Calculus Ch. 3.3 Notes: First Derivative Test

What does the derivative represent? ______________________________

When the function is **increasing**, what is common about the derivatives at those points? ______________

When the function is **decreasing**, what is common about the derivatives at those points? ______________

When $f'(x) > 0$, ______________________

When $f'(x) < 0$, ______________________

When $f'(x) = 0$, ______________________

First Derivative Test Steps (Finds inc/dec and relative max/min)

1. Find f '(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
   b. Remember, critical points also exist where function is not differentiable (sharp point)
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f '(x) to determine slope
      i. Positive (+) means increasing slope
      ii. Negative(–) means decreasing slope
4. Write Because Statements
   a. f(x) increasing in interval (a,b) b/c $f'(x) > 0$
   b. f(x) decreasing in interval (a,b) b/c $f'(x) < 0$
   c. Relative max at (**a, f(a)**) b/c f '(x) changes from + to –
   d. Relative min at (**a, f(a)**) b/c f '(x) changes from – to +

Example 1: Determine the intervals at which the function $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 - 6x - 3$ is increasing and decreasing.. Locate the relative extrema.

First Derivative Test Steps (Finds inc/dec and relative max/min)

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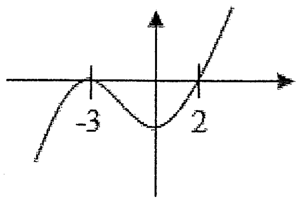
Example 2: Determine the intervals at which the function $f(x) = \dfrac{5x+2}{x-3}$ is increasing and decreasing. Locate the relative extrema.

Example 3: Make a first derivative sign line for the following graph of f '(x):



Calculus Ch. 3.3 **Notes: First Derivative Test**

What does the derivative represent? slope of tangent line to function (curve)

When the function is **increasing**, what is common about the derivatives at those points? $f'(x) > 0$

When the function is **decreasing**, what is common about the derivatives at those points? $f'(x) < 0$

When $f'(x) > 0$, f(x) is increasing

When $f'(x) < 0$, f(x) is decreasing.

When $f'(x) = 0$, f(x) is constant (horizontal or changing direction)

First Derivative Test Steps (Finds inc/dec and relative max/min)

1. Find f '(x), set equal to zero
   a. Find critical points from BOTH numerator and denominator
   b. Remember, critical points also exist where function is not differentiable (sharp point)
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into f '(x) to determine slope
      i. Positive (+) means increasing slope
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4. * Write Because Statements *
   a. f(x) increasing in interval (a,b) b/c $f'(x) > 0$
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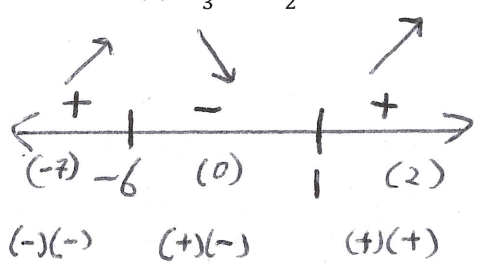
   c. Relative max at (**a, f(a)**) b/c f '(x) changes from + to –
   d. Relative min at (**a, f(a)**) b/c f '(x) changes from – to +

Example 1: Determine the intervals at which the function $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 - 6x - 3$ is increasing and decreasing. Locate the relative extrema.

$f'(x) = x^2 + 5x - 6$

$0 = (x+6)(x-1)$

$x = 1, \; x = -6$



f(x) is increasing on $(-\infty, -6) \cup (1, \infty)$ b/c $f'(x) > 0$

f(x) is decreasing on $(-6, 1)$ b/c $f'(x) < 0$

Relative max at $(-6, 51)$ b/c $f'(x)$ changes from + to −

Relative min at $(1, -6.167)$ b/c $f'(x)$ changes from − to +

First Derivative Test Steps (Finds inc/dec and relative max/min)

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   a. Find critical points from BOTH numerator and denominator
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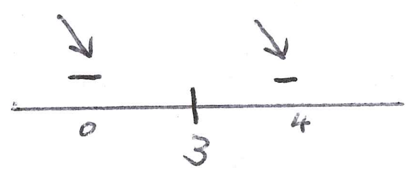
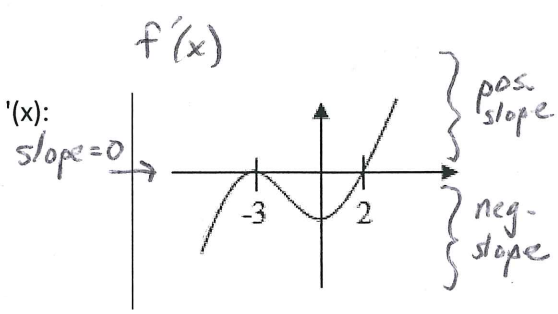
Example 2: Determine the intervals at which the function $f(x)=\dfrac{5x+2}{x-3}$ is increasing and decreasing. Locate the relative extrema.

$$f'(x)=\frac{5(x-3)-(5x+2)(1)}{(x-3)^2}=\frac{5x-15-5x-2}{(x-3)^2}=\frac{-17}{(x-3)^2}$$

critical pt: $x=3$

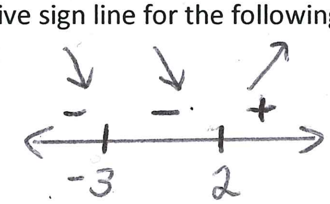


f(x) is decreasing on $(-\infty,3)\cup(3,\infty)$ b/c $f'(x)<0$
No relative extrema for f(x)



Example 3: Make a first derivative sign line for the following graph of f '(x):

1) Make sign line:



2) sketch graph using sign line