# 3.4 Concavity, POI, 2nd Derivative Test

p.195–197 #11, 13, 15, 19, 29, 31, 35, 37, 49, 51, 53, 55, 68

15) $f(x) = x(x-4)^3$

$f'(x) = (1)(x-4)^3 + x \cdot 3(x-4)^2(1)$

$= [x-4]^2[x-4+3x] = (x-4)^2(4x-4)$

$f''(x) = 2(x-4)(4x-4) + (x-4)^2(4)$

$= 2(x-4)[4x-4+2(x-4)]$

$= 2(x-4)[6x-12] = 12(x-4)(x-2)$

$2(x-4)6(x-2) \longrightarrow$ $x=4,\ x=2$

$\cup \quad \cap \quad \cup$

$+ \quad - \quad +$

$2 \quad 4$

$f(x)$ concave up $(-\infty, 2) \cup (4, \infty)$ b/c $f''(x) > 0$

$f(x)$ concave down $(2, 4)$ b/c $f''(x) < 0$

POI: $(2, -16)$ and $(4, 0)$ b/c $f''(x)$ change signs

19) $f(x) = \dfrac{x}{x^2+1}$

$f'(x) = \dfrac{(1)(x^2+1) - (x)(2x)}{(x^2+1)^2} = \dfrac{x^2+1-2x^2}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$

$x^2+1+2-2x^2$

$f''(x) = \dfrac{(-2x)(x^2+1)^2 - (1-x^2)2(x^2+1)(2x)}{(x^2+1)^4} = \dfrac{-2x(x^2+1)[x^2+1+2(1-x^2)]}{(x^2+1)^4}$

$= \dfrac{-2x(x^2+1)(3-x^2)}{(x^2+1)^4} = \dfrac{-2x(3-x^2)}{(x^2+1)^3}$ $\quad x = 0, \pm\sqrt{3}$

$\cap \quad \cup \quad \cap \quad \cup$

$-\sqrt{3} \quad 0 \quad \sqrt{3}$

$f(x)$ concave down $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$ b/c $f''(x) < 0$

$f(x)$ concave up $(-\sqrt{3}, 0) \cup (\sqrt{3}, 0)$ b/c $f''(x) > 0$

POI at $(-\sqrt{3}, -\sqrt{3}/4)$, $(0,0)$, $(\sqrt{3}, \sqrt{3}/4)$

b/c $f''(x)$ change signs

53) $f(2)=f(4)=0$

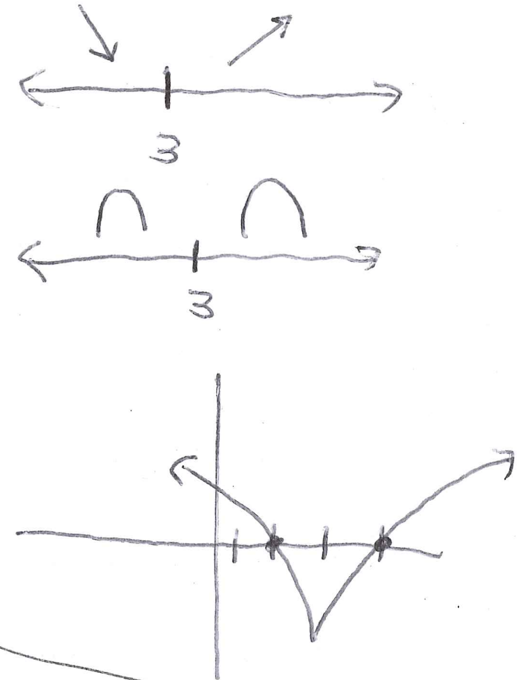
$f(3)$ is defined

$f'(x)<0$ if $x<3$

$f'(3)$ DNE

$f'(x)>0$ if $x>3$

$f''(x)<0,\ x\neq 3$



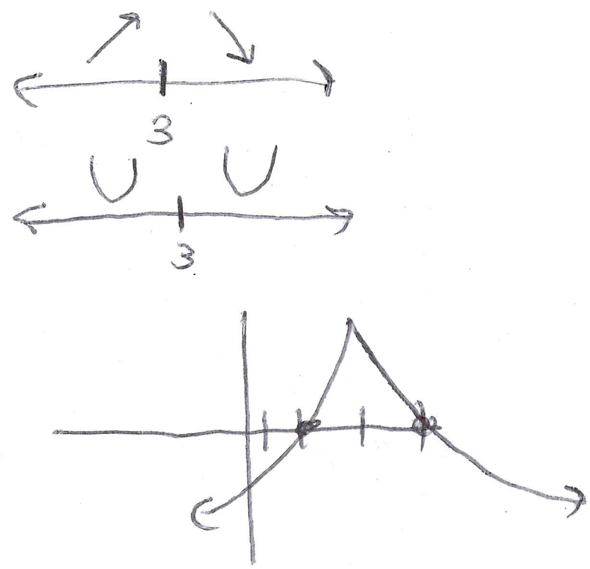

55) $f(2)=f(4)=0$

$f'(x)>0$ if $x<3$

$f'(3)$ DNE

$f'(x)<0$ if $x>3$

$f''(x)>0,\ x\neq 3$




Ex. $f(2)=1$

$f(-3)=-4$

$f(4)=6$

$f'(x)<0$ if $x<-3,\ x>4$

$f'(x)>0$ if $-3<x<4$

$f''(x)>0$ if $x<2$

$f''(x)<0$ if $x>2$

$f''(x)$ continuous everywhere