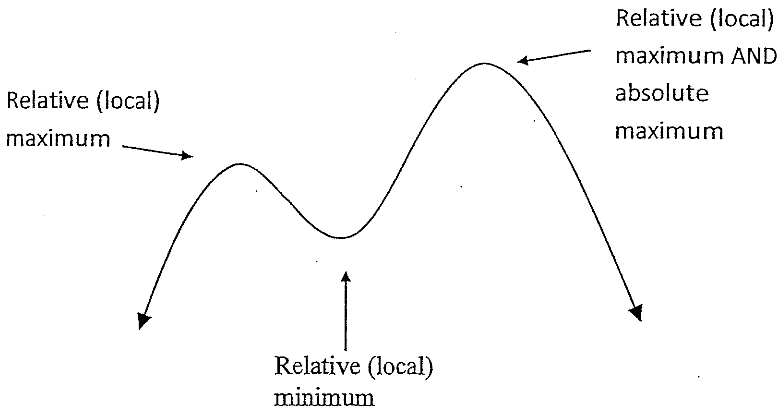
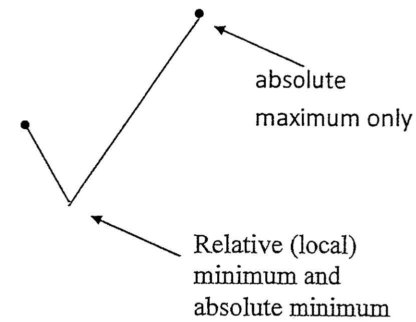
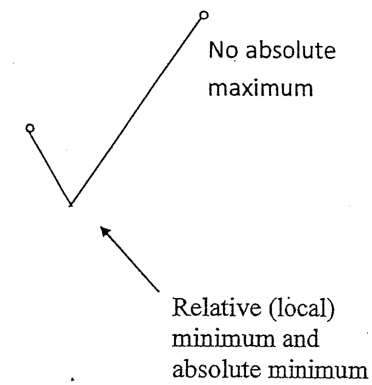
Extrema : maximums and minimums



Closed interval



Open Interval



Relative (local) extrema: any "hills and valleys" of graph

Absolute (global) extrema: highest or lowest points on the entire graph

*holes and $\pm\infty$ can **not** be considered as absolute extrema.

(EVT)

Extreme Value Theorem: If a function is continuous on a closed interval, then it has **both** an (absolute) minimum and an (absolute) maximum on that interval.

Fermat's Theorem: If a function is continuous on a closed interval, then the absolute extreme will **either** be at the **a)** critical numbers or **b)** at an endpoint.

Critical numbers (values) : x-values in the domain of a function where the derivative of a function is either 0 or undefined.

*Relative extrema ONLY occur at critical numbers, but not all critical numbers are where relative extrema occur.

*Maximum and minimum values refer to the **y-values** of the point.

Steps: * Confirm continuous function on closed interval

1. Find critical points
   a. Set $f'(x) = 0$
   b. Find where $f'(x)$ is undefined (Set denominator of $f'(x) = 0$)

2. Plug all critical points and endpoints into f(x)

3. Compare y-values to determine absolute maximum(s) and absolute minimum(s)

Find all critical numbers for each. What are the values of the absolute extrema?

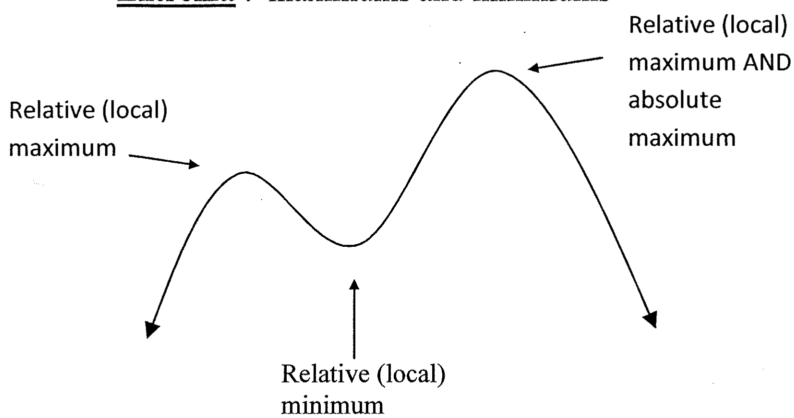
Example 1: $f(x) = 3x^4 - 4x^3$ on [0, 2]

