AB Calculus Ch. 2 Test Review

1. The average rate of change of $f(x)=4x-x^2$ on the interval [1,3] is _______.

   a.) -2 b.) -1 c.) 0 d.) 1 e.) 2

2. The instantaneous rate of change of f(x) at the endpoints are : ________________________

3. Find $\frac{d^2y}{dx^2}$ for $y=\sqrt{(x^2+1)}$.

   a.) $\frac{1}{(x^2+1)^{\frac{3}{2}}}$ b.) $\frac{1}{(x^2+1)^{\frac{1}{2}}}$ c.) $\frac{x}{(x^2+1)^{\frac{1}{2}}}$ d.) $\frac{x}{(x^2+1)^{\frac{3}{2}}}$ e.) $\frac{1}{2(x^2+1)^{\frac{3}{2}}}$

4. If the line tangent to the graph of the function f at the point (1,7) passes through the point (-2,-2), then $f'(1)$ =

   a.) - 5 b.) 1 c.) 3 d.) 7 e.) undefined

5. Find a and b such that

$$f(x)=\begin{cases} ax^3 & x\le 3 \\ x^2+b & x>3 \end{cases}$$ is differentiable everywhere.

6. A particle moves along a straight line according to the given equation: $s(t) = \frac{2}{3}t^3 - 2t^2 - 1$, $t \geq 0$.

a) Find when the particle is moving to the left.	b.) Find when the velocity is increasing.
c.) Find when the speed is decreasing.	d.) Where is the particle located when the velocity is zero?
e) Find particle's displacement from t = 0 to t = 3	f) Find particle's distance from t = 0 to t = 3
g)Is the velocity increasing or decreasing at t=2?	h) Find the velocity and position when acceleration is zero.

7. Find $\lim_{h\to 0} \frac{2(-1+h)^3 - 3(-1+h) - 1}{h}$

8. If $f(x) = x^4 - 4x$, evaluate $\lim_{x\to -1} \frac{f(x) - f(-1)}{x+1}$.

a.) -8 b.) 0 c.) 1 d.) 2 e.) 4

9. Find the points at which the graph of the equation has a vertical or horizontal tangent line:

$4x^2 + y^2 - 8x + 4y + 4 = 0$

AB Calculus Ch. 2 Test Review Session

1. The average rate of change of $f(x) = 4x - x^2$ on the interval [1,3] is________.

a.) -2 b.) -1 $\boxed{\text{c.) } 0}$ d.) 1 e.) 2

$f(1) = 4 - 1 = 3$

$f(3) = 4(3) - 9 = 3$

$\frac{f(3) - f(1)}{3-1} = \frac{3-3}{2} = \frac{0}{2}$

2. The instantaneous rate of change of f(x) at the endpoints are : ______________________

$f'(x) = 4 - 2x$

$f'(1) = 4 - 2(1) = 2$

$f'(3) = 4 - 2(3) = -2$

$\boxed{\begin{array}{l} f'(1) = 2 \\ f'(3) = -2 \end{array}}$

3. Find $\frac{d^2y}{dx^2}$ for $y = \sqrt{(x^2+1)}$. $= (x^2+1)^{1/2}$ $f'(x) = \frac{1}{2}(x^2+1)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+1}}$

$\boxed{\text{a.)}}$ $\frac{1}{(x^2+1)^{\frac{3}{2}}}$ b.) $\frac{1}{(x^2+1)^{\frac{1}{2}}}$ c.) $\frac{x}{(x^2+1)^{\frac{1}{2}}}$ d.) $\frac{x}{(x^2+1)^{\frac{3}{2}}}$ e.) $\frac{1}{2(x^2+1)^{\frac{3}{2}}}$

$f'(x) = \frac{x}{\sqrt{x^2+1}} = \frac{x}{(x^2+1)^{1/2}}$

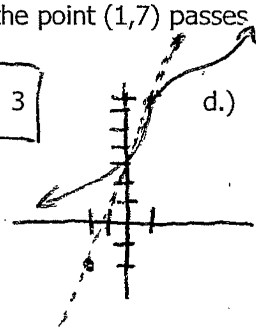
$f''(x) = \frac{1(x^2+1)^{1/2} - x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x)}{x^2+1}$

$= \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$

$\frac{x^2+1-x^2}{(x^2+1)^{3/2}} = \boxed{\frac{1}{(x^2+1)^{3/2}}}$

4. If the line tangent to the graph of the function f at the point (1,7) passes through the point (-2,-2), then $f'(1) =$

a.) - 5 b.) 1 $\boxed{\text{c.) } 3}$ d.) 7 e.) undefined



Find slope: $m = \frac{-2-7}{-2-1} = \frac{-9}{-3} = 3$

5. Find a and b such that

$$f(x) = \begin{cases} ax^3 & x \le 3 \\ x^2 + b & x > 3 \end{cases}$$ is differentiable everywhere.

