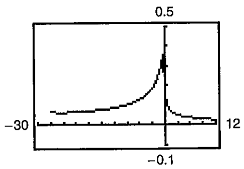
5. (a) $m = -\frac{12}{5}$ (b) $y = \frac{5}{12}x - \frac{169}{12}$

(c) $m_x = \dfrac{-\sqrt{169 - x^2} + 12}{x - 5}$

(d) $\frac{5}{12}$; It is the same as the slope of the tangent line found in (b).

7. (a) Domain: $[-27, 1) \cup (1, \infty)$

(b)

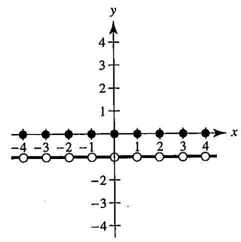


(c) $\frac{1}{14}$ (d) $\frac{1}{12}$

The graph has a hole at $x = 1$.

9. (a) g_1, g_4 (b) g_1 (c) g_1, g_3, g_4

11.



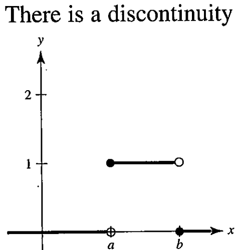
The graph jumps at every integer.

(a) $f(1) = 0,\quad f(0) = 0,\quad f(\frac{1}{2}) = -1,\quad f(-2.7) = -1$

(b) $\lim_{x\to1^-} f(x) = -1,\quad \lim_{x\to1^+} f(x) = -1,\quad \lim_{x\to1/2} f(x) = -1$

(c) There is a discontinuity at each integer.

13. (a)



(b) (i) $\lim_{x\to a^+} P_{a,b}(x) = 1$

(ii) $\lim_{x\to a^-} P_{a,b}(x) = 0$

(iii) $\lim_{x\to b^+} P_{a,b}(x) = 0$

(iv) $\lim_{x\to b^-} P_{a,b}(x) = 1$

(c) Continuous for all positive real numbers except a and b

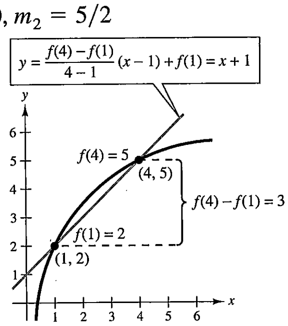
(d) The area under the graph of U and above the x-axis is 1.

Chapter 2

Section 2.1 *(page 103)*

1. $m_1 = 0, m_2 = 5/2$

3. (a)–(c)



5. $m = -5$

7. $m = 4$

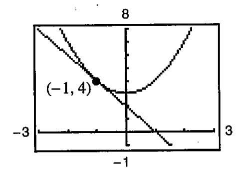
9. $m = 3$ **11.** $f'(x) = 0$ **13.** $f'(x) = -10$

15. $h'(s) = \frac{2}{3}$ **17.** $f'(x) = 2x + 1$ **19.** $f'(x) = 3x^2 - 12$

21. $f'(x) = \dfrac{-1}{(x-1)^2}$ **23.** $f'(x) = \dfrac{1}{2\sqrt{x+4}}$

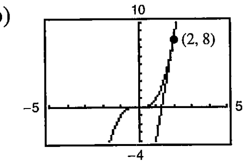
25. (a) Tangent line: $y = -2x + 2$

(b)



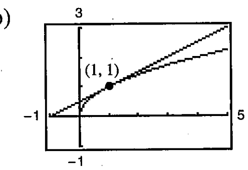
27. (a) Tangent line: $y = 12x - 16$

(b)



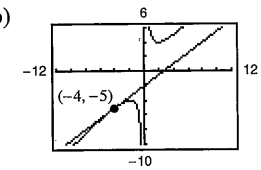
29. (a) Tangent line: $y = \frac{1}{2}x + \frac{1}{2}$

(b)



31. (a) Tangent line: $y = \frac{3}{4}x - 2$

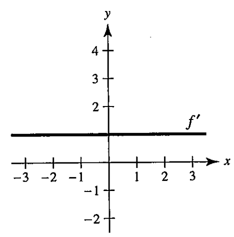
(b)



33. $y = 2x - 1$ **35.** $y = 3x - 2;\ y = 3x + 2$

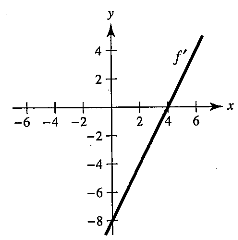
37. $y = -\frac{1}{2}x + \frac{3}{2}$

39.



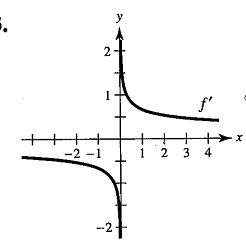
The slope of the graph of f is 1 for all x-values.

41.



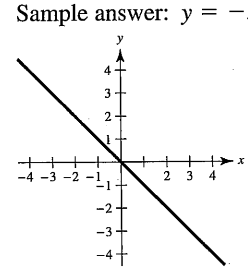
The slope of the graph of f is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.

43.



The slope of the graph of f is negative for $x < 0$ and positive for $x > 0$. The slope is undefined at $x = 0$.

45. Answers will vary.
Sample answer: $y = -x$

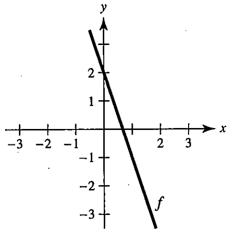


47. $g(4) = 5;\ g'(4) = -\frac{5}{3}$

49. $f(x) = 5 - 3x$
$c = 1$

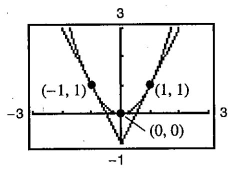
51. $f(x) = -x^2$
$c = 6$

53. $f(x) = -3x + 2$



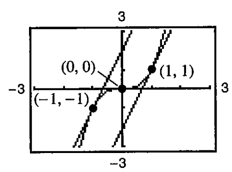
55. $y = 2x + 1;\ y = -2x + 9$

57. (a)



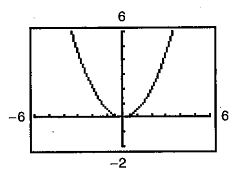
For this function, the slopes of the tangent lines are always distinct for different values of x.

