Ch. 3.2 Mean Value Theorem (MVT) and Rolle's Theorem

p. 176-177 #1-9 odd, 15, 17, 29, 33-43 odd, 51, 53, 58

7) Show that $f'(x)=0$ at some point between 2 x-intercepts (Rolle's Theorem)

$f(x) = x\sqrt{x+4}$

$0 = x(x+4)^{1/2}$

x-ints: $x=0$, $x=-4$ (endpoints)

$f(0)=0$

$f(-4)=0$

$m_{Avg} = \frac{0-0}{0-(-4)} = \boxed{0}$

* $f(x)$ is continuous and differentiable $(-4,0)$ on $[-4,0]$

$f'(x) = (1)(x+4)^{1/2} + x\cdot\frac{1}{2}(x+4)^{-1/2}(1)$

$= (x+4)^{1/2} + \frac{x}{2(x+4)^{1/2}} = \frac{2(x+4)+x}{2(x+4)^{1/2}}$

$f'(x) = \frac{3x+8}{2(x+4)^{1/2}}$ * set $f'(x)=0$

$3x+8=0$

$x=-8/3$

$\boxed{c=-8/3 \text{ and is between } -4 \text{ and } 0}$

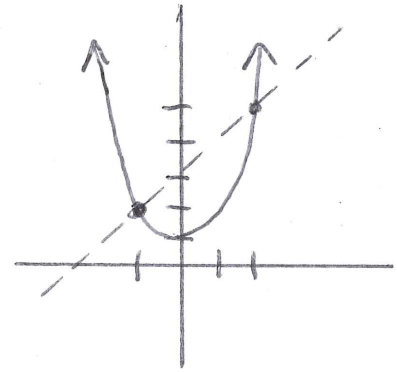
15) $f(x) = x^{2/3}-1$ $[-8,8]$

$f'(x) = \frac{2}{3}x^{-1/3}$

$f'(x) = \frac{2}{3x^{1/3}}$ $x \neq 0$, $f(x)$ not differentiable at $x=0$

Since $x=0$ lies between interval $x=-8$ and $x=8$, Rolle's Theorem fails because $f(x)$ not differentiable on $[-8,8]$

37) $f(x)=x^2+1 \quad [-1,2]$



a) Find equation of secant line thru $(-1,2)$ and $(2,5)$

$m=\frac{5-2}{2--1}=\frac{3}{3}=1 \qquad y-5=1(x-2)$

$\boxed{y=x+3}$

b) Use MVT to find c:

f(x) is continuous on $[-1,2]$ and differentiable on $(-1,2)$

*set $f'(x)=m_{Avg}$

$f'(x)=2x \quad | \quad 2x=1 \quad | \quad \boxed{c=1/2}$

$m_{avg}=1 \quad | \quad x=1/2$

c) Find equation of tangent line:

$f(1/2)=\left(\frac{1}{2}\right)^2+1=\frac{1}{4}+\frac{4}{4}=\frac{5}{4}$

point: $\left(\frac{1}{2},5/4\right)$

slope: $m=1$

$\boxed{y-5/4=1(x-1/2) \quad \text{or} \quad y=x+\frac{3}{4}}$

41) Use MVT to find c-value

$f(x)=x^{2/3} \quad [0,1]$

$f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3x^{1/3}} \quad x\neq 0$

f(x) is continuous on $[0,1]$ and differentiable on $(0,1)$

$f(0)=0 \qquad m_{Avg}=\frac{1-0}{1-0}=1$

$f(1)=1$

$\frac{2}{3x^{1/3}}=1 \qquad 2=3x^{1/3} \quad | \quad x=\left(\frac{2}{3}\right)^3=\frac{8}{27} \qquad \boxed{c=\frac{8}{27}}$

$\frac{2}{3}=x^{1/3}$

Use MVT to find c

43) $f(x) = \sqrt{2-x}$, $[-7,2]$

$f(x)$ is continuous on $[-7,2]$ and differentiable on $(-7,2)$

$f'(x) = \frac{1}{2}(2-x)^{-1/2}(-1)$

$f'(x) = -\frac{1}{2\sqrt{2-x}}$ $x \neq 2$

$f(-7) = \sqrt{9} = 3$

$f(2) = 0$

$m_{Avg} = \frac{3-0}{-7-2} = \frac{3}{-9} = \frac{-1}{3}$

set $f'(x) = m_{Avg}$

$\frac{-1}{2\sqrt{2-x}} = \frac{-1}{3}$

$2\sqrt{2-x} = 3$

$\sqrt{2-x} = \frac{3}{2}$

$2-x = \left(\frac{3}{2}\right)^2$

$2-x = 9/4$

$2 - 9/4 = x$

$-\frac{1}{4} = x$

$\boxed{c = -1/4}$

51) $s(t) = -4.9t^2 + 500$

a) Avg. velocity $[0,3]$

$s(0) = 500$

$s(3) = 455.9$

$\rightarrow \frac{455.9-500}{3-0} = -14.7 \text{ m/s}$

b) find time using MVT where $s'(t) =$ avg. velocity

$s'(t) = -9.8t$

$-9.8t = -14.7$

$\boxed{t = 1.5 \text{ sec}}$

58) $T(0) = 1500°$

$T(5) = 390°$

Avg. ROC temp change $= \frac{390-1500}{5-0} = \frac{-1110}{5} = -222°F/hr.$

By MVT, since Avg. ROC is $-222°F/hr$, there exists a time where $T'(c) = -222°F/hr.$