Mean Value Theorem (MVT): If a function, f(x), is **continuous** on [a, b] and **differentiable** on (a, b), then there must be at least one point, c in (a, b) where the slope of the tangent (derivative) is equal to the slope of the secant. $f'(c)=\dfrac{f(b)-f(a)}{b-a}$

*In other words, set the derivative equal to the slope between endpoints ($m_{avg.}$) *

MVT Steps:

1. Check Continuity (no breaks between endpoints)
   a. Does f(x) have variables in the denominator? (V.A. or holes)
   b. If so, then look to see if the x-value lies in the **closed** interval [a, b]
   c. If the x lies between the interval, then function is not continuous on the interval, MVT fails
2. Check Differentiability (smooth curve between endpoints)
   a. Does f'(x) have variables in the denominator? (sharp points, slope undefined)
   b. If yes, then look to see if the x-value lies in the **open** interval (a, b)
   c. If the x lies between the interval, then function is not differentiable on the interval, MVT fails

Note, all polynomials are continuous and differentiable everywhere

3. Find $m_{avg.}$ (This is the slope between your endpoints, slope of secant line)
4. Set f'(x) = m_{avg} and solve for x

Example 1: Determine if the mean value theorem can be applied to $f(x) = 2x^3 + x + 4$ on the interval [-2, 1]. If so, find the value of c based on the theorem.

Rolle's Theorem: If a function, f(x) is continuous on [a, b], differentiable on (a, b), and f(a) = f(b), then there must be at least one point on the function where the slope of the tangent (derivative) is 0.

*In other words, if the endpoints have the same y-values, then we can guarantee a relative maximum or relative minimum somewhere between the endpoints

*Rolle's Theorem is just a specific case of the Mean Value Theorem

Rolle's Theorem Steps:

1. Check Continuity (no breaks between endpoints)
2. Check Differentiability (smooth curve between endpoints)

Note, all polynomials are continuous and differentiable everywhere

3. Test endpoints. Does f(a) = f(b)? If not ,then Rolle's fails / does not apply
4. If yes, then set f'(x) = 0 and solve for x

Example 2: Determine if Rolle's theorem can be applied to $f(x) = x^2 - 3x + 2$ on the interval [1, 2]. If so, find the value of c such that f '(c) = 0.

Example 3: Determine if Rolle's theorem can be applied for $f(x) = 3 - |x - 3|$ on [0, 6]

Notes: Mean Value Theorem (MVT) and Rolle's Theorem

Mean Value Theorem (MVT): If a function, f(x), is **continuous** on [a, b] and **differentiable** on (a, b), then there must be at least one point, c in (a, b) where the slope of the tangent (derivative) is equal to the slope of the secant. $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

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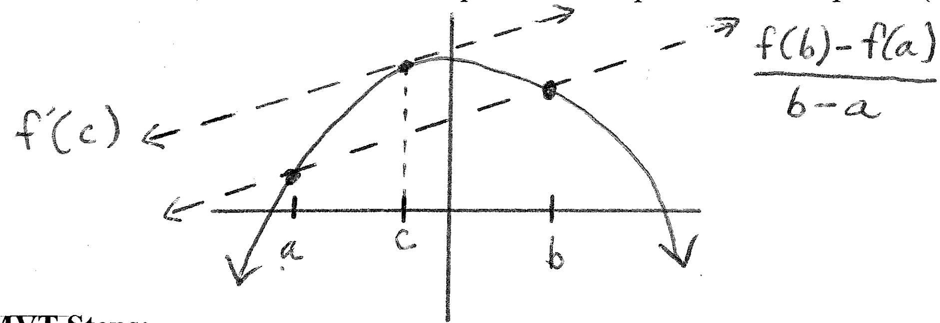

MVT Steps:

1. Check Continuity (no breaks between endpoints)
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2. Check Differentiability (smooth curve between endpoints)
   a. Does f'(x) have variables in the denominator? (sharp points, slope undefined)
   b. If yes, then look to see if the x-value lies in the **open** interval (a, b)
   c. If the x lies between the interval, then function is not differentiable on the interval, MVT fails

Note, all polynomials are continuous and differentiable everywhere

3. Find m_{avg}. (This is the slope between your endpoints, slope of secant line)
4. Set $f'(x) = m_{avg}$ and solve for x

Example 1: Determine if the mean value theorem can be applied to $f(x) = 2x^3 + x + 4$ on the interval [-2, 1]. If so, find the value of c based on the theorem.

f(x) is continuous, differentiable on [-2, 1]

$f(-2) = -14$

$f(1) = 7$

$m_{Avg} = \dfrac{-14 - 7}{-2 - 1}$

$m_{Avg} = \dfrac{-21}{-3} = 7$

$f'(x) = 6x^2 + 1$

set $f'(x) = m_{Avg}$

$6x^2 + 1 = 7$

$6x^2 = +6$

$x = \pm 1$

* we are only looking for c-value in the open interval (-2, 1)

$\cancel{c = 1}$, $\boxed{c = -1}$

Rolle's Theorem: If a function, f(x) is continuous on [a, b], differentiable on (a, b), and f(a) = f(b), then there must be at least one point on the function where the slope of the tangent (derivative) is 0.

*In other words, if the endpoints have the same y-values, then we can guarantee a relative maximum or relative minimum somewhere between the endpoints

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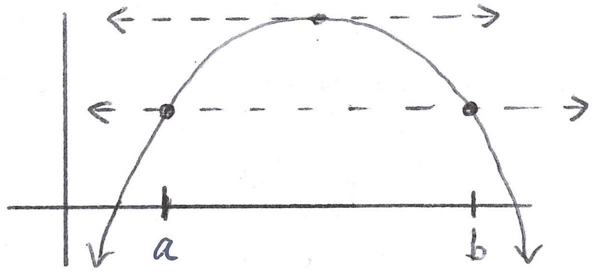

$$\frac{f(b)-f(a)}{b-a} = 0$$

Rolle's Theorem Steps:

1. Check Continuity (no breaks between endpoints)
2. Check Differentiability (smooth curve between endpoints)

Note, all polynomials are continuous and differentiable everywhere

3. Test endpoints. Does f(a) = f(b)? If not ,then Rolle's fails / does not apply
4. If yes, then set f'(x) = 0 and solve for x

Example 2: Determine if Rolle's theorem can be applied to $f(x) = x^2 - 3x + 2$ on the interval [1, 2]. If so, find the value of c such that f '(c) = 0.

f(x) is continuous and differentiable on [1, 2]

$f(1) = 0$

$f(2) = 0$

$m_{Avg} = \frac{0-0}{2-1} = 0$

set $f'(c) = m_{Avg}$

$f'(x) = 2x - 3$

$2x - 3 = 0$

$x = 3/2$

$\boxed{c = 3/2}$

Example 3: Determine if Rolle's theorem can be applied for f(x) = 3 – |x – 3| on [0, 6]

V(3, 3)

f(x) not differentiable at x=3, not differentiable on interval [0, 6]

Rolle's theorem does not apply.