Calculus Ch. 3.4 **Notes: "Concavity Test" and the 2nd Derivative Test**

Are both of these functions increasing? ________ What do we know about their derivatives? ________

1) If f "(x) is > 0, then f '(x) is increasing and f(x) is concave up.
2) If f "(x) is < 0, then f '(x) is decreasing and f(x) is concave down.
3) A Point of Inflection (POI) occurs whenever f "(x) changes sign. (f(x) changes concavity)

"Concavity Test" Steps (Finding interval Concave Up/Down and POI)

1. Find $f''(x)$, set equal to zero
   a. Find critical points from BOTH numerator and denominator
2. Put all critical points on sign line
3. Test intervals
   a. Plug values into $f''(x)$ to determine concavity
      i. Positive (+) means concave up
      ii. Negative(–) means concave down
4. Write Because Statements
   a. Concave up in interval (a,b) b/c f''(x) > 0
   b. Concave down in interval (a,b) b/c f''(x) < 0
5. Point of Inflection at (**a, f(a)**) b/c $f''(x)$ changes signs

*Note: POI may exist on graph where $f''(x)$ does not exist (sharp point). POI exists as long as graph is continuous and $f''(x)$ changes concavity (change of signs)

Example 1: Find the points of inflection if $f(x) = -2x^5 + \frac{5}{3}x^3$

The 2nd Derivative Test

The 2nd derivative test is a test for relative extrema (max/min) and NOT for Point of Inflection

*The 2nd derivative test achieves the same as the 1st derivative test.

1) If you plug a critical number from f'(x) into $f''(x)$ and if $f''(x) > 0$, then that is the x-value of the relative **minimum**

2) If you plug a critical number from f'(x) into $f''(x)$ and if $f''(x) < 0$, then that is the x-value of the relative **maximum**

3) If you plug a critical number from f'(x) into $f''(x)$ and if $f''(x) = 0$, then the test is inconclusive. We need the first derivative test to determine if critical number is a relative extrema.

2nd Derivative Test Steps (Test for Relative Extrema, **NOT** Point of Inflection)

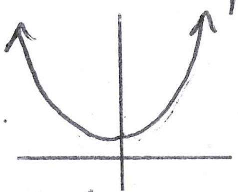
1. Find f'(x) , set equal to zero
   a. Find critical points. Set numerator and denominator of f '(x) = 0. (These are candidates for relative max/min)
2. Find $f''(x)$
3. Plug the critical points (from step #1) into $f''(x)$.
   a. If result is positive value, then f''(x) > 0, concave up, and therefore relative minimum exists at x-value
   b. If result is negative value, then f''(x) < 0, concave down, and therefore relative maximum exists at x-value
   c. f result is zero, then since f''(x) = 0, then this test is inconclusive. We cannot determine whether relative extrema exists. (Use First Derivative Test)

Example 2: Find the relative extrema of $f(x) = x^3 - 4x^2 - 3x$

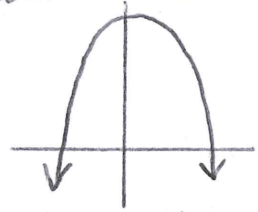
Calculus Ch. 3.4 **Notes: "Concavity Test" and the 2nd Derivative Test**

Are both of these functions increasing? yes What do we know about their derivatives? rate of slope is changing

concave up



concave down



slope is becoming more positive

slope is becoming more negative

1) If f "(x) is > 0, then f '(x) is increasing and f(x) is concave up.
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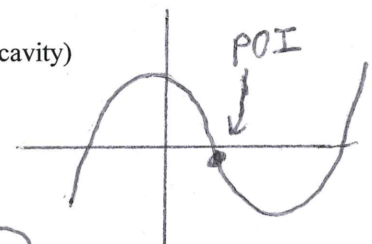

"Concavity Test" Steps (Finding interval Concave Up/Down and POI)

1. Find $f''(x)$, set equal to zero
   a. Find critical points from BOTH numerator and denominator
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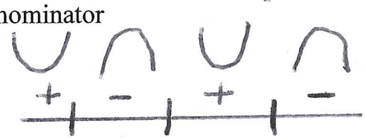
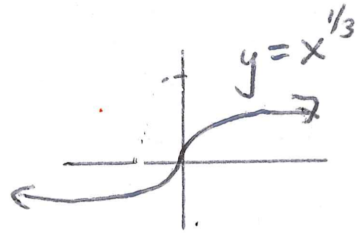