(b)



For this function, the slopes of the tangent lines are sometimes the same.

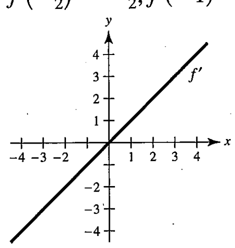
59. (a)



$f'(0) = 0, f'(\frac{1}{2}) = \frac{1}{2}, f'(1) = 1, f'(2) = 2$

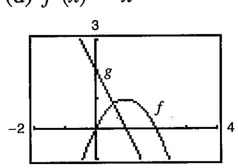
(b) $f'(-\frac{1}{2}) = -\frac{1}{2}, f'(-1) = -1, f'(-2) = -2$

(c)



(d) $f'(x) = x$

61.



$g(x) \approx f'(x)$

63. $f(2) = 4; f(2.1) = 3.99; f'(2) \approx -0.1$ **65.** 6 **67.** 4

69. $g(x)$ is not differentiable at $x = 0$.

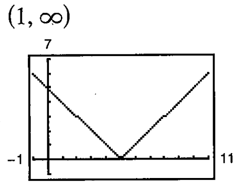
71. $f(x)$ is not differentiable at $x = 6$.

73. $h(x)$ is not differentiable at $x = -7$.

75. $(-\infty, 3) \cup (3, \infty)$ **77.** $(-\infty, -4) \cup (-4, \infty)$

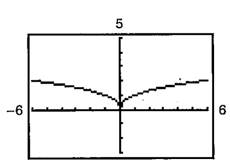
79. $(1, \infty)$

81.



$(-\infty, 5) \cup (5, \infty)$

83.



$(-\infty, 0) \cup (0, \infty)$

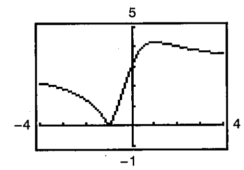
85. The derivative from the left is -1 and the derivative from the right is 1, so f is not differentiable at $x = 1$.

87. The derivatives from both the right and the left are 0, so $f'(1) = 0$.

89. f is differentiable at $x = 2$.

91. (a) $d = (3|m + 1|)/\sqrt{m^2 + 1}$

(b)



Not differentiable at $m = -1$

93. False. The slope is $\lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$.

95. False. For example, $f(x) = |x|$. The derivative from the left and the derivative from the right both exist but are not equal.

97. Proof

Section 2.2 *(page 114)*

1. (a) $\frac{1}{2}$ (b) 3 **3.** 0 **5.** $7x^6$ **7.** $-5/x^6$

9. $1/(5x^{4/5})$ **11.** 1 **13.** $-4t + 3$ **15.** $2x + 12x^2$

17. $3t^2 + 10t - 3$ **19.** $\frac{\pi}{2}\cos\theta + \sin\theta$ **21.** $2x + \frac{1}{2}\sin x$

23. $-\frac{1}{x^2} - 3\cos x$

	Function	Rewrite	Derivative	Simplify
25.	$y = \frac{5}{2x^2}$	$y = \frac{5}{2}x^{-2}$	$y' = -5x^{-3}$	$y' = -\frac{5}{x^3}$
27.	$y = \frac{6}{(5x)^3}$	$y = \frac{6}{125}x^{-3}$	$y' = -\frac{18}{125}x^{-4}$	$y' = -\frac{18}{125x^4}$
29.	$y = \frac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\frac{1}{2}x^{-3/2}$	$y' = -\frac{1}{2x^{3/2}}$

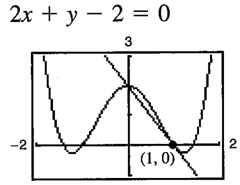
31. -2 **33.** 0 **35.** 8 **37.** 3 **39.** $2x + 6/x^3$

41. $2t + 12/t^4$ **43.** $8x + 3$ **45.** $(x^3 - 8)/x^3$

47. $3x^2 + 1$ **49.** $\frac{1}{2\sqrt{x}} - \frac{2}{x^{2/3}}$ **51.** $\frac{3}{\sqrt{x}} - 5\sin x$

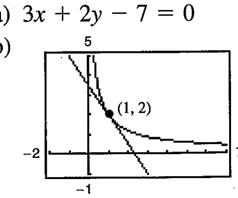
53. (a) $2x + y - 2 = 0$

(b)



55. (a) $3x + 2y - 7 = 0$

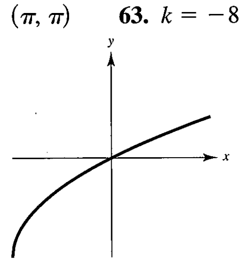
(b)



57. $(-1, 2), (0, 3), (1, 2)$ **59.** No horizontal tangents

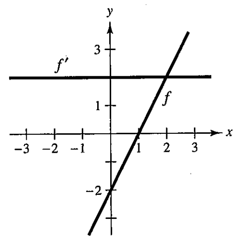
61. (π, π) **63.** $k = -8$ **65.** $k = 3$ **67.** $k = 4/27$

69.



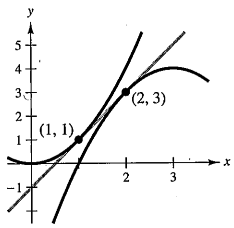
71. $g'(x) = f'(x)$ **73.** $g'(x) = -5f'(x)$

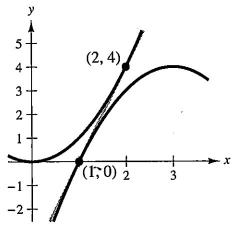
75.



The rate of change of f is constant, and therefore f' is a constant function.

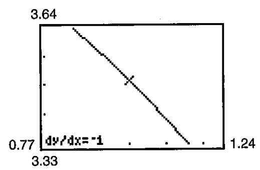
77. $y = 2x - 1$ $\qquad$ $y = 4x - 4$





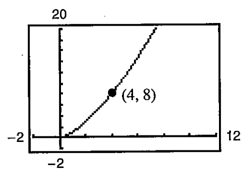
79. $f'(x) = 3 + \cos x \neq 0$ for all x. **81.** $x - 4y + 4 = 0$

83.



