AP Calculus AB — Chapter 3.1-3.4 Concept Review

I. Extreme Value Theorem

1. Check continuity
2. Check f(x) is a closed function
3. Find f'(x).
   a. Find Critical Points:
   b. Set numerator and denominator of f'(x) = 0
4. Plug critical points and endpoints into f(x) to find absolute max/min

* max and min values refer to y-values

II. Mean Value Theorem

1. Check continuity on closed interval [a,b]
   a. Does f(x) have variables in the denominator? (V.A. or holes)
   b. If so, then look to see if the x-value lies in the **closed** interval [a, b]
   c. If the x lies between the interval, then function is not continuous on the interval, MVT fails
2. Check Differentiability on open interval (a, b)
   a. Does f'(x) have variables in the denominator? (sharp points, slope undefined)
   b. If so, then look to see if the x-value lies in the **open** interval (a, b)
   c. If the x lies between the interval, then function is not differentiable on the interval, MVT fails
3. Find m_{avg}. (This is the slope between your endpoints)
4. Set f'(x) = m_{avg} and solve for x
5. Make sure the x value(s) lies in the open interval (a, b)

III. Rolle's Theorem

1. Check continuity on closed interval [a,b]
2. Check Differentiability on open interval (a, b)
3. Check endpoints. Does f(a) = f(b) ? If not , then Rolle's fails
4. Set f'(x) = 0 and solve for x
5. Make sure the x value(s) lies in the open interval (a, b)

IV. 1st Derivative Test (Finds inc/dec and relative max/min)

1. Find f '(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f '(x) to determine slope
      i. Positive (+) means increasing slope
      ii. Negative(–) means decreasing slope
4. Write Because Statements
   a. f(x) increasing in interval (a,b) b/c f '(x) > 0
   b. f(x) decreasing in interval (a,b) b/c f '(x) < 0
   c. Relative max at (a, f(a)) b/c f '(x) changes from + to –
   d. Relative min at (a, f(a)) b/c f '(x) changes from – to +

V. Finding Intervals of Concave Up/Down and Points of Inflection (POI) / "Concavity Test"

1. Find f ''(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f ''(x) to determine concavity
      i. Positive (+) means concave up
      ii. Negative(–) means concave down
4. Write Because Statements
   a. Concave up in interval (a,b) b/c f ''(x) > 0
   b. Concave down in interval (a,b) b/c f ''(x) < 0
   c. Point of Inflection at (a, f(a)) b/c f ''(x) changes signs

VI. 2nd Derivative Test (Finding relative max/min)

1. Find f '(x) , set equal to zero
   a. Find critical points . (These are candidates for relative max/min)
2. Find f ''(x)
3. Plug the critical points (from step #1) into f ''(x).
   a. If result is positive value, then f''(x) > 0, concave up, and therefore relative minimum exists at x-value
      - **Relative Minimum at x = a because f '(a) = 0 and f ''(a) > 0**
   b. If result is negative value, then f''(x) < 0, concave down, and therefore relative maximum exists at x-value
      - **Relative Maximum at x = b because f '(b) = 0 and f ''(b) < 0**
   c. If result is zero, then since f''(x) = 0, then this test is inconclusive. We cannot determine whether relative extrema exists. (Use First Derivative Test)

VII. Absolute Extrema on an Open Interval

1. Find f '(x)
2. Find critical number (only 1)
3. Make sign line
4. Write because statements
5. An Absolute Min occurs at (a, f(a)) b/c f ' (x) < 0 for all x < a and f '(x) > 0 for all x > a
6. An Absolute Max occurs at (a, f(a)) b/c f ' (x) > 0 for all x < a and f '(x) < 0 for all x > a