*Note: POI may exist on graph where $f''(x)$ does not exist (sharp point). POI exists as long as graph is continuous and $f''(x)$ changes concavity (change of signs) or vertical tangent line

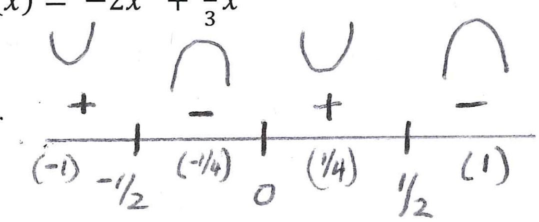
Example 1: Find the points of inflection if $f(x) = -2x^5 + \frac{5}{3}x^3$ * and find intervals concave up/down

$f'(x) = -10x^4 + 5x^2$

$f''(x) = -40x^3 + 10x$

$0 = -10x(4x^2 - 1)$

$x = 0, \; x = \pm\frac{1}{2}$



$f''(x)$ concave up on $(-\infty, -\frac{1}{2}) \cup (0, \frac{1}{2})$ b/c $f''(x) > 0$

$f''(x)$ concave down on $(-\frac{1}{2}, 0) \cup (\frac{1}{2}, \infty)$ b/c $f''(x) < 0$

POI at $(-\frac{1}{2}, -0.146), (0,0), (\frac{1}{2}, 0.146)$ b/c $f''(x)$ changes signs.

The 2^{nd} Derivative Test

The 2^{nd} derivative test is a test for relative extrema (max/min) and NOT for Point of Inflection

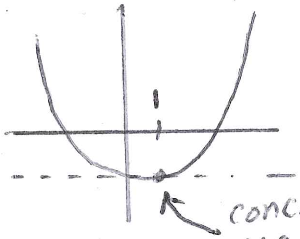
*The 2^{nd} derivative test achieves the same as the 1^{st} derivative test.

1) If you plug a critical number from f'(x) into $f''(x)$ and if $f''(x) > 0$, then that is the x-value of the relative **minimum**

2) If you plug a critical number from f'(x) into $f''(x)$ and if $f''(x) < 0$, then that is the x-value of the relative **maximum**

3) If you plug a critical number from f'(x) into $f''(x)$ and if $f''(x) = 0$, then the test is inconclusive. We need the first derivative test to determine if critical number is a relative extrema.

*critical numbers from f'(x) indicate candidates for relative max/min.

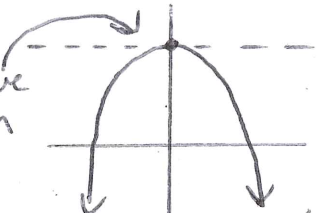


$f'(1)=0$

$f''(1)>0$

concave up

so the graph has relative min at $x=1$



concave down

$f'(0)=0$

$f''(0)<0$

* graph has relative max at $x=0$

2^{nd} Derivative Test Steps (Test for Relative Extrema, **NOT** Point of Inflection)

1. Find f'(x) , set equal to zero
   a. Find critical points. Set numerator and denominator of f '(x) = 0. (These are candidates for relative max/min)
2. Find $f''(x)$
3. Plug the critical points (from step #1) into $f''(x)$.
   a. If result is positive value, then f''(x) > 0, concave up, and therefore relative minimum exists at x-value
   b. If result is negative value, then f''(x) < 0, concave down, and therefore relative maximum exists at x-value
   c. f result is zero, then since f''(x) = 0, then this test is inconclusive. We cannot determine whether relative extrema exists. (Use First Derivative Test)

Example 2: Find the relative extrema of $f(x) = x^3 - 4x^2 - 3x$ *using 2^{nd} derivative test.

$f'(x) = 3x^2 - 8x - 3$

$0 = (3x+1)(x-3)$

$x = -1/3, 3$

$f''(x) = 6x - 8$

*Test critical points using $f''(x)$.

$f''(x) = 6x - 8$

$f''(-\frac{1}{3}) = 6(-\frac{1}{3}) - 8 < 0$

Relative max at $x = -\frac{1}{3}$
b/c $f'(-1/3) = 0$ and $f''(-\frac{1}{3}) < 0$

$f''(3) = 6(3) - 8 = 10 > 0$

Relative min at $x = 3$
b/c $f'(3) = 0$ and $f''(3) > 0$