$f'(1)$ appears to be close to -1.
$f'(1) = -1$

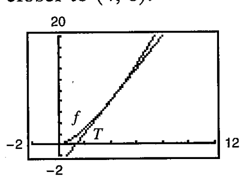
85. (a)



$(3.9, 7.7019)$,
$S(x) = 2.981x - 3.924$

(b) $T(x) = 3(x - 4) + 8 = 3x - 4$
The slope (and equation) of the secant line approaches that of the tangent line at $(4, 8)$ as you choose points closer and closer to $(4, 8)$.

(c)



The approximation becomes less accurate.

(d)

Δx	-3	-2	-1	-0.5	-0.1	0
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8
$T(4 + \Delta x)$	-1	2	5	6.5	7.7	8

Δx	0.1	0.5	1	2	3
$f(4 + \Delta x)$	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	8.3	9.5	11	14	17

87. False. Let $f(x) = x$ and $g(x) = x + 1$.

89. False. $dy/dx = 0$ **91.** True

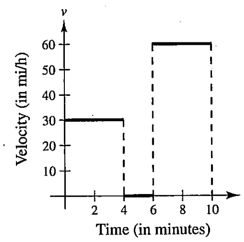
93. Average rate: 4
Instantaneous rates:
$f'(1) = 4; f'(2) = 4$

95. Average rate: $\frac{1}{2}$
Instantaneous rates:
$f'(1) = 1; f'(2) = \frac{1}{4}$

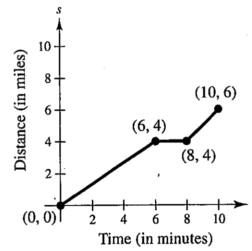
97. (a) $s(t) = -16t^2 + 1362$; $v(t) = -32t$ (b) -48 ft/sec
(c) $s'(1) = -32$ ft/sec; $s'(2) = -64$ ft/sec
(d) $t = \dfrac{\sqrt{1362}}{4} \approx 9.226$ sec (e) -295.242 ft/sec

99. $v(5) = 71$ m/sec; $v(10) = 22$ m/sec

101.

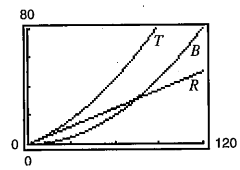


103.



105. $V'(6) = 108\ \text{cm}^3/\text{cm}$

107. (a) $R(v) = 0.417v - 0.02$
(b) $B(v) = 0.0056v^2 + 0.001v + 0.04$
(c) $T(v) = 0.0056v^2 + 0.418v + 0.02$
(d)



(e) $T'(v) = 0.0112v + 0.418$
$T'(40) = 0.866$
$T'(80) = 1.314$
$T'(100) = 1.538$

(f) Stopping distance increases at an increasing rate.

109. Proof **111.** $y = 2x^2 - 3x + 1$

113. $9x + y = 0,\ 9x + 4y + 27 = 0$ **115.** $a = \frac{1}{3}, b = -\frac{4}{3}$

117. $f_1(x) = |\sin x|$ is differentiable for all $x \neq n\pi$, n an integer.
$f_2(x) = \sin|x|$ is differentiable for all $x \neq 0$.

119. Putnam Problem A2, 2010

Section 2.3 *(page 125)*

1. $2(2x^3 - 6x^2 + 3x - 6)$ **3.** $(1 - 5t^2)/(2\sqrt{t})$

5. $x^2(3\cos x - x \sin x)$ **7.** $(1 - x^2)/(x^2 + 1)^2$

9. $(1 - 5x^3)/[2\sqrt{x}(x^3 + 1)^2]$ **11.** $(x \cos x - 2 \sin x)/x^3$

13. $f'(x) = (x^3 + 4x)(6x + 2) + (3x^2 + 2x - 5)(3x^2 + 4)$
$= 15x^4 + 8x^3 + 21x^2 + 16x - 20$
$f'(0) = -20$

15. $f'(x) = \dfrac{x^2 - 6x + 4}{(x - 3)^2}$
$f'(1) = -\dfrac{1}{4}$

17. $f'(x) = \cos x - x \sin x$
$f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{8}(4 - \pi)$

	Function	Rewrite	Differentiate	Simplify
19.	$y = \dfrac{x^2 + 3x}{7}$	$y = \frac{1}{7}x^2 + \frac{3}{7}x$	$y' = \frac{2}{7}x + \frac{3}{7}$	$y' = \dfrac{2x + 3}{7}$
21.	$y = \dfrac{6}{7x^2}$	$y = \frac{6}{7}x^{-2}$	$y' = -\frac{12}{7}x^{-3}$	$y' = -\dfrac{12}{7x^3}$
23.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}$, $x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}$, $x > 0$

25. $\dfrac{3}{(x + 1)^2}$, $x \neq 1$ **27.** $(x^2 + 6x - 3)/(x + 3)^2$

29. $(3x + 1)/(2x^{3/2})$ **31.** $6s^2(s^3 - 2)$

33. $-(2x^2 - 2x + 3)/[x^2(x - 3)^2]$

35. $10x^4 - 8x^3 - 21x^2 - 10x - 30$

37. $-\dfrac{4xc^2}{(x^2 - c^2)^2}$ **39.** $t(t \cos t + 2 \sin t)$

41. $-(t \sin t + \cos t)/t^2$ **43.** $-1 + \sec^2 x = \tan^2 x$

45. $\dfrac{1}{4t^{3/4}} - 6 \csc t \cot t$ **47.** $\dfrac{3}{2} \sec x(\tan x - \sec x)$

49. $\cos x \cot^2 x$ **51.** $x(x \sec^2 x + 2 \tan x)$

53. $4x \cos x + (2 - x^2) \sin x$

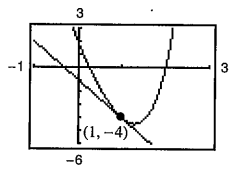
55. $\dfrac{2x^2 + 8x - 1}{(x + 2)^2}$ **57.** $\dfrac{1 - \sin\theta + \theta \cos\theta}{(1 - \sin\theta)^2}$

59. $y' = \dfrac{-2 \csc x \cot x}{(1 - \csc x)^2}, \quad -4\sqrt{3}$

61. $h'(t) = \sec t(t \tan t - 1)/t^2, \quad 1/\pi^2$

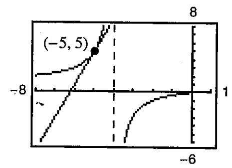
63. (a) $y = -3x - 1$

(b)