Example 2: $f(x) = (x-1)^{\frac{2}{3}}$ on [-1, 0]

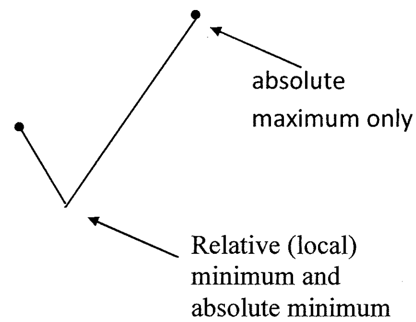
Example 3: $f(x) = \frac{4}{3}x\sqrt{3-x}$ on [0, 3]

Notes: Extrema and Critical Points

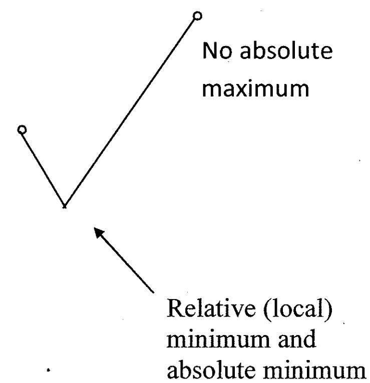
<u>Extrema</u> : maximums and minimums



<u>Closed interval</u>



<u>Open Interval</u>



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Steps:

1. Find critical points
   a. Set f'(x) = 0
   b. Find where f'(x) is undefined (Set denominator of f'(x) = 0)

2. Plug all critical points and endpoints into f(x)

3. Compare y-values to determine absolute maximum(s) and absolute minimum(s)

Find all critical numbers for each. What are the values of the absolute extrema?

Example 1: $f(x) = 3x^4 - 4x^3$ on [0, 2]

$f'(x) = 12x^3 - 12x^2$

$0 = 12x(x-1)$

$x = 0, 1$

$f(0) = 0$

$f(1) = -1$ (min)

$f(2) = 16$ (max)

Example 2: $f(x) = (x-1)^{\frac{2}{3}}$ on [-1, 0] f(x) continuous on [-1, 0]

$f'(x) = \frac{2}{3}(x-1)^{-1/3}(1)$

$f'(x) = \frac{2}{3(x-1)^{1/3}}$

$x = 1$ (critical pt.)

$f(-1) = \sqrt[3]{4}$ (Abs. max is $\sqrt[3]{4}$ at $x=-1$)

$f(0) = 1$ (Abs. min is 1 at $x=0$)

f(x) continuous on [0, 3]

Example 3: $f(x) = \frac{4}{3}x\sqrt{3-x}$ on [0, 3] $f(x) = \frac{4}{3}x(3-x)^{1/2}$

$f'(x) = \frac{4}{3}(3-x)^{1/2} + \frac{4}{3}x \cdot \frac{1}{2}(3-x)^{-1/2}(-1)$

$0 = \frac{4\sqrt{3-x}}{3} - \frac{2x}{3\sqrt{3-x}}$

$= \frac{4(3-x) - 2x}{3\sqrt{3-x}} = \frac{12-4x-2x}{3\sqrt{3-x}}$

$f'(x) = \frac{12-6x}{3\sqrt{3-x}}$

$12 - 6x = 0$ $6x = 12$ $\underline{x = 2}$

$3\sqrt{3-x} = 0$ $\underline{x = 3}$

$f(0) = 0$
$f(3) = 0$] (Abs. min)

$f(2) = \frac{4}{3}(2)\sqrt{1} = \frac{8}{3}$ (Abs. max)