↳ continuous and differentiable

$\boxed{\text{at } x=3}$

$ax^3 = x^2 + b$

$a(3)^3 = 3^2 + b$

$27a = 9 + b$

$27(2/9) = 9 + b$

$6 = 9 + b$

$\boxed{-3 = b}$

at $x = 3$

$3ax^2 = 2x$

$3a(9) = 2(3)$

$\boxed{a = 2/9}$

$v(t) = 2t^2 - 4t$

$a(t) = 4t - 4$

$v(t)$: 0 $-$ 2 $+$

$0 = 2t(t-2)$

$t = 0, 2$

6. A particle moves along a straight line according to the given equation: $s(t) = \frac{2}{3}t^3 - 2t^2 - 1$, $t \geq 0$.

a) Find when the particle is moving to the left.

$0 < t < 2$

$\boxed{(0,2)}$

b.) Find when the velocity is increasing. $\boxed{a(t) > 0}$

$a(t)$: 0 $-$ 1 $+$

$\boxed{(1, \infty)}$

$0 = 4t - 4$ $\quad 0 = 4(t-1)$

c.) Find when the speed is decreasing.

opposite signs for $v(t)$ and $a(t)$

$1 < t < 2$

$\boxed{(1,2)}$

d.) Where is the particle located when the velocity is zero?

$t = 0, 2$

$s(2) = \frac{2}{3}(2)^3 - 2(2)^2 - 1$

$= \frac{16}{3} - 8 - 1 = \frac{16}{3} - \frac{27}{3} = \frac{-11}{3}$

$\boxed{s(0) = -1 \quad s(2) = -11/3}$

e) Find particle's displacement from t = 0 to t = 3

$s(0) = -1$

$s(3) = -1$

displacement $= 0$

f) Find particle's distance from t = 0 to t = 3

$s(0) = -1$

$s(2) = -11/3$

$s(3) = -1$

$8/3$, $8/3$

$= \frac{16}{3} \approx 5.3\overline{3}$

g) Is the velocity increasing or decreasing at t=2?

$a(2) = 8 - 4 = 4$

since $a(2) > 0$ velocity is increasing at $t = 2$.

h) Find the velocity and position when acceleration is zero.

$0 = 4t - 4$

$1 = t$

$v(1) = 2(1)^2 - 4(1) = -2$

$s(1) = -7/3$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\quad f'(-1) = \frac{f(-1+h) - f(-1)}{h}$

7. Find $\lim_{h \to 0} \frac{2(-1+h)^3 - 3(-1+h) - 1}{h}$

Find $f'(-1)$

$f(x) = 2x^3 - 3x$ $\quad f'(x) = 6x^2 - 3$

$f'(-1) = 6(-1)^2 - 3 = 6 - 3 = 3$

$\boxed{f'(-1) = 3}$

8. If $f(x) = x^4 - 4x$, evaluate $\lim_{x \to -1} \frac{f(x) - f(-1)}{x+1}$. Find $f'(-1)$

$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

$f'(x) = 4x^3 - 4$

$f'(-1) = 4(-1)^3 - 4 = \boxed{-8}$

$\boxed{\text{a.) } -8}$ b.) 0 c.) 1 d.) 2 e.) 4

9. Find the points at which the graph of the equation has a vertical or horizontal tangent line:

$4x^2 + y^2 - 8x + 4y + 4 = 0$

$8x + 2y\left(\frac{dy}{dx}\right) - 8 + 4\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(2y + 4) = 8 - 8x$

$\frac{dy}{dx} = \frac{2(4 - 4x)}{2(y+2)} = \frac{4 - 4x}{y + 2}$

horizontal tangent

set numerator $f'(x) = 0$

$4 - 4x = 0$ $\quad 4 = 4x$

$x = 1$

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$y^2 + 4y = 0$ $\quad y(y+4) = 0$

$y = 0, -4$

$\boxed{(1,0) \text{ and } (1,-4)}$

vertical tangent

set denominator $f'(x) = 0$

$y + 2 = 0$ $\quad y = -2$

$4x^2 + 4 - 8x - 8 + 4 = 0$

$4x^2 - 8x = 0$

$4x(x-2) = 0$ $\quad x = 0, 2$

$\boxed{(0,-2) \text{ and } (2,-2)}$