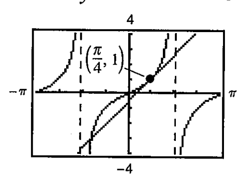
65. (a) $y = 4x + 25$

(b)



67. (a) $4x - 2y - \pi + 2 = 0$

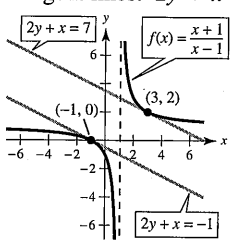
(b)



69. $2y + x - 4 = 0$

71. $25y - 12x + 16 = 0$ **73.** $(1, 1)$ **75.** $(0, 0), (2, 4)$

77. Tangent lines: $2y + x = 7$; $2y + x = -1$



79. $f(x) + 2 = g(x)$ **81.** (a) $p'(1) = 1$ (b) $q'(4) = -1/3$

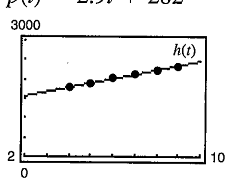
83. $(18t + 5)/(2\sqrt{t})\ \text{cm}^2/\text{sec}$

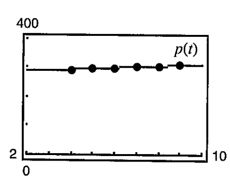
85. (a) $-\$38.13$ thousand/100 components
(b) $-\$10.37$ thousand/100 components
(c) $-\$3.80$ thousand/100 components
The cost decreases with increasing order size.

87. Proof

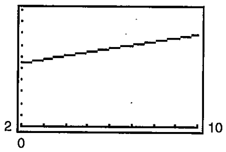
89. (a) $h(t) = 112.4t + 1332$
$p(t) = 2.9t + 282$

(b)





(c) $A = \dfrac{112.4t + 1332}{2.9t + 282}$



A represents the average health care expenditures per person (in thousands of dollars).

(d) $A'(t) = \dfrac{27{,}834}{8.41t^2 + 1635.6t + 79{,}524}$

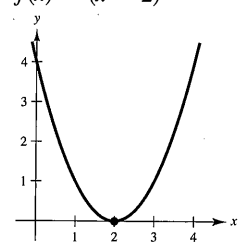
$A'(t)$ represents the rate of change of the average health care expenditures per person for the given year t.

91. $12x^2 + 12x - 6$ **93.** $3/\sqrt{x}$ **95.** $2/(x - 1)^3$

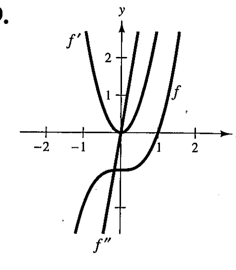
97. $2 \cos x - x \sin x$ **99.** $2x$ **101.** $1/\sqrt{x}$ **103.** 0

105. -10

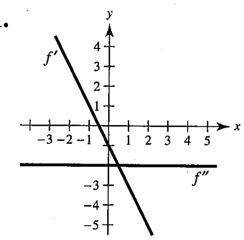
107. Answers will vary.
Sample answer:
$f(x) = (x - 2)^2$



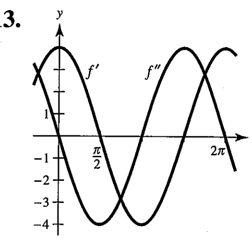
109.



111.



113.



115. $v(3) = 27$ m/sec
$a(3) = -6$ m/sec^2
The speed of the object is decreasing.

117.

t	0	1	2	3	4
$s(t)$	0	57.75	99	123.75	132
$v(t)$	66	49.5	33	16.5	0
$a(t)$	−16.5	−16.5	−16.5	−16.5	−16.5

The average velocity on [0, 1] is 57.75, on [1, 2] is 41.25, on [2, 3] is 24.75, and on [3, 4] is 8.25.

119. $f^{(n)}(x) = n(n - 1)(n - 2) \cdots (2)(1) = n!$

121. (a) $f''(x) = g(x)h''(x) + 2g'(x)h'(x) + g''(x)h(x)$
$f'''(x) = g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)$
$f^{(4)}(x) = g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)$

(b) $f^{(n)}(x) = g(x)h^{(n)}(x) + \dfrac{n!}{1!(n-1)!} g'(x)h^{(n-1)}(x) + \dfrac{n!}{2!(n-2)!} g''(x)h^{(n-2)}(x) + \cdots + \dfrac{n!}{(n-1)!1!} g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)$

123. $n = 1$: $f'(x) = x \cos x + \sin x$
$n = 2$: $f'(x) = x^2 \cos x + 2x \sin x$
$n = 3$: $f'(x) = x^3 \cos x + 3x^2 \sin x$
$n = 4$: $f'(x) = x^4 \cos x + 4x^3 \sin x$
General rule: $f'(x) = x^n \cos x + nx^{(n-1)} \sin x$

125. $y' = -1/x^2, y'' = 2/x^3,$
$x^3y'' + 2x^2y' = x^3(2/x^3) + 2x^2(-1/x^2)$
$= 2 - 2 = 0$

127. $y' = 2\cos x, y'' = -2\sin x,$
$y'' + y = -2\sin x + 2\sin x + 3 = 3$

129. False. $dy/dx = f(x)g'(x) + g(x)f'(x)$ **131.** True

133. True **135.** $f'(x) = 2|x|; f''(0)$ does not exist.

137. Proof

Section 2.4 *(page 136)*

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (5x-8)^4$	$u = 5x - 8$	$y = u^4$
3.	$y = \sqrt{x^3 - 7}$	$u = x^3 - 7$	$y = \sqrt{u}$
5.	$y = \csc^3 x$	$u = \csc x$	$y = u^3$

7. $12(4x-1)^2$ **9.** $-108(4-9x)^3$ **11.** $-1/(2\sqrt{5-t})$

13. $4x/\sqrt[3]{(6x^2+1)^2}$ **15.** $-x/\sqrt[4]{(9-x^2)^3}$

17. $-1/(x-2)^2$ **19.** $-2/(t-3)^3$

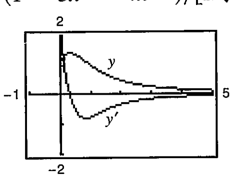
21. $-3/[2\sqrt{(3x+5)^3}]$ **23.** $2x(x-2)^3(3x-2)$

