AP Calculus AB - Quiz Review #3.1 – 3.4

1. Find the critical numbers of the function $f(x) = 2x^3 - 8x^2 + 10x$

2. Find the absolute extrema of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 3]$.

3. If $f(x) = \dfrac{-x^2 + 4}{4x}$ on $[-2, 2]$, determine if Rolle's Theorem applies. If yes, find the value(s) of c defined by Rolle's Theorem. If no, explain why not.

4. If $f(x) = -(25 - 5x)^{\frac{1}{2}}$ on $[3, 5]$, determine if the Mean Value Theorem applies. If yes, find the value(s) of c defined by the Mean Value Theorem. If no, explain why not.

5. If $f(x) = x^3 - 4x^2 + 5x$, find the interval(s) where *f(x)* is Increasing and Decreasing

6. If $f(x) = x^3 - 12x^2 - 27x$, use the First Derivative Test to find all relative extrema (x-values).

7. If $f(x) = x^3 + x^2 - 21x$, find the intervals where *f(x)* is concave up and concave down, and find all points of inflection.

8. If $f(x) = 2x^3 - x^2 + 20x - 10$, use the Second Derivative Test to find all relative extrema.

9. Sketch a labeled graph of a function, f, with the following characteristics:

$f(0) = f(2) = 0$
$f'(x) > 0$ if $x < 1$
$f'(1)$ does not exist
$f'(x) < 0$ if $x > 1$
$f''(x) > 0, x \neq 1$

AP Calculus AB $\boxed{\text{Review}}$ - Quiz #3.1 – 3.4 Review KEY

1. Find the critical numbers of the function $f(x) = 2x^3 - 8x^2 + 10x$

$f'(x) = 6x^2 - 16x + 10$ $\boxed{x = 5/3,\ 1}$

$= 2(3x^2 - 8x + 5)$

$0 = 2(3x - 5)(x - 1)$

$f(x)$ is continuous on $[-2, 3]$

2. Find the absolute extrema of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 3]$.

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$0 = 6(x-2)(x+1)$

$x = 2, -1$

$f(-2) = -3$

$f(2) = -19$

$f(3) = -8$

$f(-1) = 8$

$\boxed{\begin{array}{l}\text{Absolute minimum is } -19 \text{ at } x = 2 \\ \text{Absolute maximum is } 8 \text{ at } x = -1\end{array}}$

3. If $f(x) = \dfrac{-x^2 + 4}{4x}$ on $[-2, 2]$, determine if Rolle's Theorem applies. If yes, find the value(s) of c defined by Rolle's Theorem. If no, explain why not.

Check conditions: not continuous on $[-2, 2]$ since $f(x)$ is not continuous at $x = 0$. Rolle's theorem does not apply.

4. If $f(x) = -(25 - 5x)^{\frac{1}{2}}$ on [3, 5], determine if the Mean Value Theorem applies. If yes, find the value(s) of c defined by the Mean Value Theorem. If no, explain why not.

$f'(x) = -\frac{1}{2}(25 - 5x)^{-1/2}(-5)$

$= \dfrac{+5}{2\sqrt{25 - 5x}}$

$f(x)$ is continuous on $[3, 5]$ and differentiable on $(3, 5)$ so MVT applies

$25 - 5x = 0$

$5x = 25 \quad x = 5$

$f(3) = -\sqrt{10}$

$f(5) = 0$

$m_{Avg} = \dfrac{-\sqrt{10} - 0}{3 - 5} = \dfrac{-\sqrt{10}}{-2} = \dfrac{\sqrt{10}}{2}$

* set $f'(x) = m_{Avg}$ $\quad \dfrac{+5}{2\sqrt{25 - 5x}} = \dfrac{\sqrt{10}}{2}$

$+10 = 2\sqrt{10(25 - 5x)}$

$+5 = \sqrt{10(25 - 5x)} \rightarrow (-5)^2 = \left(\sqrt{10(25 - 5x)}\right)^2$

$25 = 10(25 - 5x)$

$5 = 2(25 - 5x)$

$5 = 50 - 10x$

$-45 = -10x$

$\frac{9}{2} = x$

$\boxed{c = 4.5}$

5. If $f(x) = x^3 - 4x^2 + 5x$, find the interval(s) where $f(x)$ is Increasing and Decreasing

$f'(x) = 3x^2 - 8x + 5$

$0 = (3x - 5)(x - 1)$

$x = 5/3,\ 1$

$+ \quad - \quad +$

$1 \quad 5/3$

$f(x)$ increasing $(-\infty, 1) \cup (5/3, \infty)$ b/c $f'(x) > 0$

$f(x)$ decreasing $(1, 5/3)$ b/c $f'(x) < 0$

$x^3 - 12x^2 - 27x$

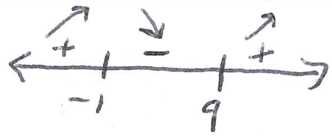
6. If $f(x) = \cancel{x^4 - 12x^2 + 16x}$, use the First Derivative Test to find all relative extrema. (x-values)

$f'(x) = 3x^2 - 24x - 27$

$0 = 3(x^2 - 8x - 9)$

$= 3(x-9)(x+1)$

$x = 9, -1$



Relative max at $x = 1$ b/c $f'(x)$ change signs from + to −
Relative min at $x = 9$ b/c $f'(x)$ change signs from − to +

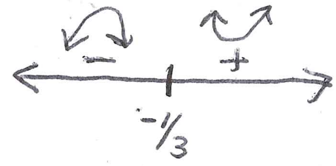
7. If $f(x) = x^3 + x^2 - 21x$, find the intervals where *f(x)* is concave up and concave down, and find all points of inflection.

$f'(x) = 3x^2 + 2x - 21$

$f''(x) = 6x + 2$

$0 = 6x + 2$

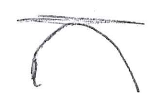
$x = -1/3$



$f(x)$ concave up $(-1/3, \infty)$ b/c $f''(x) > 0$
$f(x)$ concave down $(-\infty, -1/3)$ b/c $f''(x) < 0$
POI at $(-1/3, \underline{7.074})$ b/c $f''(x)$ change signs

Find Rel max/min

8. If $f(x) = 2x^3 - x^2 - 20x - 10$, use the Second Derivative Test to find all relative extrema.



$f'(x) = 6x^2 - 2x - 20$

$= 2(3x^2 - x - 10)$

$2(3x + 5)(x - 2)$

$x = -5/3, 2$

$f''(x) = 12x - 2$

$f''(-5/3) = 12(-5/3) - 2 = -22 < 0$, concave down
Relative max at $x = -5/3$

$f''(2) = 12(2) - 2 = 22 > 0$, concave up
Relative min at $x = 2$

9. Sketch a labeled graph of a function, f, with the following characteristics:

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$f'(1)$ does not exist
$f'(x) < 0$ if $x > 1$
$f''(x) > 0, x \neq 1$