 0 = 0$

$2y\left(\frac{dy}{dx}\right) + 4\left(\frac{dy}{dx}\right) = 8 - 8x$

$\frac{dy}{dx}(2y + 4) = 8 - 8x$

$\boxed{\frac{dy}{dx} = \frac{8-8x}{2y+4}}$

horizontal tangent: $8 - 8x = 0$ $x = 1$

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$y^2 + 4y = 0$

$y(y+4) = 0$

$y = 0, -4$

$\boxed{\text{horizontal tangents: } (1,0) \text{ and } (1,-4)}$

vertical tangents: $2y + 4 = 0$

$2y = -4$ $y = -2$

$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$

$4x^2 + 4 - 8x - 8 + 4 = 0$

$4x^2 - 8x = 0$

$4x(x-2) = 0$

$x = 0, 2$

$\boxed{\text{vertical tangents } (0,-2) \text{ and } (2,-2)}$

Key (5)
2000

AP Calculus AB–5 / BC–5

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

$2\begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) When $x = 1$, $y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y - 3)(y + 2) = 0$

$y = 3,\ y = -2$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x - 1)$

$4\begin{cases} 1: y^2 - y = 6 \\ 1: \text{solves for } y \\ 2: \text{tangent lines} \end{cases}$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x(2y - x^2) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with ~~x-coordinate 0.~~

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$-\dfrac{1}{4}x^5 = 6$

$x = \sqrt[5]{-24}$

$3\begin{cases} 1: \text{sets denominator of } \dfrac{dy}{dx} \text{ equal to } 0 \\ 1: \text{substitutes } y = \dfrac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \quad \text{into the equation for the curve} \\ 1: \text{solves for } x\text{-coordinate} \end{cases}$