25. $\dfrac{1-2x^2}{\sqrt{1-x^2}}$ **27.** $\dfrac{1}{\sqrt{(x^2+1)^3}}$

29. $\dfrac{-2(x+5)(x^2+10x-2)}{(x^2+2)^3}$ **31.** $\dfrac{-9(1-2v)^2}{(v+1)^4}$

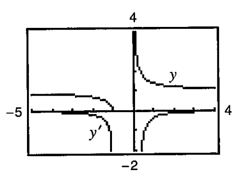
33. $20x(x^2+3)^9 + 2(x^2+3)^5 + 20x^2(x^2+3)^4 + 2x$

35. $(1 - 3x^2 - 4x^{3/2})/[2\sqrt{x}(x^2+1)^2]$



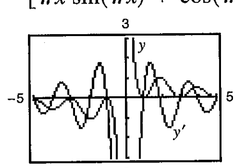
The zero of y' corresponds to the point on the graph of the function where the tangent line is horizontal.

37. $-\dfrac{\sqrt{\dfrac{x+1}{x}}}{2x(x+1)}$



y' has no zeros.

39. $-[\pi x\sin(\pi x) + \cos(\pi x) + 1]/x^2$



The zeros of y' correspond to the points on the graph of the function where the tangent lines are horizontal.

41. (a) 1 (b) 2; The slope of $\sin ax$ at the origin is a.

43. $-4\sin 4x$ **45.** $15\sec^2 3x$ **47.** $2\pi^2 x\cos(\pi x)^2$

49. $2\cos 4x$ **51.** $(-1 - \cos^2 x)/\sin^3 x$

53. $8\sec^2 x\tan x$ **55.** $10\tan 5\theta\sec^2 5\theta$

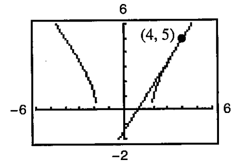
57. $\sin 2\theta\cos 2\theta = \dfrac{1}{2}\sin 4\theta$ **59.** $\dfrac{6\pi\sin(\pi t - 1)}{\cos^3(\pi t - 1)}$

61. $\dfrac{1}{2\sqrt{x}} + 2x\cos(2x)^2$ **63.** $2\sec^2 2x\cos(\tan 2x)$

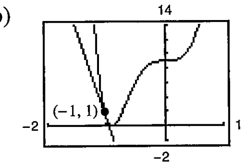
65. $y' = \dfrac{x+4}{\sqrt{x^2+8x}}, \dfrac{5}{3}$ **67.** $f'(x) = \dfrac{-15x^2}{(x^3-2)^2}, -\dfrac{3}{5}$

69. $f'(t) = \dfrac{-5}{(t-1)^2}, -5$ **71.** $y' = -12\sec^3 4x\tan 4x, 0$

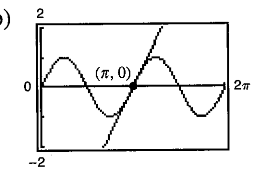
73. (a) $8x - 5y - 7 = 0$
(b)



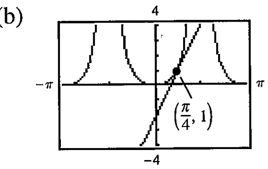
75. (a) $24x + y + 23 = 0$
(b)



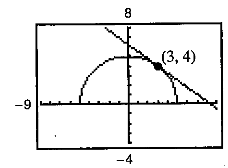
77. (a) $2x - y - 2\pi = 0$
(b)



79. (a) $4x - y + (1 - \pi) = 0$
(b)



81. $3x + 4y - 25 = 0$



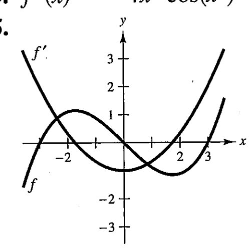
83. $\left(\dfrac{\pi}{6}, \dfrac{3\sqrt{3}}{2}\right), \left(\dfrac{5\pi}{6}, -\dfrac{3\sqrt{3}}{2}\right), \left(\dfrac{3\pi}{2}, 0\right)$ **85.** $2940(2-7x)^2$

87. $\dfrac{2}{(x-6)^3}$ **89.** $2(\cos x^2 - 2x^2\sin x^2)$

91. $h''(x) = 18x + 6, 24$

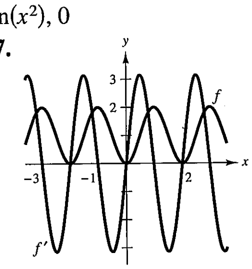
93. $f''(x) = -4x^2\cos(x^2) - 2\sin(x^2), 0$

95.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

97.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

99. The rate of change of g is three times as fast as the rate of change of f.

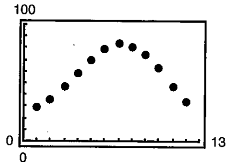
101. (a) $g'(x) = f'(x)$ (b) $h'(x) = 2f'(x)$
(c) $r'(x) = -3f'(-3x)$ (d) $s'(x) = f'(x+2)$

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	-1	-2	-4		

103. (a) $\frac{1}{2}$
(b) $s'(5)$ does not exist because g is not differentiable at 6.

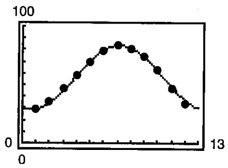
105. (a) 1.461 (b) -1.016 **107.** 0.2 rad, 1.45 rad/sec

109. (a)



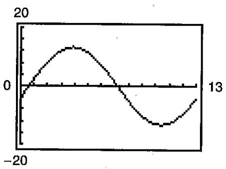
$T(t) = 56.1 + 27.6 \sin(0.48t - 1.86)$

(b)



The model is a good fit.

(c) $T'(t) \approx 13.25 \cos(0.48t - 1.86)$



(d) The temperature changes most rapidly around spring (March–May) and fall (Oct.–Nov.)
The temperature changes most slowly around winter (Dec.–Feb.) and summer (Jun.–Aug.)
Yes. Explanations will vary.

111. (a) 0 bacteria per day (b) 177.8 bacteria per day
(c) 44.4 bacteria per day (d) 10.8 bacteria per day
(e) 3.3 bacteria per day
(f) The rate of change of the population is decreasing as time passes.

