

Chapter 2 Derivatives Test Review #2

1. Given $x^2y + y^2 = 2x$

a. Find dy/dx

b. Find a point on the graph where there is a horizontal tangent

c. Find the points on the graph where there is a vertical tangent

2. Find d^2y/dx^2 for $\frac{x+2}{3-x}$

3. Find dy/dx for $\left(\frac{3x+1}{1-x^2}\right)^3$

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Key

1. Given $x^2y + y^2 = 2x$

a. Find dy/dx

$$2xy + x^2 \left(\frac{dy}{dx}\right) + 2y \left(\frac{dy}{dx}\right) = 2$$

$$x^2 \frac{dy}{dx} + 2y \frac{dy}{dx} = 2 - 2xy$$

$$\frac{dy}{dx} (x^2 + 2y) = 2 - 2xy$$

$$\frac{dy}{dx} = \frac{2 - 2xy}{x^2 + 2y}$$

b. Find the points on the graph where there is a horizontal tangent

* set numerator of derivative = 0

$$2 - 2xy = 0 \quad | \quad xy = 1$$

$$2 = 2xy \quad | \quad x = 1, y = 1$$

* check with equation $x^2y + y^2 = 2x$

$$(1)^2(1) + (1)^2 = 2(1) \checkmark$$

$(1, 1)$

c. Find the points on the graph where there is a vertical tangent

* set denominator of derivative = 0

$$x^2 + 2y = 0$$

$$0^2 + 2(0) = 0$$

$$(-2)^2 + 2(-2) = 0$$

* check with equation

$$0^2(0) + 0^2 = 2(0) \checkmark$$

$$(-2)^2(-2) + (-2)^2 = 2(-2) \checkmark$$

$(0, 0)$ and $(-2, -2)$

2. Find d^2y/dx^2 for $\frac{x+2}{3-x}$

$$\frac{dy}{dx} = \frac{(1)(3-x) - (x+2)(-1)}{(3-x)^2}$$

$$\frac{dy}{dx} = \frac{-2(5)(3-x)^{-3}(-1)}{(3-x)^3} = \frac{10}{(3-x)^3}$$

$$\frac{dy}{dx} = \frac{3-x+x+2}{(3-x)^2} = \frac{5}{(3-x)^2}$$

OR

$$\frac{0(3-x)^2 - 5(2)(3-x)(-1)}{(3-x)^4} = \frac{10}{(3-x)^3}$$

3. Find dy/dx for $\left(\frac{3x+1}{1-x^2}\right)^3$

* chain (first)

* quotient

$$\frac{dy}{dx} = 3 \left(\frac{3x+1}{1-x^2}\right)^2 \left[\frac{3(1-x^2) - (3x+1)(-2x)}{(1-x^2)^2} \right] = \frac{3(3x+1)^2(3x^2+2x+3)}{(1-x^2)^2(1-x^2)^2}$$

$$\frac{dy}{dx} = \frac{3(3x+1)^2(3x^2+2x+3)}{(1-x^2)^4}$$