Ch. 3.5 Notes Limits at Infinity (End behavior)

A. Checking for horizontal asymptotes: (H.A.)

<u>Method 1:</u>

If $f(x) = \frac{p(x)}{q(x)}$, compare degrees between numerator and denominator

- If denominator degree > numerator degree, H.A. is $y = 0$
- If denominator degree = numerator degree, H.A. is $y = \frac{a}{b}$ (leading coefficients)
- If numerator degree > denominator degree, then <u>no</u> Horizontal Asymptote: limit at infinity is either $+\infty$ or $-\infty$.

<u>Method 2:</u>

Divide every term by variable with highest degree

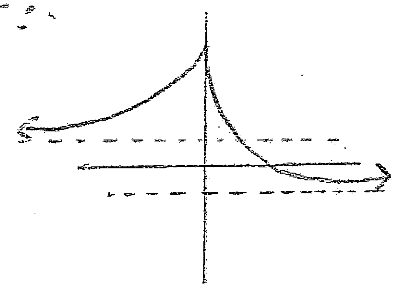
Ex.1 $f(x) = \dfrac{3x^2+2}{4x^3-5x}$

Ex.2 $f(x) = \dfrac{2x^3+x^2}{3x^2-2x}$

Ex.3 $f(x) = \dfrac{4x^3-2x^2}{5x^3-3x+9}$

* Horizontal Asymptote is a description of end behavior, <u>not</u> a boundary that the graph cannot cross. A function can never cross a vertical asymptote, but it might cross horizontal asymptote.

B. Finding H.A. with radicals in denominator:



Ex. 4) Find H.A. for $y = \dfrac{-2x+6}{\sqrt{5x^2+1}}$

* Test $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$

* Divide each term by variable with degree of highest order

$$\lim_{x\to\infty} \frac{-2x+6}{\sqrt{5x^2+1}} =$$

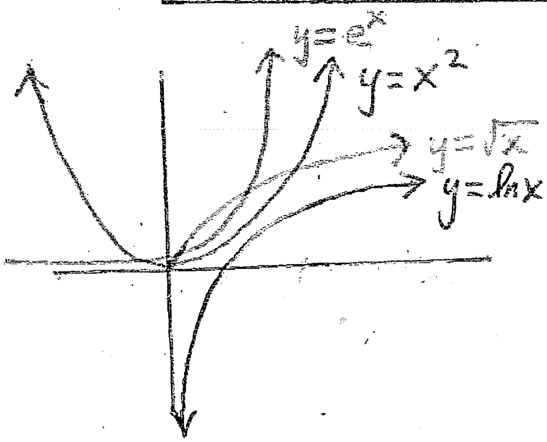
$$\lim_{x\to-\infty} \frac{-2x+6}{\sqrt{5x^2+1}} =$$

*Need to add a negative sign when evaluating limit for $x \to -\infty$

C. Comparative Growth Rates

* Different families of functions grow at different rates as x approaches $+\infty$.

$$\boxed{\text{Logs} < \text{Radicals} < \text{Polynomial} < \text{Exponential}}$$



Ex. 5 $\lim\limits_{x\to\infty} \dfrac{\sqrt{50000x+10000}}{x^2} =$

Ex. 6 $\lim\limits_{x\to\infty} \dfrac{-e^{2x}}{10000x^4+x^5} =$