113. (a) $f'(x) = \beta \cos \beta x$
$f''(x) = -\beta^2 \sin \beta x$
$f'''(x) = -\beta^3 \cos \beta x$
$f^{(4)}(x) = \beta^4 \sin \beta x$
(b) $f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2(\sin \beta x) = 0$
(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$
$f^{(2k-1)}(x) = (-1)^{k+1} \beta^{2k-1} \cos \beta x$

115. (a) $r'(1) = 0$ (b) $s'(4) = \frac{5}{8}$

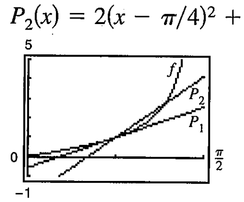
117. (a) and (b) Proofs

119. $g'(x) = 3\left(\dfrac{3x-5}{|3x-5|}\right), \quad x \neq \dfrac{5}{3}$

121. $h'(x) = -|x| \sin x + \dfrac{x}{|x|} \cos x, \quad x \neq 0$

123. (a) $P_1(x) = 2(x - \pi/4) + 1$
$P_2(x) = 2(x - \pi/4)^2 + 2(x - \pi/4) + 1$

(b)



(c) P_2

(d) The accuracy worsens as you move away from $x = \pi/4$.

125. False. If $y = (1-x)^{1/2}$, then $y' = \frac{1}{2}(1-x)^{-1/2}(-1)$.

127. True **129.** Putnam Problem A1, 1967

Section 2.5 *(page 145)*

1. $-x/y$ **3.** $-\sqrt{y/x}$ **5.** $(y - 3x^2)/(2y - x)$

7. $(1 - 3x^2y^3)/(3x^3y^2 - 1)$

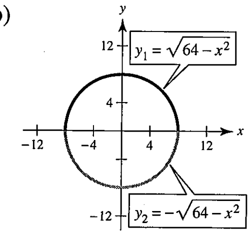
9. $(6xy - 3x^2 - 2y^2)/(4xy - 3x^2)$ **11.** $\cos x/[4 \sin(2y)]$

13. $(\cos x - \tan y - 1)/(x \sec^2 y)$

15. $[y \cos(xy)]/[1 - x \cos(xy)]$

17. (a) $y_1 = \sqrt{64 - x^2}$; $y_2 = -\sqrt{64 - x^2}$

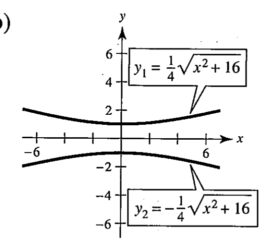
(b)



(c) $y' = \mp \dfrac{x}{\sqrt{64 - x^2}} = -\dfrac{x}{y}$ (d) $y' = -\dfrac{x}{y}$

19. (a) $y_1 = \dfrac{\sqrt{x^2 + 16}}{4}$; $y_2 = \dfrac{-\sqrt{x^2 + 16}}{4}$

(b)



(c) $y' = \dfrac{\pm x}{4\sqrt{x^2 + 16}} = \dfrac{x}{16y}$ (d) $y' = \dfrac{x}{16y}$

21. $-\dfrac{y}{x}, -\dfrac{1}{6}$ **23.** $\dfrac{98x}{y(x^2 + 49)^2}$, Undefined

25. $-\dfrac{y(y + 2x)}{x(x + 2y)}, -1$ **27.** $-\sin^2(x + y)$ or $-\dfrac{x^2}{x^2 + 1}, 0$

29. $-\dfrac{1}{2}$ **31.** 0 **33.** $y = -x + 7$ **35.** $y = -x + 2$

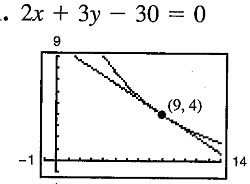
37. $y = \sqrt{3}x/6 + 8\sqrt{3}/3$ **39.** $y = -\frac{2}{11}x + \frac{30}{11}$

41. (a) $y = -2x + 4$ (b) Answers will vary.

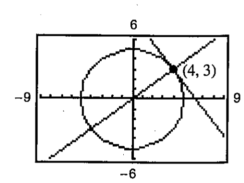
43. $\cos^2 y, \; -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}, \; \dfrac{1}{1 + x^2}$ **45.** $-4/y^3$

47. $-36/y^3$ **49.** $(3x)/(4y)$

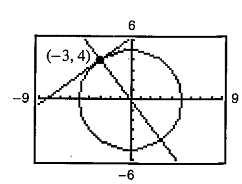
51. $2x + 3y - 30 = 0$



53. At $(4, 3)$:
Tangent line: $4x + 3y - 25 = 0$
Normal line: $3x - 4y = 0$



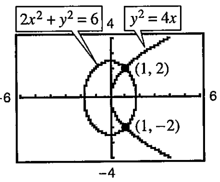
At $(-3, 4)$:
Tangent line: $3x - 4y + 25 = 0$
Normal line: $4x + 3y = 0$



55. $x^2 + y^2 = r^2 \Rightarrow y' = -x/y \Rightarrow y/x =$ slope of normal line. Then for (x_0, y_0) on the circle, $x_0 \neq 0$, an equation of the normal line is $y = (y_0/x_0)x$, which passes through the origin. If $x_0 = 0$, the normal line is vertical and passes through the origin.

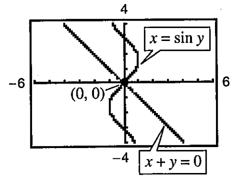
57. Horizontal tangents: $(-4, 0), (-4, 10)$
Vertical tangents: $(0, 5), (-8, 5)$

59.



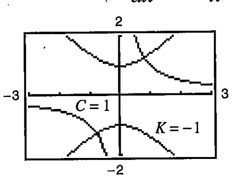
At $(1, 2)$:
Slope of ellipse: -1
Slope of parabola: 1
At $(1, -2)$:
Slope of ellipse: 1
Slope of parabola: -1

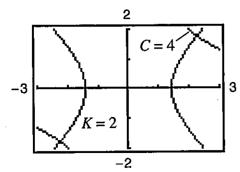
61.



At $(0, 0)$:
Slope of line: -1
Slope of sine curve: 1

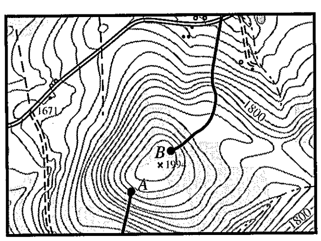
63. Derivatives: $\dfrac{dy}{dx} = -\dfrac{y}{x}, \dfrac{dy}{dx} = \dfrac{x}{y}$





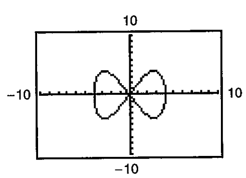
65. Answers will vary. In the explicit form of a function, the variable is explicitly written as a function of x. In an implicit equation, the function is only implied by an equation. An example of an implicit function is $x^2 + xy = 5$. In explicit form, it would be $y = (5 - x^2)/x$.

67.

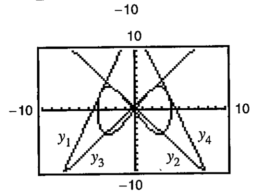


Use starting point B.

69. (a)



(b)

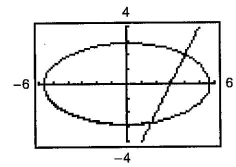


$y_1 = \frac{1}{3}\left[(\sqrt{7} + 7)x + (8\sqrt{7} + 23)\right]$
$y_2 = -\frac{1}{3}\left[(-\sqrt{7} + 7)x - (23 - 8\sqrt{7})\right]$
$y_3 = -\frac{1}{3}\left[(\sqrt{7} - 7)x - (23 - 8\sqrt{7})\right]$
$y_4 = -\frac{1}{3}\left[(\sqrt{7} + 7)x - (8\sqrt{7} + 23)\right]$

(c) $\left(\dfrac{8\sqrt{7}}{7}, 5\right)$

71. Proof **73.** $y = -\dfrac{\sqrt{3}}{2}x + 2\sqrt{3}, y = \dfrac{\sqrt{3}}{2}x - 2\sqrt{3}$

75. (a) $y = 2x - 6$
(b)



(c) $\left(\frac{28}{17}, -\frac{46}{17}\right)$

Section 2.6 *(page 153)*

1. (a) $\frac{3}{4}$ (b) 20 **3.** (a) $-\frac{5}{8}$ (b) $\frac{3}{2}$
5. (a) -8 cm/sec (b) 0 cm/sec (c) 8 cm/sec
7. (a) 12 ft/sec (b) 6 ft/sec (c) 3 ft/sec
9. In a linear function, if x changes at a constant rate, so does y. However, unless $a = 1$, y does not change at the same rate as x.
11. (a) 64π cm²/min (b) 256π cm²/min
13. (a) 972π in.³/min; $15{,}552\pi$ in.³/min
(b) If dr/dt is constant, dV/dt is proportional to r^2.
15. (a) 72 cm³/sec (b) 1800 cm³/sec
17. $8/(405\pi)$ ft/min **19.** (a) 12.5% (b) $\frac{1}{144}$ m/min
21. (a) $-\frac{7}{12}$ ft/sec; $-\frac{3}{2}$ ft/sec; $-\frac{48}{7}$ ft/sec
(b) $\frac{527}{24}$ ft²/sec (c) $\frac{1}{12}$ rad/sec
23. Rate of vertical change: $\frac{1}{5}$ m/sec
Rate of horizontal change: $-\sqrt{3}/15$ m/sec
25. (a) -750 mi/h (b) 30 min
27. $-50/\sqrt{85} \approx -5.42$ ft/sec
29. (a) $\frac{25}{3}$ ft/sec (b) $\frac{10}{3}$ ft/sec
31. (a) 12 sec (b) $\frac{1}{2}\sqrt{3}$ m (c) $\sqrt{5}\pi/120$ m/sec
33. Evaporation rate proportional to $S \Rightarrow \dfrac{dV}{dt} = k(4\pi r^2)$
$V = \left(\dfrac{4}{3}\right)\pi r^3 \Rightarrow \dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$. So $k = \dfrac{dr}{dt}$.
35. 0.6 ohm/sec **37.** $\dfrac{dv}{dt} = \dfrac{16r}{v}\sec^2\theta\dfrac{d\theta}{dt}, \dfrac{d\theta}{dt} = \dfrac{v}{16r}\cos^2\theta\dfrac{dv}{dt}$
39. $\dfrac{2\sqrt{21}}{525} \approx 0.017$ rad/sec
41. (a) $\dfrac{200\pi}{3}$ ft/sec (b) 200π ft/sec
(c) About 427.43π ft/sec
43. About 84.9797 mi/h
45. (a) $\dfrac{dy}{dt} = 3\dfrac{dx}{dt}$ means that y changes three times as fast as x changes.
(b) y changes slowly when $x \approx 0$ or $x \approx L$. y changes more rapidly when x is near the middle of the interval.
47. -18.432 ft/sec² **49.** About -97.96 m/sec

Review Exercises for Chapter 2 *(page 157)*

1. $f'(x) = 0$ **3.** $f'(x) = 2x - 4$ **5.** 5
7. f is differentiable at all $x \neq 3$. **9.** 0 **11.** $3x^2 - 22x$
13. $\dfrac{3}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$ **15.** $-\dfrac{4}{3t^3}$ **17.** $4 - 5\cos\theta$
19. $-3\sin\theta - (\cos\theta)/4$ **21.** -1 **23.** 0
25. (a) 50 vibrations/sec/lb (b) 33.33 vibrations/sec/lb

27. (a) $s(t) = -16t^2 - 30t + 600$
$v(t) = -32t - 30$
(b) -94 ft/sec
(c) $v'(1) = -62$ ft/sec; $v'(3) = -126$ ft/sec
(d) About 5.258 sec (e) About -198.256 ft/sec

29. $4(5x^3 - 15x^2 - 11x - 8)$ **31.** $\sqrt{x}\cos x + \sin x/(2\sqrt{x})$

33. $\dfrac{-(x^2+1)}{(x^2-1)^2}$ **35.** $\dfrac{4x^3\cos x + x^4\sin x}{\cos^2 x}$

37. $3x^2\sec x\tan x + 6x\sec x$ **39.** $-x\sin x$

41. $y = 4x + 10$ **43.** $y = -8x + 1$ **45.** $-48t$

47. $\frac{225}{4}\sqrt{x}$ **49.** $6\sec^2\theta\tan\theta$

51. $v(3) = 11$ m/sec; $a(3) = -6$ m/sec^2 **53.** $28(7x+3)^3$

55. $-\dfrac{2x}{(x^2+4)^2}$ **57.** $-45\sin(9x+1)$

59. $\frac{1}{2}(1 - \cos 2x) = \sin^2 x$ **61.** $(36x+1)(6x+1)^4$

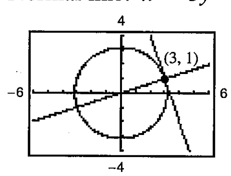
63. $\dfrac{3}{(x^2+1)^{3/2}}$ **65.** $\dfrac{-3x^2}{2\sqrt{1-x^3}}$; -2 **67.** $-\dfrac{8x}{(x^2+1)^2}$; 2

69. $-\csc 2x\cot 2x$; 0 **71.** $384(8x+5)$ **73.** $2\csc^2 x\cot x$

75. (a) $-18.667°/\text{h}$ (b) $-7.284°/\text{h}$
(c) $-3.240°/\text{h}$ (d) $-0.747°/\text{h}$

77. $-\dfrac{x}{y}$ **79.** $\dfrac{y(y^2-3x^2)}{x(x^2-3y^2)}$ **81.** $\dfrac{y\sin x + \sin y}{\cos x - x\cos y}$

83. Tangent line: $3x + y - 10 = 0$
Normal line: $x - 3y = 0$



85. (a) $2\sqrt{2}$ units/sec (b) 4 units/sec (c) 8 units/sec

87. 450π km/h

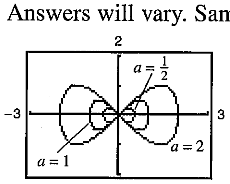
P.S. Problem Solving *(page 159)*

1. (a) $r = \frac{1}{2}$; $x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$
(b) Center: $(0, \frac{5}{4})$; $x^2 + (y - \frac{5}{4})^2 = 1$

3. $p(x) = 2x^3 + 4x^2 - 5$

5. (a) $y = 4x - 4$ (b) $y = -\frac{1}{4}x + \frac{9}{2}$; $\left(-\frac{9}{4}, \frac{81}{16}\right)$
(c) Tangent line: $y = 0$ (d) Proof
Normal line: $x = 0$

7. (a) Graph $\begin{cases} y_1 = \dfrac{1}{a}\sqrt{x^2(a^2-x^2)} \\ y_2 = -\dfrac{1}{a}\sqrt{x^2(a^2-x^2)} \end{cases}$ as separate equations.
(b) Answers will vary. Sample answer:



The intercepts will always be $(0, 0)$, $(a, 0)$, and $(-a, 0)$, and the maximum and minimum y-values appear to be $\pm\frac{1}{2}a$.
(c) $\left(\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right)$, $\left(\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2}, \dfrac{a}{2}\right)$, $\left(-\dfrac{a\sqrt{2}}{2}, -\dfrac{a}{2}\right)$

9. (a) When the man is 90 ft from the light, the tip of his shadow is $112\frac{1}{2}$ ft from the light. The tip of the child's shadow is $111\frac{1}{9}$ ft from the light, so the man's shadow extends $1\frac{7}{18}$ ft beyond the child's shadow.
(b) When the man is 60 ft from the light, the tip of his shadow is 75 ft from the light. The tip of the child's shadow is $77\frac{7}{9}$ ft from the light, so the child's shadow extends $2\frac{7}{9}$ ft beyond the man's shadow.
(c) $d = 80$ ft
(d) Let x be the distance of the man from the light, and let s be the distance from the light to the tip of the shadow.
If $0 < x < 80$, then $ds/dt = -50/9$.
If $x > 80$, then $ds/dt = -25/4$.
There is a discontinuity at $x = 80$.

11. (a) $v(t) = -\frac{27}{5}t + 27$ ft/sec (b) 5 sec; 73.5 ft
$a(t) = -\frac{27}{5}$ ft/sec^2
(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.

13. Proof. The graph of L is a line passing through the origin $(0, 0)$.

15. (a) j would be the rate of change of acceleration.
(b) $j = 0$. Acceleration is constant, so there is no change in acceleration.
(c) a: position function, d: velocity function, b: acceleration function, c: jerk function

Chapter 3

Section 3.1 *(page 167)*

1. $f'(0) = 0$ **3.** $f'(2) = 0$ **5.** $f'(-2)$ is undefined.

7. 2, absolute maximum (and relative maximum)

9. 1, absolute maximum (and relative maximum);
2, absolute minimum (and relative minimum);
3, absolute maximum (and relative maximum)

11. $x = 0, x = 2$ **13.** $t = 8/3$ **15.** $x = \pi/3, \pi, 5\pi/3$

17. Minimum: $(2, 1)$
Maximum: $(-1, 4)$

19. Minimum: $(2, -8)$
Maximum: $(6, 24)$

21. Minimum: $\left(-1, -\frac{5}{2}\right)$
Maximum: $(2, 2)$

23. Minimum: $(0, 0)$
Maximum: $(-1, 5)$

25. Minimum: $(0, 0)$
Maxima: $\left(-1, \frac{1}{4}\right)$ and $\left(1, \frac{1}{4}\right)$

27. Minimum: $(1, -1)$
Maximum: $\left(0, -\frac{1}{2}\right)$

29. Minimum: $(-1, -1)$
Maximum: $(3, 3)$

31. Minimum value is -2 for $-2 \le x < -1$.
Maximum: $(2, 2)$

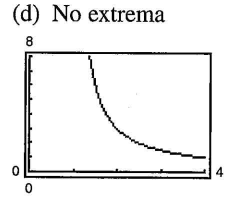
33. Minimum: $(3\pi/2, -1)$
Maximum: $(5\pi/6, 1/2)$

35. Minimum: $(\pi, -3)$
Maxima: $(0, 3)$ and $(2\pi, 3)$

37. (a) Minimum: $(0, -3)$;
Maximum: $(2, 1)$
(b) Minimum: $(0, -3)$
(c) Maximum: $(2, 1)$
(d) No extrema

39. (a) Minimum: $(1, -1)$;
Maximum: $(-1, 3)$
(b) Maximum: $(3, 3)$
(c) Minimum: $(1, -1)$
(d) Minimum: $(1, -1)$

41.



Minimum: $(4, 1)$